NOTES ON BUILDING CONSTRUCTION

PART IV

CALCULATIONS FOR BUILDING STRUCTURES—COURSE FOR HONOURS

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NOTES

ON

BUILDING CONSTRUCTION

ARRANGED TO MEET THE REQUIREMENTS OF THE SYLLABUS OF THE SCIENCE & ART DEPARTMENT OF THE COMMITTEE OF COUNCIL ON EDUCATION, SOUTH KENSINGTON

PART IV.

CALCULATIONS FOR BUILDING STRUCTURES— COURSE FOR HONOURS

FOURTH EDITION, REVISED

LONGMANS, GREEN, AND CO

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PREFACE TO PART IV.

THE writer desires to express to the Publishers, and to those who have done him the honour of reading the previous Parts of this work, his regret for the great delay that has occurred in the preparation of this volume for the Press.

More than one serious illness, and the pressure of his duties, have obliged him again and again to lay this work entirely aside. Finding that he was not likely to have leisure or strength to complete the volume, he has, with the consent of the Publishers, entrusted most of it to a friend, by which arrangement he feels that he has conferred a great benefit upon his readers.

The object in view has been to explain fully and clearly the simplest and best methods for arriving at the proper forms and dimensions to be given to Timber and Iron Beams, Cantilevers, Plate, Box, and Trussed Girders, Roofs, Walls, Arches, Water pipes, etc. etc., showing by examples the calculations required even for the details.

In carrying out this object the use of Advanced Mathematics has been as far as possible avoided. The methods explained are chiefly those that can be effected almost entirely by drawing and measuring lines, and the calculations given require but a very slight knowledge of Mathematics. Short rules are added for the use of prac-

tical men, and also tables by which many calculations are altogether avoided.

The great advantage of using simple graphic methods instead of elaborate calculations is that the former, if incorrectly carried out, at once proclaim the fact by the polygons of forces refusing to "close," so that an error has to be faced at once and rectified; whereas an error in a figure, or even of a decimal point, in calculations quite correct in other respects, may escape notice, and lead to an utterly erroneous result, which, if acted upon, may endanger the stability of the structure to which it is applied.

It is hoped that this volume will be found to explain not only the calculations that may be called for in the Honours Examination at South Kensington, but also all that can possibly be required in connection with ordinary buildings. Upon the more complicated engineering structures it does not profess to enter.

THIRD EDITION, 1895.

THIS edition has been revised to the extent of correcting any misprints and clerical errors that have been found in it.

Some further attention has been given to the use of steel rolled joists in Chap. IV., and the table of framing of iron roofs has been re-written to include some more recent examples.

NOTE

The following is an extract from the Syllabus of the Science and Art Department of the Committee of Council on Education, South Kensington.

It shows the heads of the examination in connection with the calculation of structures for Honours, and opposite to each subject, the portion of this volume in which the information required is to be found.

FIRST STAGE, OR ELEMENTARY COURSE

No examination as to the calculation of structures.

SECOND STAGE, OR ADVANCED COURSE

All that is required will be found in Part II.

EXAMINATION FOR HONOURS

REQUIREMENTS OF SYLLABUS.	WHERE DEALT WITH IN THIS VOLUME.
He must be able to solve simple problems in the theory of construction, and to determine the safe dimensions of iron or wooden beams subjected to dead loads.	Simple problems in the theory of construction, equilibrium, chap. ii.; beams, chap. iii.; dimensions of timber beams, pp. 51, 75–80; wrought iron beams, chap. iv.; cast iron beams, chap. v.
In ordinary roof trusses and framed structures of a similar description, he must be able to trace the stresses, brought into action by the loads, from the points of application to the points of support, as well as to determine the nature and amount of the stresses on the different members of the truss, and, consequently, the quantity of material required in each part.	Stresses on frames in general, chap. ix.; open webbed girders, chap. x.; trussed beams, chap. xi.; roofs, chap. xii.; determination of the quantity of material required in each part, i.e. of the dimensions of the parts, tension and compression bars, chap. vi.; joints and connections, chap. vii.

LIST OF BOOKS CONSULTED.

Adams' Designing Wrought and Cast Iron Structures.

Aide Mémoire for the use of Officers of the Royal Engineers.

Bow's Economics of Construction.

Clark's Manual of Rules, Tables, and Data.

Cunningham's Applied Mechanics.

Fidler's Practical Treatise on Bridge Construction

Hurst's Architectural Surveyor's Hand Book

Molesworth's Pocket-book of Engineering Formulæ

Proceedings of the Institute of Civil Engineers

Rankine's Civil Engineering

Rankine's Useful Rules and Tables

Ritter's Elementary Theory and Calculation of Iron Bridges and Roofs

Stoney on Strains

Transactions of the Society of Engineers.

Unwin's Elements of Machine Design—Part I

Unwin's Wrought-Iron Bridges and Roofs

Wray's Application of Theory to the Practice of Construction (revised by Seddon)

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EXAMPLES

PLATES

CHAPTER I

INTRODUCTORY.

THE object of this Part of the Course is to teach a student how to design those parts of buildings which require to be carefully proportioned, in order that the cost of material and labour may be a *minimum*, consistently with the structure being of sufficient strength and permanence; but it does not deal in any way with the question of artistic design or appearance. It is assumed that the student is familiar with Parts I, II, and III of this Course; in fact, that he understands the usual forms of parts of ordinary buildings, and the nature of the various materials in use for the construction of buildings.

In designing any structure care should be taken that *each part* is strong enough to resist the forces that *may* act upon it. To insure this result it is necessary to know—

1. *The nature and direction of the stresses*[1] that each part may be called upon to resist.

In ordinary buildings these stresses are produced by (*a*) The force of gravity (*i.e.* the weight of the parts of the structure and of any load they may have to carry). (*b*) The force of the wind.

2. *The most suitable description of material.*

3. *The best form and dimensions to be given to each part* in order that it may be able to resist the stress that may act upon it.

When the student has ascertained these particulars, and designed the structure accordingly, he may be sure not only that each part will be strong enough, but that no more material will be used than is necessary to give the strength required; in fact that the materials will be used without waste.

It should, however, be remembered that in practice cases frequently occur, especially where the parts are small, in which the *minimum dimensions that can practically be allowed are greater than the maximum required by theory.*

[1] See p. 6 for definition of this term.

It is of course easy in many cases to make a structure strong enough by using plenty of material; but an engineer or architect who understands his work will use only sufficient material to make sure of a safe and permanent structure, taking care to dispose it so as to obtain a maximum of strength at a minimum cost.

The calculations required for designing the different parts of buildings are not difficult, and they require very little knowledge of mathematics. In many books on the subject formulæ are given without any explanation as to how they are obtained, and are therefore rather bewildering, and tend to wrap the subject in a veil of mystery which does not properly belong to it.

An attempt will be made in this chapter to explain the principles upon which the calculations depend, and then, in subsequent chapters, to show how these principles are applied to ordinary cases which occur in practice. If the student understands the principles, he will always be able to apply them to any unusual case which may arise in his own experience.

TERMS IN USE.

In order to clear the ground, it is desirable first to explain the meanings of the various terms that have to be used in considering the subject. Many of these definitions have been given in Part III., but they are now repeated in a somewhat different form.

Load.—The external forces that act upon any structure, together with the weight of the structure itself, are called the "load." The term "external forces" includes also the reactions at the points of support. See p. 15.

Thus in the case of a beam A B (Fig. 1) supported at the ends, and

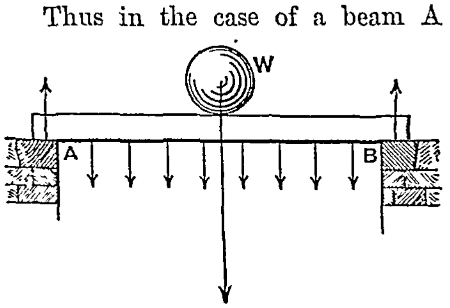

Fig. 1.

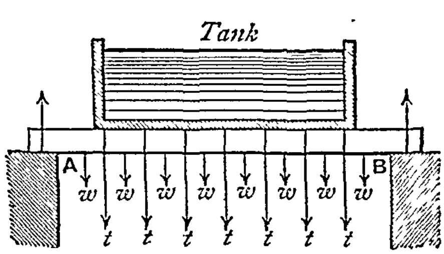

Fig. 2.

carrying a load W at its middle point, the external forces acting are (1) the weight of the beam itself, (2) the weight of the load, (3) the upward reactions at A and B. The direction of these forces is shown by the arrows. In Fig. 2 the beam is uniformly loaded throughout part of its length, the

forces acting being (1) its own weight, *w w w*, etc. (2) the weight of the tank and its contents, *t t t*, etc. (3) the upward reactions at A and B.

On an inclined rafter (Fig. 3) there may be the weight of the rafter and roof-covering acting vertically, and the force of the wind acting normally, *i.e.* at right angles to the slope (air, being a fluid, exerts a normal pressure)

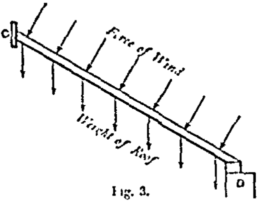

Fig. 3.

Fig. 4 shows two men carrying a ladder with a boy sitting upon it, the load they have to bear is (1) the weight of the ladder, (2) the weight of the boy.

Again, in the case of a column carrying a girder (Fig 5) which supports a building, the weight (10 tons) on the head of the column is the load to which it is subjected. The load on the foundation of the column will be 10 tons + the weight of the column.

Or in the case of a wire from which a lamp is suspended (Fig. 6), the weight of the lamp, say 50 lbs., is the load upon the wire.

The load in each case is the total of the external forces, *i.e.*

Fig. 4.

of the weight of beam + the weight of tank and its contents; the weight of rafter + force of wind, etc.

The weight of the structure itself is in some cases small compared with the load upon it and can in practice be ignored, as for instance that of the beam supporting a tank of water, or the wire supporting the lamp. In others the weight of the structure itself is important, as in the case of the ladder, which forms a considerable part of the total load.

Distribution of Load.—The load may be *concentrated at the centre* of the beam (Fig. 1), or *concentrated at any point* (Fig. 18),

or *uniformly distributed over the whole* of the beam (Fig. 16), or *over a portion* (Figs. 2 and 20).

DEAD LOAD is that which is very gradually applied, and which remains steady.

Thus the water might be poured into the tank very gradually, and when once poured in would remain quiet, forming a dead load.

In the same way the weight quietly placed and remaining steadily on the column, and the lamp on the rod, would be dead loads.

Of course the weight of a structure itself is a dead load.

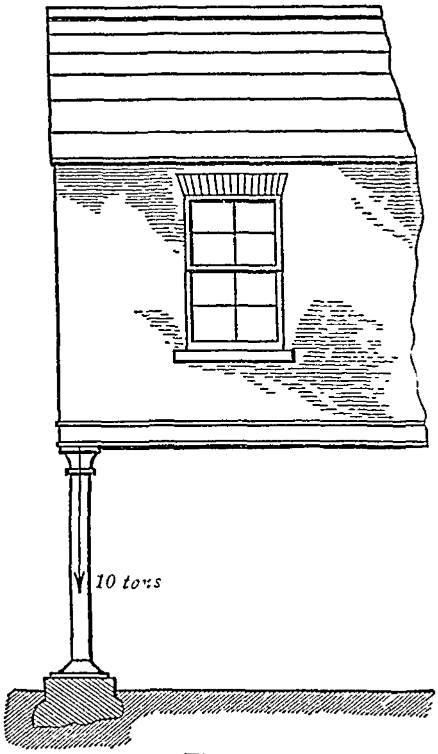

Fig. 5.

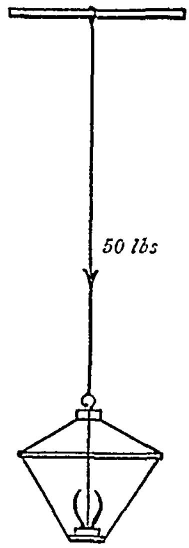

Fig. 6.

LIVE LOADS are those which are suddenly applied or are accompanied by shocks or vibration.

Thus the boy might jump suddenly upon the ladder (Fig. 4), causing a shock. An excited crowd upon a balcony or floor might make sudden and fitful movements causing jars and vibration—in either case the load would be called a "live load," not a dead or steady load.

There are but few cases in which the parts of ordinary buildings are subject to live loads—the chief instance being in floors subject, as before mentioned, to suddenly moving loads. In engineering structures, however, railway bridges are subject to the live loads caused by swiftly passing trains, breakwaters and sea walls are liable to the sudden impact of the waves; and so in parts of

machines, and in other cases, the loads acting upon a structure are often suddenly applied

Comparative Effect of Dead and Live Loads.—It has been found by experiment that a live load produces nearly twice the effect that a dead load of the same weight would produce.

Therefore to find the dead load which would produce the same effect as a given live load, the latter must be multiplied by 2.

This operation is called converting the live load into an equivalent dead load.

Thus a floor girder may weigh 30 lbs. per foot run; if the load upon it of 250 lbs. per foot run be a live load, the total equivalent dead load will be (30 lbs. + 2 × 250 lbs.) = 530 lbs. per foot run.

The BREAKING LOAD for any structure or piece of material is that dead load which will just produce fracture in the structure or material.

The WORKING LOAD or SAFE LOAD is the greatest dead load which the structure or material can safely be permitted to bear in practice.

It may be useful to know the load that would cause rupture in the structure, i.e. the breaking load, but the load that should actually be applied to it, i.e. the working load, must be so much smaller as to put all danger of rupture out of the question.

The breaking load or the working load may be either live load or dead load, or a combination of both; but for convenience it is usual to reduce it all to an equivalent dead load, by doubling the live load and adding it to the dead load, as in the example given above.

Strain is the alteration in the shape of a body produced by a stress.

Thus a stress of tension, or tensile stress, produces a stretching or strain of elongation, a compressive stress leads to a shortening or squeezing strain, a transverse stress to a bending strain, and so on.

At one time the word "strain" was generally used instead of the word "stress" to denote the forces of tension, compression, etc., and it is still so used in many treatises on applied mechanics; but under the high authority and guidance of the late Professor Rankine, the best writers on the subject use the word "stress" *to signify the forces acting upon a body*,[1] and the word "strain" to mean the alteration of figure that takes place under the action of those stresses.

---

[1] "The word 'stress' has been adopted as a general term to comprehend various forces which are exerted between contiguous bodies or parts of bodies, and which are distributed over the surface of contact of the masses between which they act."—*Civil Engineering*, by Professor Rankine, p. 161.

It should be noticed that this use of the word "strain" is different from the more usual one, in which it is implied that some injury has been done to the material of the body which has been strained.

Stress.—When a force acts upon a structure or piece of material it produces an alteration of form or *strain* which may be merely temporary, lasting only while the force is applied, or it may be permanent, or may eventually end in rupture. This alteration in form calls out a resistance in the internal structure of the material which is called the stress upon the body.[1]

Thus the weight of the lamp (Fig. 6) tends to alter the form of the rod which supports it by making it elongate, tending to tear it in two ; but the strength of the fibres composing the rod resists this tendency to alter its form ; and it is subject to a stress in the direction of its length.[2]

Again, the weight on the girder in Fig. 5 tends to shorten the column, and would, if sufficiently great, crush it. The weight (Fig. 1) tends to bend the beam, and would, if sufficiently great, break it (Fig. 31). The tendency, however, in each case is resisted by the internal strength of the body.

The various kinds of stress are as follows :—

TENSION.—This stress elongates the body upon which it acts, and tends to cause rupture by tearing asunder.

There are many parts of ordinary structures subject to a tensile stress ;

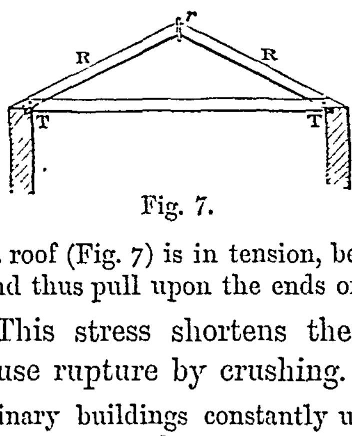

Fig. 7.

e.g. the tie beam TT of a roof (Fig. 7) is in tension, because the rafters tend to spread out at the feet and thus pull upon the ends of the tie beam.

COMPRESSION.—This stress shortens the body upon which it acts and tends to cause rupture by crushing.

Many parts of ordinary buildings constantly undergo compression, for example, columns and story posts, also the struts of roofs, which are compressed in the direction of their length by the weight of the roof that they support.

[1] Theoretically the smallest force acting upon a body produces a permanent alteration in its form (see p. 10). Practically, however, the permanent change of form produced by forces which are very small in proportion to the strength of the structure may be ignored.

[2] Strictly, the stress in the rod increases towards the supporting beam, owing to the weight of the rod itself, and in the same way the stress in the column is greater near the ground than at the top, on account of the weight of the column itself, but the increase is so small that it may practically be neglected.

TRANSVERSE STRESS.—This stress bends the body on which it acts and tends to break it across.

Instances of this stress in connection with buildings will at once occur to the student, *e.g.* in *lintels or bressumers carrying walls, joists and girders of floors*, etc., the rafters of a roof under the influence of wind and load.

SHEARING STRESS is that produced when one part of a body is forcibly pressed or pulled so as to tend to make it slide over another part.

As, for example, when two plates riveted together (Fig. 8), are severed

Fig. 8

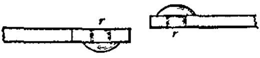

Fig. 9

by pulling or pushing in opposite directions, the rivet *r* is *sheared*—one plate *sliding upon the other* (*Fig. 9*).

BEARING STRESS is that which occurs when one body presses against another, so as to tend to produce indentation or cutting.[1]

As, for example, when a rivet holding a plate cuts into the plate, making the hole larger: thus in Fig. 10 the plates A, B, being pulled in opposite

Fig. 10

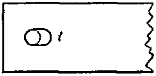

Fig. 11

directions, the rivet *c*, being of harder iron than the plate B, has borne upon it and cut into it, making the hole larger as shown at *d*, Fig. 11.

TORSION is the stress produced by twisting the parts of a body in opposite directions, or, what comes to the same thing, by fixing one part and twisting another around the axis of the fixed part.[2]

This stress is common in machinery, but not in ordinary building structures, and it will therefore not be further considered.

Stresses classified as regards Description.—Summing up therefore, it appears that there are six different kinds of stress, namely—

Tensile	Shearing	Transverse
Compressive	Bearing	Torsional

[1] Bearing stress resolves itself into stresses of compression and shear, *i.e.* in Fig. 11, compression of the metal in front of the bolt and shearing of the metal at the sides.

[2] It can be shown that this stress resolves itself into tension, compression and shear.

The three latter can, however, be resolved into combinations of the three former.

The various stresses are also classified as follows:—

Stresses classified as regards Intensity.—Breaking Stress is that stress at which a material will just give way.

Limiting Stress is the term applied to a stress which is purposely limited to less than the breaking stress of the material.

Thus it may be known that a bar of iron will bear a stress of 20 tons per square inch before breaking, but yet it may be determined, in order to be quite sure that the bar will be safe, not to load it with more than 5 tons per square inch. In this case, 5 tons would be the limiting stress; any less stress might be applied, but not a greater stress than the limiting stress, *i.e.* 5 tons per square inch.

The limiting stress should in all cases be less than the elastic limit of the material (see p. 10).

Safe Working Stress is the stress that may in practice be safely allowed upon the parts of a structure.

The amount of this stress depends upon the material, the nature of the stress, or the nature of load, whether live or dead, to which the structure is subjected. It is ascertained by dividing the breaking stress by a factor of safety, which varies under the several conditions mentioned above, and is determined by experience (see p. 9 and Appendix I.)

Intensity of Stress is the amount of the stress, expressed in units of weight (such as tons), divided by the area of the surface over which it acts, expressed in units of area (such as inches); provided of course that the stress is uniformly distributed over the surface. If it is not uniformly distributed, the intensity of stress will of necessity be greater at some points than at others. It is generally expressed in tons per square inch, sometimes in lbs. per square inch.

Thus in the case of the lamp hanging by a rod, or rather wire (p. 4), the stress is uniformly distributed over the cross section of the rod. Assuming that this cross section has an area of $\frac{1}{224}$th square inch,

$$\text{Intensity of stress} = \frac{\text{weight of lamp}}{\text{area of cross section}} = \frac{50 \text{ lbs.}}{\frac{1}{224} \text{ square inch}}$$

$$= 50 \times 224 \text{ lbs. per square inch,}$$

or 5 tons per square inch.

Again, if a bar 2 inches square, that is, with a sectional area of 4 square inches, has 40 tons hanging from it, the total tensile stress on the bar is 40 tons, but the intensity of stress is 10 tons per square inch.

The cast iron column shown in Fig. 5 may be taken as another example. The load is 10·0 tons, and assuming that the area of the cross section is 8 square inches, the *average* intensity of stress is $\frac{10}{8} = 1{\cdot}25$ ton per square inch.[1]

[1] As will be explained in Chapter VI., unless the column is very short, it cannot be assumed that the stress is uniformly distributed over the cross section, and therefore the intensity of stress will vary from point to point, and the *maximum* intensity may be far greater than the *average*.

The Factor of Safety is the ratio in which the breaking load exceeds the working load

This ratio depends upon the nature of the load and that of the material, and it is found by experience

The following Table shows the Factors of Safety given by Professor Rankine in his *Useful Rules and Tables*

TABLE A

	Factors of Safety	
	Dead Load	Live Load
For *perfect* materials and workmanship	2	4
For good ordinary *materials* and workmanship — Metals	3	6
— Timber	4 to 5	8 to 10
— Masonry	4	8

It will be seen that, for the reasons given above, the factor of safety for a live load is taken at double that for a dead load

When a load is mixed, *i e* partly live and partly dead, the live portion may be converted into an equivalent amount of dead load, and the factor of safety for dead load then applied to the whole

The factors of safety shown in Table A are lower than can be safely used for the work ordinarily met with

Factors of safety which have been used in practice for different special structures are given in Appendix I

Fracture —When a body is subjected to a stress, and the stress *at any point* is greater than the material can withstand, *fracture* ensues

The nature of the fracture depends of course upon the kind of stress to which the body has been subjected

Thus a tensile stress carried far enough produces fracture by tearing, a compressive stress by crushing, a transverse stress by cross breaking, etc

Table of Stresses and Strains and Modes of Fracture.

The following Table shows the various stresses and the strains and description of fracture to which they lead when carried sufficiently far

TABLE B.

Stresses.	Strains	Modes of Fracture
Tensile or pulling	Extension	Tearing
Compressive or thrusting	Compression	Crushing
Transverse	Bending	Breaking across
Shearing	Distortion	Cutting asunder
Bearing	Indentation	Cutting and crushing
Torsional	Twisting	Wrenching asunder

rapidly than the load. Thus 13 tons will produce an elongation slightly greater than $\frac{13}{12000}$ths of the length, and so on in an increasing ratio.

The MODULUS OF ELASTICITY is a number representing the ratio of the intensity of stress (of any kind) to the intensity of strain[1] (of the same kind) produced by that stress, so long as the elastic limit is not passed.

Thus the modulus of tensile elasticity of any material is found by dividing the tensile stress in lbs. per square inch of sectional area by the elongation (produced by that stress) expressed as a portion of the length of the body.

For instance, if a weight of 1 ton hung from an iron bar produce an elongation of $\frac{1}{12000}$th of the length of the bar, the modulus of elasticity of the bar will be 2240 lbs. $\div \frac{1}{12000} = 26{,}880{,}000$ lbs. This is rather lower than the modulus of average wrought iron.

Similarly, the modulus of compressive elasticity is found by dividing the compressive stress in lbs. per square inch of section by the shortening (produced by that stress) expressed as a fraction of the length of the body.

In most building materials the modulus of tensile elasticity and that of compressive elasticity are practically equal to one another so long as the stresses do not exceed the elastic limit.

In advanced works on applied mechanics other moduli are used which, however, are not required in ordinary calculations, except the modulus of rupture, see p. 52, and need not be further referred to in these notes.

Moduli of elasticity for different materials are given in Table I.

Deflection is the bending of any body caused by a transverse stress.

Thus the ladder shown in Fig. 14 is bent by the weight of the boy sitting upon it.

If the intensity of the stress be below the elastic limit, the deflection will disappear when the stress is removed; but if the intensity of stress be in excess of the elastic limit, a permanent *set* will remain.

Stiffness in a material is the power that it may possess to resist being strained out of its proper shape; this is a very different thing from the strength, or power to resist rupture. Compare, for instance, glass and wrought iron.

The stiffness of the material depends upon its elasticity; the greater the value of the modulus of elasticity, the greater the stiffness of the body.

[1] By "intensity of strain" is meant the amount of alteration of figure per unit of length; thus if a bar be elongated under tensile stress, the intensity of strain would be the amount of elongation (expressed as a fraction of a foot) per foot of the bar. Expressing the total elongation as a fraction of the length of the bar amounts to the same thing.

The stiffness of a structure depends not only upon the elasticity of the material of which it is composed, but upon its arrangement Thus a floor with deep narrow joists is much stiffer than one of the same strength with wide and shallow joists

This quality has an importance only second to that of strength in fact, *strength* and *stiffness* must be considered together

In a floor, for example, although the joists may be strong enough to resist breaking, if they are not stiff enough, the floor will be springy and uncomfortable, and if they have a ceiling attached to them it will be cracked, and may be rendered dangerous by the deflection or bending of the joists

In roofs, the rafters, if not stiff enough, will bend or sag, causing ugly hollows, and in some cases lodgment of wet upon the roof covering.

Other Physical Properties of bodies do not affect the calculations about to be entered upon, a discussion of them would tend only to confuse the student, and they will therefore not be alluded to in this volume.

CHAPTER II.

EQUILIBRIUM.

WHEN external forces act upon a body, its powers of resistance are called out until they are just sufficient to balance the external forces; when this balance is maintained, the external forces and those of resistance are said to be in *equilibrium*.

When the external or active forces require, for equilibrium, greater resistances than the body can offer, movement ensues in the form of rupture of the material if the internal resistances are ineffectual, or of movement of the body itself if the external resistances are insufficient.

Thus if a rope be fastened to a man, and a boy pull at it with a force of 30 lbs., the man passively resists until there is a stress upon him of 30 lbs., and equilibrium is established. If, however, the man begins to pull the rope with a force of 80 lbs., equilibrium is disturbed and movement commences—the boy is dragged forward; or if he ties the rope to a wall, and the man pulls with sufficient force, the rope is broken.

Every structure is under the action of two distinct sets of forces, namely—

1. The external forces. 2. The internal forces, or the stresses.

And in order that the structure may not move, or, as it is termed, may be "in equilibrium," it is necessary in the first place that the external forces be in equilibrium amongst themselves; and secondly, that the internal forces, or stresses, be not greater than the resistance the material of the body is able to offer.

The external forces on a structure consist of the various loads it has to bear, and of the reactions at the points of support.

It will thus be seen that, to design a structure as regards strength, three distinct operations are required, namely—

1. To find the external forces acting on the structure.
2. To find the stresses produced by those forces.
3. To calculate the dimensions of the various parts of the structure so that the stresses (or, more strictly speaking, the inten-

sity of stress) shall nowhere be greater than the material is safely able to resist.

The resistance is always exactly equal and opposite to the stress up to the point of rupture, and the resistance offered at that moment is called the *ultimate resistance*. This introduces the important subject of resistance of materials on which ultimately depend all calculations made in connection with the strength of structures, and unfortunately it is a subject of which our knowledge is by no means as full as might be wished. At the end of this book will be found a Table giving the *ultimate resistance* to the various stresses of tension, compression, shearing, etc., of some of the materials usually employed in building construction,[1] but more detailed information is given in Part III.

These operations involve the application of the conditions of equilibrium of a body, the general consideration of which requires a greater knowledge of mathematics than is assumed for this Course; the large majority of cases that occur in building practice can, however, be treated in an elementary manner, which we now proceed to do in the order mentioned above.

EXTERNAL FORCES ACTING ON A STRUCTURE.

The *loads* a structure has to bear may be given, but usually they have to be ascertained in each case, and it will be shown in the sequel how to do this.

For instance, it might be *given* that a certain beam has to bear a weight of 10 tons, but in designing a roof, the weight of the roof-covering, the loads caused by the wind and snow, etc., would have to be ascertained or estimated.

The *reactions* at the points of support are determined, as soon as the loads are known, by the application of the following principle.

Whenever a force acts upon a body tending to move it in a particular direction, this force must be opposed by an equal and opposite force, or else the body will move—that is, it will not be in equilibrium.

This is Newton's third law, namely, that to every action there is always an equal and contrary reaction; or, the mutual actions of any two bodies are always equal and oppositely directed in the same straight line. Or, putting it another way,

If a body is in equilibrium under the action of two forces, it is self-evident that the two forces must be equal and oppositely directed.

[1] See Table I.

Taking, for instance, the case of two men pulling at a rope (Fig. 12).

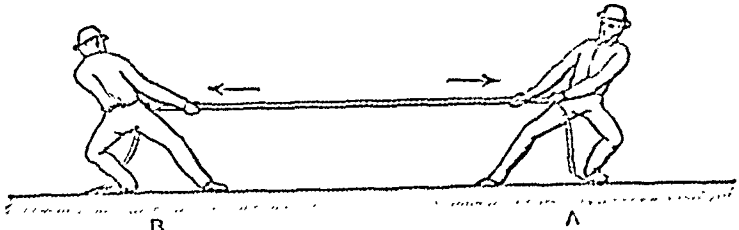

Fig. 12.

Supposing the man A pulls with a force of 30 lbs., B must pull in the opposite direction also with a force of 30 lbs. or the rope would move toward A.

Now, supposing B ties his end of the rope to a ring in the wall (Fig. 13) and goes away, while A goes on pulling with a force of 30 lbs. The wall will now supply a force of 30 lbs. to oppose the pull of 30 lbs. The force supplied by the wall is called the reaction of the wall.

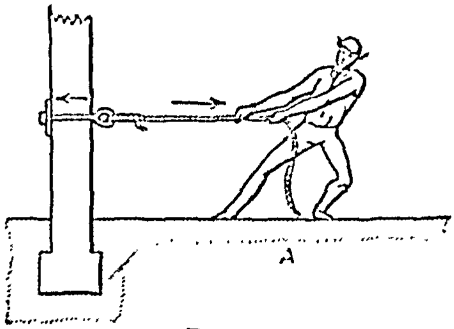

Fig. 13.

The next case to consider is that of a body in equilibrium under the action of parallel forces, as in Fig. 14, showing two men carrying a boy on a ladder.

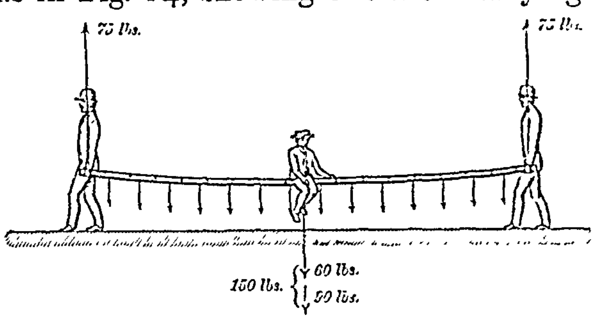

Fig. 14.

The boy's weight and the weight of the ladder act vertically downwards (Fig. 14); the men therefore have to supply vertical forces acting upwards to maintain equilibrium.

Supposing the boy to weigh 60 lbs. and the ladder 90 lbs., the whole weight to be carried by the men is 150 lbs. That is, there are forces amounting to 150 lbs. acting vertically downwards, and the men have to supply the reactions to resist these forces, and prevent the ladder from moving downwards. If the boy is in the middle, it is clear that each man will have to bear half the total weight, amounting for each to 75 lbs.

In the same way, when a horizontal beam supporting a single load placed at its centre (Fig. 15), or loaded symmetrically throughout its length (Fig. 16), rests upon two walls, half the total weight is borne by each wall, or, in other words, each wall has to supply a reaction equal to half the weight.

When the loads to be carried are symmetrically placed with regard to the supporting bodies, and are symmetrical in themselves, the reaction afforded by each of these supporting bodies is equal, as in the illustrations given above.

When, however, the weights are not symmetrically placed with regard to the supporting bodies, then the reaction to be

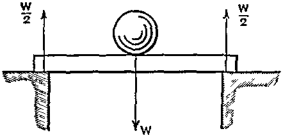

Fig 15

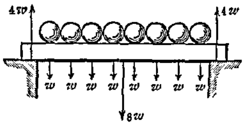

Fig 16

afforded by one support differs from that to be afforded by the other support

Reverting to the illustration of the boy carried on the ladder, if the boy does not sit in the centre of the ladder (Fig 17), then the man A, nearer to whom he sits, will bear more of his weight than the other man—that is, he has to supply a greater reaction.

In order to ascertain the amount of weight each man will have to bear, a simple calculation is necessary, founded upon the principle of the lever, which no doubt is familiar to the student

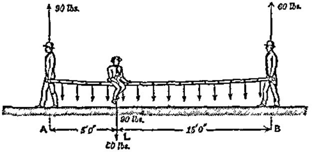

Fig 17.

By the principle of the lever we know that to produce equilibrium the upward force or reaction acting at A, multiplied by the leverage AL, must equal the reaction at B multiplied by BL

We have, therefore, considering now only the weight of the boy,

$$\text{Reaction at A} \times \text{AL} = \text{reaction B} \times \text{BL},$$
$$\text{Reaction A} \times 5 = \text{reaction B} \times 15,$$
$$\text{Reaction A} = \tfrac{15}{5} \times \text{reaction B},$$
$$= 3 \times \text{reaction B}$$
$$\text{Reaction A} + \text{reaction B} = 4 \times \text{reaction B}$$

But we know that the two reactions together must be equal to the weight[1]

$$4 \times \text{Reaction B} = 60 \text{ lbs}$$
$$\text{Reaction B} = 15 \text{ lbs}$$

And
$$\text{Reaction A} = 45 \text{ lbs}$$

Therefore we see that, because the boy is three times as far from B as from A, A *has three times as much of his weight to carry as* B

---

[1] From the conditions of equilibrium (see Appendix II) it appears that—
by taking moments about A,
$$W \times 5 - R_B \times 20 = 0$$

We have, however, ignored the weight of the ladder, which, being a uniform load of 90 lbs., produces a reaction of 45 lbs. at A and 45 lbs. at B.

	Reaction caused by Boy.	Ladder.	
The total reaction at A =	45 +	45 =	90
B =	15 +	45 =	60
Total reactions caused by weight of boy and ladder =			150

The general rule for finding the reaction at either support, caused by a load, placed anywhere on a beam supported at both ends, may be stated as follows :—

Rules for finding the Reactions at Supports.

Rule I.—The reaction at either support, caused by a single load (W) *placed anywhere upon a beam supported at both ends, is equal to the load multiplied by the distance from its centre of gravity to the other support, and divided by the length of the beam between the supports.*

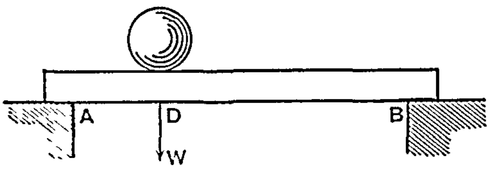

Fig. 18.

SINGLE LOAD.—Thus with reference to Fig. 18

$$\text{Reaction at A} = R_A = \frac{W \times DB}{AB}$$

Where W is *in the centre*, we have $DB = \frac{1}{2}AB$, and

$$\text{Reaction at A} = R_A = \frac{W \times \frac{1}{2}AB}{AB} = \tfrac{1}{2}W.$$

General Case of Single Load.—If the load = W, the span = l, the distance of W from A = a, we have

$$R_A = \frac{l-a}{l} W \quad . \quad . \quad (1). \qquad R_B = \frac{a}{l} W \quad . \quad . \quad . \quad (1\ A).$$

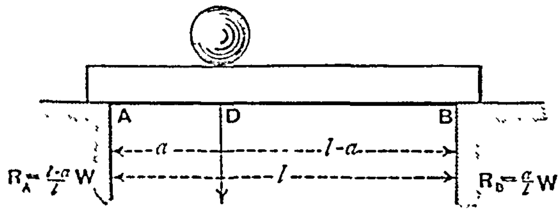

Fig. 19.

If W be in the centre, then

$$R_A = R_B = \frac{W\frac{l}{2}}{l} = \frac{W}{2} \quad (1\ B).$$

TWO OR MORE LOADS.—When a number of weights rest upon a beam the reactions can be found either by calculating the reactions produced by each weight separately, and adding them together, or by considering that they produce the same effects as would be produced by a weight equal to all of them put together, acting through their common centre of gravity.

The second method is to be adopted in preference when the position of the centre of gravity is self-evident. Thus in Fig. 20 it is clear that the four weights, each of 100 lbs., produce the same reactions that would be produced

by one weight of 400 lbs. placed midway between them, that is, at the centre of gravity of the weights (Fig 21).

Uniform Load.—Again the reactions produced by the uniformly distributed load 8 *w* (Fig 16) will be equal to those of a concentrated load W = 8 *w* acting through the centre of gravity of 8 *w*

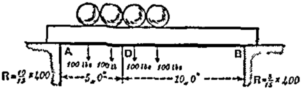

Fig 20

When the load is uniformly distributed over a portion or the whole of the beam, the reactions produced by it *will be the same as those* produced by a single weight equal to the load and placed at the centre of gravity of the load. Thus in Fig 21 the reactions produced by the load of 400 lbs. distributed along EF would be exactly the same as those produced by a single weight of 400 lbs acting at D, the *c g* of EF (see *Fig* 22).

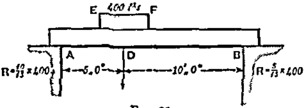

Fig 21

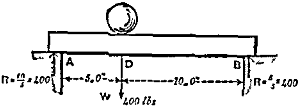

Fig 22

When the weights are unequal and placed unsymmetrically, their reactions are best found by the first method, that is, by taking each in turn and finding the reactions produced by it. The reactions produced at either support will be the sum of the reactions produced by each weight at that support

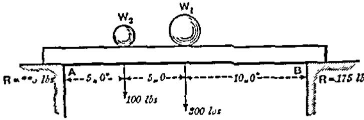

Fig 23

Thus in Fig 23 the reactions produced would be as follows —

	At A	At B
Reactions produced by W_1	$\frac{10}{20}W_1$	$\frac{10}{20}W_1$
Reactions produced by W_2	$\frac{15}{20}W_2$	$\frac{5}{20}W_2$
Total reactions caused by W_1+W_2	$\frac{10}{20}W_1+\frac{15}{20}W_2$	$\frac{10}{20}W_1+\frac{5}{20}W_2$
Or substituting the values of W_1 and W_2, we have	$\frac{10}{20}\ 300+\frac{15}{20}\ 100$	$\frac{10}{20}\ 300+\frac{5}{20}\ 100$
Total reactions caused by W_1+W_2 *i e* by 300 lbs + 100 lbs	$=150+75$ $=225$	$=150+25$ $=175$

The general case of two weights unsymmetrically placed, as shown in Fig. 24.

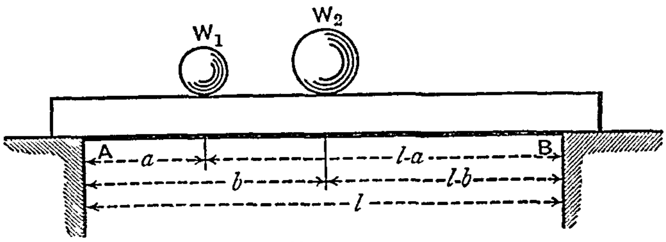

Fig. 24.

Suppose a to be the distance from the abutment A to the point of application of W_1, b the similar distance for W_2, and l the *span* or distance between the supports. Then

$$\text{Reaction at A} = \frac{l-a}{l} \times W_1 + \frac{l-b}{l} W_2,$$

$$\text{Reaction at B} = \frac{a}{l} W_1 + \frac{b}{l} W_2.$$

Several loads placed unsymmetrically.—Whatever the number of weights, the reactions are found in a similar manner.

Thus in Fig. 25 there are five weights, W_1, W_2, W_3, W_4, W_5, the distances

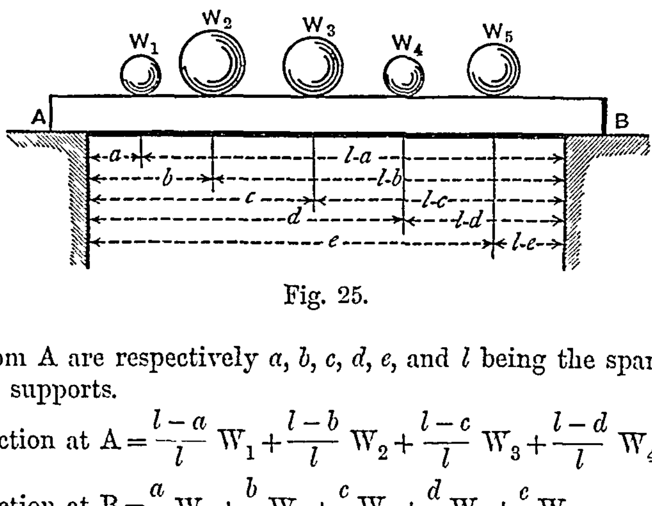

Fig. 25.

of which from A are respectively a, b, c, d, e, and l being the span or distance between the supports.

$$\text{The reaction at A} = \frac{l-a}{l} W_1 + \frac{l-b}{l} W_2 + \frac{l-c}{l} W_3 + \frac{l-d}{l} W_4 + \frac{l-e}{l} W_5.$$

$$\text{The reaction at B} = \frac{a}{l} W_1 + \frac{b}{l} W_2 + \frac{c}{l} W_3 + \frac{d}{l} W_4 + \frac{e}{l} W_5.$$

The weights W_1, W_2, W_3, W_4, W_5 being known, as also the lengths of a, b, c, d, e, and l also known, it is easy to substitute the values and calculate the reactions.

We arrive then at the following rule:—

Rule II.—The reaction at either support caused by any number of loads placed upon a beam supported at each end is equal to the sum of the reactions at that support caused by each load taken independently.

Or, stated in another way, the reactions at the supports A and B caused by a number of loads W_1, W_2, W_3, W_4, W_5, W_6, etc., will be as follows —

Total reaction at A = reaction at A of W_1 + reaction at A of W_2 + reaction at A of W_3 + reaction at A of W_4 + reaction at A of W_5 + reaction at A of W_6, etc

Similarly, the total reaction at B = reaction at B of W_1 + reaction at B of W_2 + reaction at B of W_3 + reaction at B of W_4 + reaction at B of W_5 + reaction at B of W_6, etc.

The reactions of the individual weights at A and B are of course found by Rule 1.

Load on the Supports —It is evident that the load on a support is equal and opposite to the reaction the support exerts on the beam Thus in the case of the men carrying the boy on a ladder, when the boy is sitting in the centre of the ladder the downward pressure on each man is equal to the reaction, or to 75 lbs , when, however, the boy is not sitting at the centre of the ladder, but at the position shown in Fig. 17, the man A has to bear a downward pressure of 90 lbs. and the man B of 60 lbs

In order to familiarise the student with the method of finding the reactions caused by various forms of load, it will be well to take as examples a few cases such as occur in practice.

External Forces acting on a Girder.

Example 1.—A girder AB of 30 feet span (Fig 26) supports a

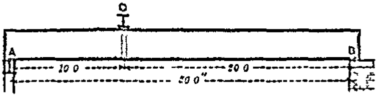

Fig 26.

beam C at a point 10 feet from the column at A The load on the beam C is uniformly distributed, and amounts, including the weight of this beam, to 46 tons The girder AB itself weighs 2 6 tons Find the proportion of weight to be borne by the column and the wall

This weight will be equal to the reaction at each point of support Since the beam C is uniformly loaded, it will impose a load of $\frac{46}{2} = 23$ tons on the girder AB at the point C The girder AB is therefore subject to two loads, namely —

1 Its own weight = 2 6 tons
2 The load upon it = 23 tons

The former is a uniformly distributed dead load, the reactions of which

can be found as at p. 17. The latter is a single load placed unsymmetrically upon the girder, and its reactions can be found by Rule I.[1]

	At A; tons.	At B; tons.
The reaction of the weight of girder itself will be . .	1·3	1·3
The reaction of the load C	15·3	7·7
Total reactions . .	16·6	9·0

If the reactions have been correctly worked out, their sum must be equal to the sum of the loads.

In this case the sum of the weights is $23 + 2{\cdot}6 = 25{\cdot}6$ tons.

The sum of the reactions as found above is $16{\cdot}6 + 9{\cdot}0 = 25{\cdot}6$ tons.

The result is the same in both cases—which shows the working to have been correct.

If in this example the beam C had been supported in the centre of AB, the reactions due to it at A and B would have been equal each to $\frac{23}{2} = 11{\cdot}5$ tons, and the total reactions would have been altered accordingly—each being equal to $1{\cdot}3 + 11{\cdot}5 = 12{\cdot}8$ tons. The column A would thus have to bear less, and the wall B more, than in the former case.

External Forces acting on a Cantilever.

Example 2.—A wrought-iron cantilever, as shown in Fig. 27, is bolted to a wall and sustains a weight of 1 cwt. Find the other external forces acting on the cantilever.

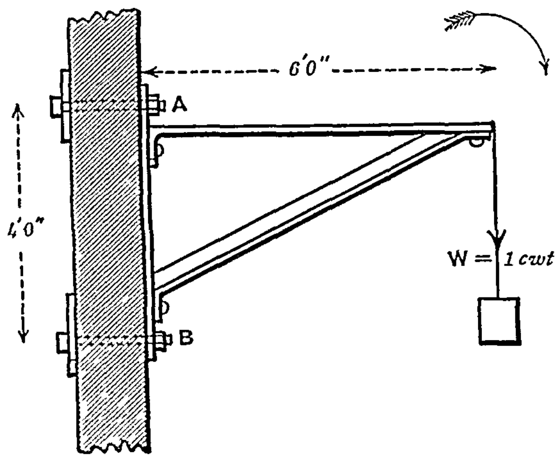

Fig. 27.

To oppose the downward tendency of W, the two bolts AB must supply an upward reaction R = W, but the proportion in which R is divided between the two bolts depends upon the method of fixing. For instance, if the lower bolt were working in a slot, the top bolt would have to supply the whole of R and *vice versâ*.

W also tends to turn the cantilever around the point B in the direction of the hands of a watch, and to oppose this the bolt at A must supply a horizontal reaction R_1. From the principle of the lever, or, as it is termed, by taking moments about B (see Appendix II.)—

$$6 \times W - 4 \times R_1 = 0,$$

or

$$R_1 = \frac{6}{4}W = 1{\cdot}5 \text{ cwt.}$$

[1] Reaction at $A = \frac{20}{30} \times 23 = 15{\cdot}3$,

Reaction at $B = \frac{10}{30} \times 23 = 7{\cdot}7$.

The bolt at A must therefore be of sufficient strength to withstand a pull of 1 5 cwt., or in other words 1 5 cwt is the stress in the top bolt

The reaction R_2 at B is clearly equal to the reaction R_1 at A_1, but acting in the opposite direction

External Forces acting on a Roof.

Example 3.—The roof shown in Fig 28 is loaded with equal

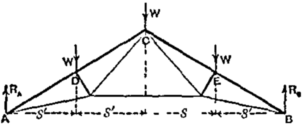

Fig 28

loads W, at the points D, C, and E Find the reactions at the supports A and B

It can be seen at once from symmetry that the reactions must each be equal to $W+\frac{W}{2}$ However, applying Rule II, p 20,

$$R_A=\frac{24}{32}W+\frac{W}{2}+\frac{8}{32}W=W+\frac{W}{2},$$
$$R_B=\frac{8}{32}W+\frac{W}{2}+\frac{24}{32}W=W+\frac{W}{2}$$

The preceding must suffice to give a general idea of the manner of finding the reactions at the supports, and of thus determining all the external forces acting on the body or structure Other cases, such for example as the reactions due to the pressure of the wind, acting on one side of a roof, will be considered in subsequent chapters

The next step (see p 14) is to find the internal stresses produced by the external forces, and for this we must consider the various structures separately, commencing with beams (Chap III)

CHAPTER III.

BEAMS.[1]

FIGS. 31 and 32 show the manner in which a rectangular wooden beam, supported at the ends, breaks when subjected to a concentrated load greater than it can bear. The beam bends,

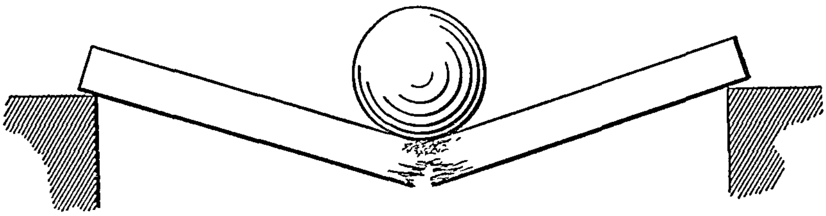

Fig. 31.

sinking most at the point under the weight, and the fibres of the upper portion of the beam are crushed, and those of the lower portion torn asunder, as shown on a larger scale in Fig. 32.

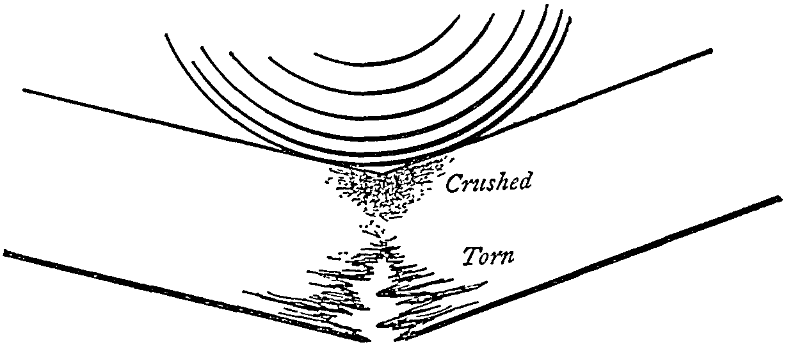

Fig. 32.

There are therefore certain external forces at work tending to break the beam across, and certain internal forces which are resisting this tendency to rupture.

In order to ascertain the strength of the beam, as compared with any load it may be called upon to bear, it is necessary to ascertain the nature and extent—

1. Of the external forces tending to break the beam across.
2. Of the internal forces tending to resist rupture.

[1] *The formulæ for practical use* are given in Equation 30, p. 54, and in Appendices VII. and XXI.

A uniform load of sufficient magnitude would also produce rupture in the same way, but with a slight difference as to the form assumed by the beam just before the time of rupture

When a beam is supported at both ends and subject to a *safe* load it will bend to a certain extent and the upper fibres will be in compression and the lower fibres in tension

Now if we examine the external forces acting upon the beam in Fig 33, we see that the weight acting downwards causes a reaction at each support acting upwards just as in *Fig 17* the weight of the boy on the ladder rendered it necessary for an upward force to be exerted by the man at each end

The downward vertical force caused by the weight acting towards the centre of the earth is balanced by the upward forces caused by the reaction at each end, and the whole structure is in a state of equilibrium (*i e* the beam supports the weight)

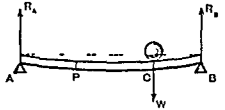

Fig 33

Conversely we may say that the reactions acting vertically upwards are balanced by the weight acting vertically downwards

They are in the position of vertical forces each acting at the end of a lever under the weight as a fulcrum and the action is the same as in bending a stick across one's knee

When we consider the effect of these reactions upon the beam we see that each acts with a leverage equal to the distance from the support to the point immediately under the weight about which the beam tends to break

The power of these reactions to bend or break the beam therefore consists of

R_A (the reaction at A) $\times$ AC

or of R_B (the reaction at B) $\times$ BC

Bending moment—This is called the *bending moment* or in some books, the *moment of flexure*

Since the bending moment of the reaction R_A *at* C (*the section of the beam under the weight*) is proportional not only to the magnitude of R_A but also *to the lever arm* AC with which it acts, it is therefore equal to

$$AC \times R_A,$$

but

$$R_A = \frac{CB}{AB} W,$$

therefore the bending moment due to R_A at C

$$= AC \frac{CB}{AB} W \qquad (2)$$

In the same way the bending moment due to the reaction R_B at C is

$$CB \times R_B$$
$$= CB \cdot \frac{AC}{AB} \cdot W \quad . \quad . \quad . \quad . \qquad (3),$$

which shows that the bending moment at C due to R_A is equal to the bending moment due to R_B, a manifestly correct result.[1]

Let it now be required to find the bending moment at any point P. Considering the left-hand part of the beam, the bending moment is seen to be

$$AP \times R_A = \frac{AP \cdot CB}{AB} \cdot W.$$

Now considering the right-hand part of the beam; the reaction R_B tends to turn that part of the beam against the direction of the hands of a watch, but the weight itself opposes and tends to turn it in the contrary direction. Thus the bending moment will be equal to the bending power of R_B, less that of W, that is

$$= PB \cdot R_B - PC \cdot W,$$
$$= (PB \cdot \frac{AC}{AB} - PC)W,$$
$$= \frac{PB \cdot AC - PC \cdot AB}{AB} \cdot W,$$
$$= \frac{(PC + CB)AC - PC(AC + CB)}{AB} \cdot W,$$
$$= \frac{CB \cdot AC - PC \cdot CB}{AB} \cdot W = \frac{(AC - PC)CB}{AB} \cdot W = \frac{AP \cdot CB}{AB} \cdot W.$$

The same result as before.

The bending moment at any point of a beam can therefore be found by the following

Rule for finding the Bending Moment at any given Point.

Consider either of the portions into which the beam is divided at the given point, and multiply each force acting on that portion by the distance of its point of application from the given point; these products can be taken as positive when the force tends to turn the portion of the beam under consideration in the direction of the hands of a watch, and negative when in the opposite direction. Distributed loads are to be reckoned as acting at their centre of gravity (see p. 19).

The *algebraic*[2] sum of these products is the bending moment required.

[1] Strictly, the distances AC and CB should be measured horizontally, but in practice the bending of a beam is so small that these distances can be measured without any appreciable error along the beam, or in other words may be taken to be, after bending, equal to what they were before bending.

[2] That is, the positive products are to be added and the negative products deducted.

In practice that portion of the beam would be chosen on which the fewest forces are acting

Example 4.—As a simple numerical example let

$W = 100$ lbs,
$\left.\begin{array}{l} AC = 6 \text{ feet} \\ CB = 14 \text{ feet} \end{array}\right\} AB = 20 \text{ feet}$

Then $R_A = \frac{14}{20} \times 100 = 70$ lbs.

$R_B = \frac{6}{20} \times 100 = 30$ lbs.

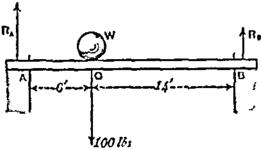

Fig 34

And the bending moment at C

$= CB \times R_B = 14 \text{ feet} \times 30 \text{ lbs}$,
$= 420$ foot lbs.

It is also equal to
$AC \times R_A = 6 \text{ feet} \times 70 \text{ lbs}$,
$= 420$ foot lbs

As will be seen subsequently, it is sometimes better to express lengths in inches, and so expressed the bending moment at C would be

$= 420 \times 12 = 5040$ inch lbs

Further, if the loads are expressed in cwts. the bending moments will be expressed in *inch-cwts.*, and in *inch tons* if the loads are expressed in tons.

BENDING MOMENT UNDER VARIOUS CONDITIONS[1]

We proceed now to consider the *Bending Moment* or *Moment of Flexure* caused in a beam fixed or supported in various ways by the different distributions of load that are most likely to occur in practice

The following will be the notation used throughout —

M = Bending moment or moment of flexure in inch pounds, inch cwts, or inch tons, according to the denomination in which W and w are expressed
M_c = Bending moment at centre
M_P = Moment at point P
M_S = Moment at point S, and so on
A and B are used for the points of support
W = Total weight on the beam in lbs, cwts., or tons
w = Weight per inch run of the beam in lbs, cwts, or tons
$\left.\begin{array}{l} l = \text{length} \\ b = \text{breadth} \\ d = \text{depth} \end{array}\right\}$ of the beam, all in inches
R_A = Reaction at support A
R_B = Reaction at support B

[1] *The formulæ for practical use* will be found numbered 4 to 27 on pp 28 to 40 also in a condensed form in the Table with Appendix VII

Case 1.—BEAM FIXED AT ONE END AND LOADED AT THE OTHER END.

To find the bending moment M_P at any point P, consider the portion PE of the beam as shown in Fig. 35, then by the rule (p. 26) the bending moment is equal to the weight W multiplied by the leverage with which it acts, namely PE.

Now if x be the distance of the point P from the wall, the leverage with which W acts at P is $(l-x)$, therefore

$$M_P = W(l-x) \quad . \quad . \quad . \quad . \quad (4).$$

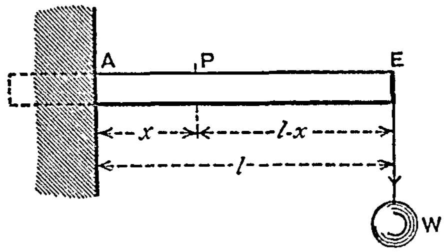

Fig. 35.

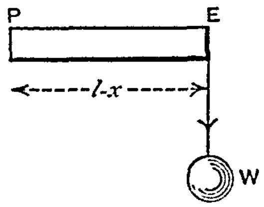

Fig. 35*a*.

At the wall A the weight will be acting with the leverage of the full length of the beam l, hence $\quad M_A = Wl \quad . \quad . \quad . \quad . \quad (5).$

At the point E the leverage is nothing, and the bending moment

$$M_E = 0$$

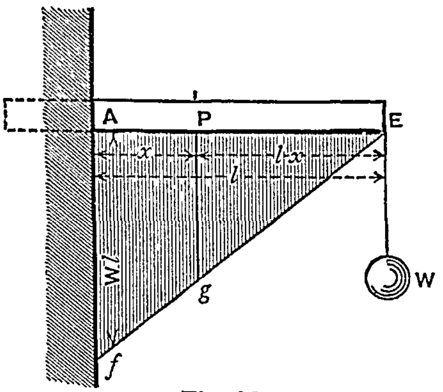

Fig. 36.

Graphic Representation.—The above can be shown graphically thus :—

Draw AE (Fig. 36) as before.[1] Plot Af vertically to represent Wl (the bending moment at A) to some convenient scale of inch-tons to an inch.[2] Join Ef and draw Pg parallel to Af; then Pg will represent the bending moment at P, for, by similar triangles,

$$\frac{Pg}{Af} = \frac{l-x}{l} = \frac{M_P}{M_A},$$

so that Pg and Af are in the same proportion as the respective bending moments, and the value in inch-tons of M_P can be ascertained by scaling the length of Pg.

Case 2.—BEAM FIXED AT ONE END, LOADED UNIFORMLY.

Here if w is the load for unit of span the whole load is wl, and at the

[1] This and the similar figures following are merely graphic representations of the results obtained algebraically. Appendix VI. shows a graphic process applied to obtain the results themselves.

[2] For instance, supposing M_A is 100 inch-tons, then if a scale of 100 inch-tons to the inch is chosen (*i.e.* one inch represents 100 inch-tons), Af will have to be made 1 inch long. Supposing Pg is measured and found to be 0·65 inch, then $M_P = 65$ inch-tons.

point A (Fig 37) it acts with a *mean* leverage $\left(\frac{l}{2}\right)$ equal to the distance from its centre of gravity to the point A

Therefore
$$M_A = wl \times \frac{l}{2},$$
$$= \frac{wl^2}{2} \qquad \qquad (6)$$

The load tending to bend the beam about any section P is that upon PE,

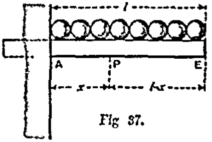

Fig 37.

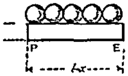

Fig 38

$= w(l-x)$, and it acts with a leverage equal to the distance of its centre of gravity from P, *i.e* $\frac{l-x}{2}$, therefore

$$M_P = w(l-x) \times \frac{l-x}{2},$$
$$= w\frac{(l-x)^2}{2} \qquad \qquad (7)$$

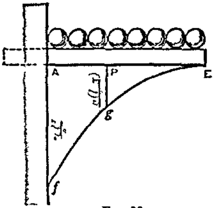

Fig 39

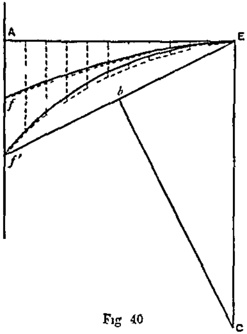

Fig 40

Graphic Representation —This case may be shown graphically, as in Fig 39

Draw AE as before, and $Af = \frac{wl^2}{2}$ to represent the bending moment at A. Draw fE a parabola with its vertex at E; then the ordinate dropped from any point P, distant x from A, will be equal to $\frac{w(l-x)^2}{2}$, which is the bending moment at that point.[1]

Simpler Construction.—To be of ready practical use, a simpler construction is needed. Now if Af is made less than $\frac{AE}{4}$, a circular arc drawn through f, and tangent to AE at E, will *practically* coincide with the required parabola, and can be used instead of it.

This is shown in Fig. 40, the circle being in full and the parabola in dotted lines. The value of the above limit as to the proportion of Af to AE is shown on the same figure, where Af is made $= \frac{AE}{4}$ and the divergence between the circle and parabola is seen to be not sufficiently great as to lead to erroneous results, but in the case of Af' which is equal to $\frac{AE}{2}$ the error is seen to be considerable.

An easy method of finding the centre of the required circle in such cases is also shown on this figure. EC is drawn perpendicular to AE, Ef_1 is joined and bisected at b, and bC is drawn perpendicular to Ef_1. The point of its intersection with EC is obviously the required centre.

Case 3.—BEAM FIXED AT ONE END AND LOADED WITH A UNIFORMLY DISTRIBUTED LOAD, AND ALSO WITH A CONCENTRATED LOAD AT ITS EXTREMITY.

This case (Fig. 41) is a combination of Cases 1 and 2.

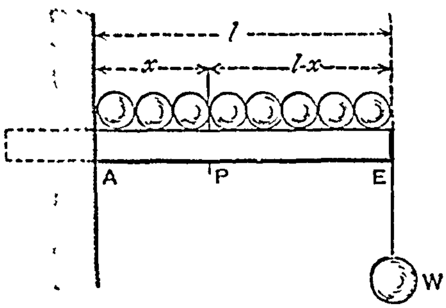

Fig. 41.

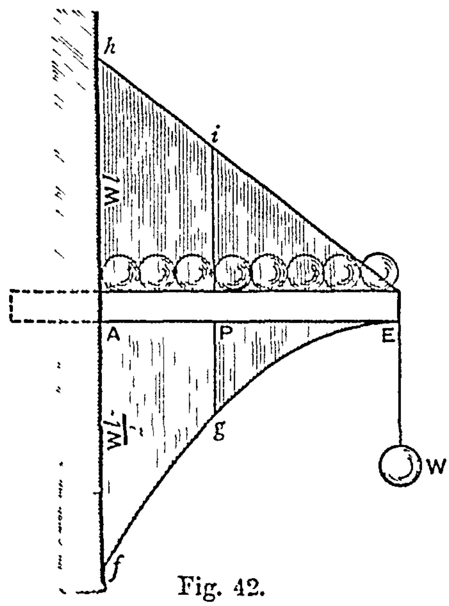

Fig. 42.

Graphic Representation.—This case is shown graphically in Fig. 42; the

[1] For a proof of this, reference may be made to any work on conic sections.

diagram from Fig 36 being plotted above, and that from Fig 39 below the beam AE.[1]

The bending moment at A will be the sum of the bending moments caused by W and by wl, so that, combining Equation 5 with Equation 6,

$$M_A = Wl + \frac{wl^2}{2} \qquad (8),$$

$= Ah + Af,$

$= hf$ (less the thickness of the beam in Fig 42).

Combining Equation 4 with Equation 7,

$$M_P = W(l - x) + \frac{w(l - x)^2}{2} \qquad (9),$$

$= Pi + Pj = ij$ (less the thickness of the beam)

It is not absolutely correct to say that hf = the bending moment at A, and that ij = the bending moment at P, for it is obvious that the depth of the beam must be deducted from these ordinates, or the beam must be represented by a single line when drawing the diagrams.

This case would apply to a balcony crowded with people and having a very heavy railing.

Case 4 (Fig 43)—BEAM FIXED AT ONE END AND LOADED WITH SEVERAL CONCENTRATED WEIGHTS W_1, W_2, W_3, ETC

The bending moment at any point produced by all the weights is equal to the sum of the bending moments produced by each at that point.

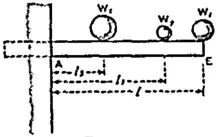

Fig. 43.

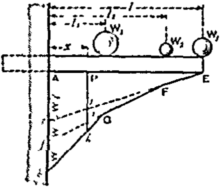

Fig 44

Graphic Representation (Fig 44).

Draw $Ae = W_1 l$ and join Fe,

$ef = W_2 l_2$ and join F (the point on Ee under W_2) with f,

$fg = W_3 l_3$ and join G (the point on Ff under W_3) with g

It is evident that

$$M_A = W_1 l + W_2 l_2 + W_3 l_3, \qquad (10),$$
$$= Ae + ef + fg = Ag$$

At any point P distant x from A

$$M_P = W_1(l - x) + W_2(l_2 - x) + W_3(l_3 - x), \qquad (11),$$
$$= Pi + ij + jk = Pk$$

[1] Both diagrams must be plotted to the same scale—that is, the same number of inch tons or inch lbs to an inch must be taken for both

Case 5 (Fig. 45).—BEAM FIXED AT ONE END AND SUPPORTING A LOAD UNIFORMLY DISTRIBUTED OVER A PART OF ITS LENGTH.

Z being the loaded portion of the beam

At the point A $\quad M_A = wz(\frac{1}{2}z + y)$ . . . . . (12).

At any point Q in AB distant x' from A

$$M_Q = wz(\tfrac{1}{2}z + y - x') \quad \text{. . . (12 A).}$$

At any point P in BD distant x from A

$$M_P = w(z + y - x) \times \tfrac{1}{2}(z + y - x),$$
$$= \tfrac{1}{2}w(z + y - x)^2 \quad \text{. . . (12 B).}$$

At the point B $\quad M_B = wz \times \frac{1}{2}z = \frac{1}{2}wz^2$ . . . (12 C).

At the point D $\quad M_D = 0.$

At any point in DE $\quad M = 0.$

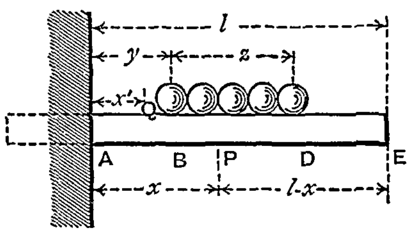

Fig. 45.

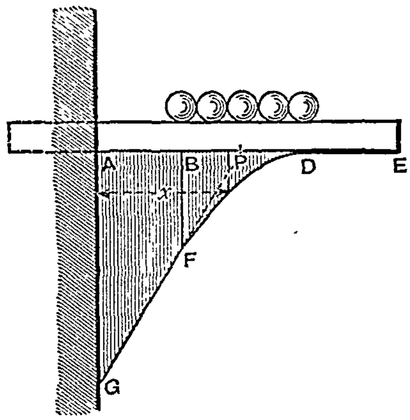

Fig. 46.

Graphic Representation (Fig. 46).

Draw $AG = M_A = wz(\frac{1}{2}z + y)$ and join G and the middle point between B and D.

From B draw the vertical line BF,[1] cutting the line from G in F.

Between the points F and D (the points immediately under the extremities of the load) draw a semi-parabola as for a beam BD uniformly loaded (Case 2).

The vertical distance between GFD and ABD at any point gives the bending moment at that point.

Case 6 (Fig. 47).—BEAM SUPPORTED AT BOTH ENDS AND LOADED IN THE CENTRE.

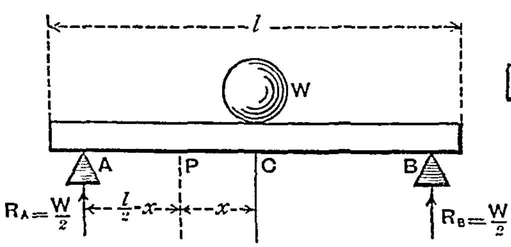

Fig. 47.

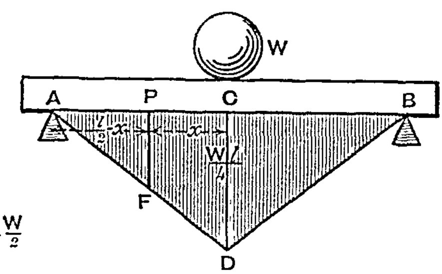

Fig. 48.

[1] If BF be made less than $\frac{BD}{4}$, the parabola can be drawn as a circle.

M_c = Reaction at A or B × leverage,

$$\text{Reaction at A or B} = \frac{W}{2},$$

$$\text{leverage} = \frac{l}{2}$$

Hence $$M_c = \frac{W}{2} \times \frac{l}{2} = \frac{Wl}{4} \qquad (13)$$

At any point P

$$M_p = \frac{W}{2}\left(\frac{l}{2} - x\right) \qquad (14),$$

$$M_A = 0,$$
$$M_B = 0$$

Graphic Representation—Fig 48 shows this Case graphically

Draw CD to represent to some convenient scale $M_c = \frac{Wl}{4}$. Join AD and BD, then the vertical distance from AB to AD or DB at any point is the bending moment at that point

Thus PP' is the bending moment at P and is equal to $\frac{W}{2}\left(\frac{l}{2} - x\right)$

Case 7 (*Fig* 49)—BEAM SUPPORTED AT BOTH ENDS AND UNIFORMLY LOADED

According to the rule given at p 26 the reaction R_a produces a positive bending moment at $C = \frac{wl}{2} \times \frac{l}{2}$, and the load on AC produces a negative bending moment in the opposite direction at $C = \frac{wl}{2} \times \frac{l}{4}$

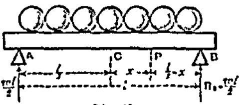

Fig 49

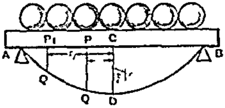

Fig 50

Hence the bending moment at C is

$$M_c = \frac{wl}{2} \times \frac{l}{2} - \frac{wl}{2} \times \frac{l}{4} = \frac{wl^2}{8} \qquad (15)$$

To find the bending moment at any point P distant x from the centre of the beam, consider the portion AP of the beam, then by the rule—

$$M_p = \frac{wl}{2}\left(\frac{l}{2} + x\right) - w\left(\frac{l}{2} + x\right)\frac{\frac{l}{2} + x}{2},$$

$$= \frac{w}{2}\left(\frac{l}{2} + x\right)\left(l - \frac{l}{2} - x\right),$$

$$= \frac{w}{2}\left(\frac{l}{2} + x\right)\left(\frac{l}{2} - x\right) = \frac{w}{2}\left(\frac{l^2}{4} - x^2\right) \qquad (16)$$

Graphic Representation (Fig. 50).—As in Case 2 the bending moments are graphically represented by a portion of a parabola ADB as shown in Fig. 50, and if a suitable scale is chosen so that CD is less than $\frac{1}{4}$ AC or $\frac{1}{8}$ AB, the arc of a circle can be used instead of the parabola without appreciable error. The method of drawing a parabola is given in Appendix III.

To obtain, therefore, the bending moment at any point in AB, plot CD to represent $\frac{wl^2}{8}$ to some convenient scale, so that CD is less than $\frac{1}{8}$ AB. Draw a circular arc through the points A, D, and B, then PQ will represent the bending moment at the point P; P_1Q_1 the bending moment at P_1, and so on.

The bending moment at P can also be obtained by considering the portion PB of the beam. It will then be found that

$$\begin{aligned} M &= w\left(\frac{l}{2}-x\right)\frac{\frac{l}{2}-x}{2} - \frac{wl}{2}\left(\frac{l}{2}-x\right), \\ &= \frac{w}{2}\left(\frac{l}{2}-x\right)\left\{\frac{l}{2}-x-l\right\}, \\ &= -\frac{w}{2}\left(\frac{l}{2}-x\right)\left(\frac{l}{2}+x\right), \\ &= -\frac{w}{2}\left(\frac{l^2}{4}-x^2\right). \end{aligned}$$

The same value as before but with a negative sign. This difference in signs expresses the fact that the forces acting on the portion AP of the beam tend to turn that portion in the positive direction, *i.e.* in the same direction as the hands of a watch, whereas the forces acting on the portion PB tend to turn that portion in the negative direction.

The maximum bending moment occurs at the centre of the beam, and is $\frac{wl^2}{8}$. Now wl is the total load, which is written W, so that $M_c = \frac{Wl}{8}$.

Comparison between Central and Distributed Load.—On referring to Case 6 it will be seen that a central load W produces a bending moment $\frac{Wl}{4}$ at the centre of the beam, that is, double the bending moment produced by a uniformly distributed load of the same amount. In other words, the safe distributed load on a beam is double the safe concentrated load at the centre.

Case 8 (Fig. 51).—BEAM SUPPORTED AT BOTH ENDS, LOADED IN CENTRE AND ALSO UNIFORMLY.

This is a combination of Cases 6 and 7.

Graphic Representation.—In the graphic representation, Fig. 52, the

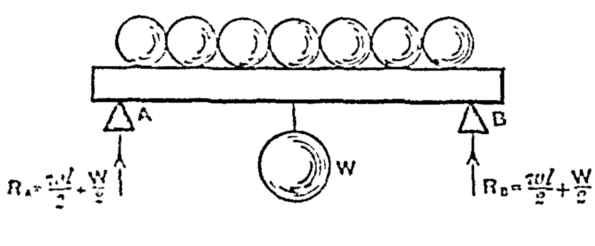

Fig. 51.

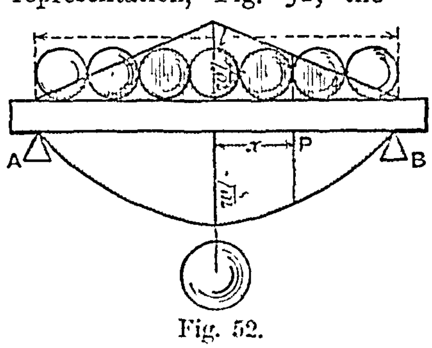

Fig. 52.

diagram of bending moments for the distributed load wl is below, and that for

the single load W is above the line AB It will be easily seen that the total bending moment at the centre $= M_c$ for the central load $+ M_c$ for the distributed load Therefore, combining Equation 13 with Equation 15,

$$M_c = M_c \text{ for } W + M_c \text{ for } wl,$$

$$= \frac{Wl}{4} + \frac{wl^2}{8} \tag{17}$$

And further, by combining Equation 14 with Equation 16,

$$M_x = M_x \text{ for } W + M_x \text{ for } wl,$$

$$= \frac{1}{2}W\left(\frac{l}{2} - x\right) + \frac{1}{2}w\left(\frac{l^2}{4} - x^2\right) \tag{18}$$

It will be observed that the bending moment at any point of the beam can be obtained by adding together the ordinate of the triangle and of the curve at that point

Case 9 (Fig 53)—BEAM SUPPORTED AT BOTH ENDS LOADED AT ANY POINT.

In this case the maximum bending moment is at D, the point of application of the load With regard to the reactions at A and B see p 17

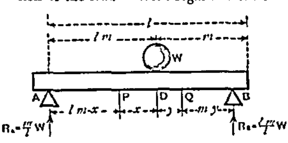

Fig 53

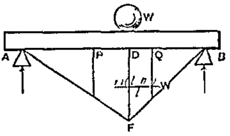

Fig 54

Here the bending moment immediately under W is

Bending moment at D $= M_D =$ reaction at A $\times$ leverage, AD,

$$= \frac{m}{l} W \times (l - m),$$

$$= \frac{m(l-m)}{l} W \tag{19}$$

Or $\qquad M_D = -$ reaction at B $\times$ leverage, BD,

$$= -\left(\frac{l-m}{l}\right) W \times m,$$

$$= -\frac{m(l-m)}{l} W$$

At any point P in AD to the left of and at a distance x from W

$$M_P = R_A \times \text{leverage},$$

$$= \frac{m}{l} W \times (l - m - x) \tag{20}$$

At any point Q in DB to the right of and at a distance y from W

$$M_Q = -R_B \times \text{leverage, BQ},$$

$$= -\frac{l-m}{l} W \times (m - y) \tag{21}$$

At A and B $\qquad M_A$ and M_B each $= 0$

Graphic Representation.—The bending moments are represented by a triangle, as shown in Fig. 54.

Case 10 (Fig. 55).—BEAM SUPPORTED AT THE ENDS AND PARTIALLY LOADED WITH A UNIFORM LOAD.

Fig. 55 shows the manner in which the beam is loaded, and the values of the reactions R_A and R_B are as follows:

$$R_A = (l - m - n)w \times \frac{\frac{l - m + n}{2}}{l} = \frac{(l - m - n)(l - m + n)}{2l} . w,$$

$$R_B = (l - m - n)w \times \frac{\frac{l + m - n}{2}}{l} = \frac{(l - m - n)(l + m - n)}{2l} . w.$$

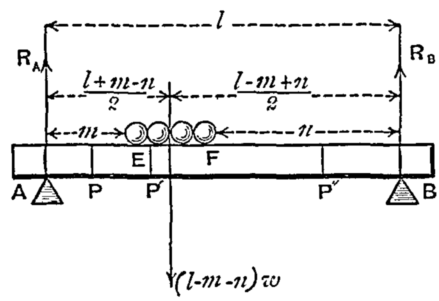

Fig. 55.

The distance of any point (P, P′, or P″) from A will be called x, and from B the distance will be called y.

The bending moment at any point P situated between A and E is

$$M_P = R_A \times x,$$

$$= \frac{(l - m + n)(l - m - n)}{2l} . wx \quad . \quad . \quad . \quad (22).$$

Similarly the bending moment at any point P″ situated between F and B at a distance y from B will be found to be (considering the portion P″B of the beam and neglecting the negative sign)

$$M_{P''} = R_B \times y = \frac{(l + m - n)(l - m - n)}{2l} . wy \quad . \quad . \quad (23).$$

Lastly, the bending moment at any point P′ situated between E and F is equal to

$$M_{P'} = R_A \times x - (x - m)w \times \frac{x - m}{2},$$

$$= \frac{(l - m + n)(l - m - n)}{2l} . wx - \frac{(x - m)^2}{2} . w \quad (24).$$

It can also be shown by the application of the differential calculus that $M_{P'}$ is a maximum when

$$x = \frac{l^2 + m^2 - n^2}{2l} \quad . \quad . \quad . \quad . \quad . \quad (25).$$

Graphic Representation.—It will be found that if the distributed load over

EF were concentrated, one half at E and the other half at F, that neither R_A nor R_B would be altered in value, and hence M_E and M_F would also have the same value. But the bending moment at P would be changed and would be equal to

$$M'_P = \frac{(l-m+n)(l-m-n)}{2l}wx - \frac{(l-m-n)w}{2}(x-m),$$

therefore

$$M_P - M'_P = \frac{(x-m)w}{2}\left\{l-m-n-(x-m)\right\}$$

Fig. 56 represents the part EF of the beam by itself, and it is clear that

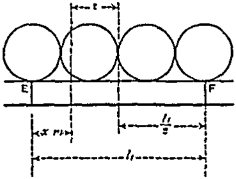

Fig. 56

$l-m-n=l_1$, and if z is the distance of P from the centre of EF,

$$x-m=\frac{l_1}{2}-z$$

Hence

$$M_P - M'_P = \frac{\left(\frac{l_1}{2}-z\right)w}{2}\left\{l_1-\frac{l_1}{2}-z\right\},$$

$$= \frac{\left(\frac{l_1}{2}-z\right)\left(\frac{l_1}{2}+z\right)}{2}w,$$

$$= \frac{w}{2}\left(\frac{l_1^2}{4}-z^2\right)$$

It will therefore be seen that the bending moment at P is greater when the load is distributed than when it is concentrated at E and F. And further, *by referring to Equation 16, it will be seen that the increase can be represented* graphically by the ordinate of a parabola as shown in Fig. 57, or practically by the ordinate of an arc of a circle, if $\frac{wl_1^2}{8}$ is represented by less than $\frac{l_1}{4}$.

Now the diagram of bending moments, when the loads are concentrated at E and F, *is as shown in Fig. 58, where*

$$EQ = R_A \times m = \frac{(l-m+n)(l-m-n)}{2l}wm,$$

and

$$FQ_1 = R_B \times n = \frac{(l+m-n)(l-m-n)}{2l}wn$$

so that the diagram, when the load is distributed, can be found by *adding* the diagrams in Figs. 57 and 58 together, as shown in Fig. 59.

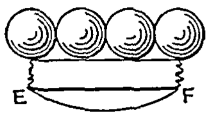

Fig. 57.

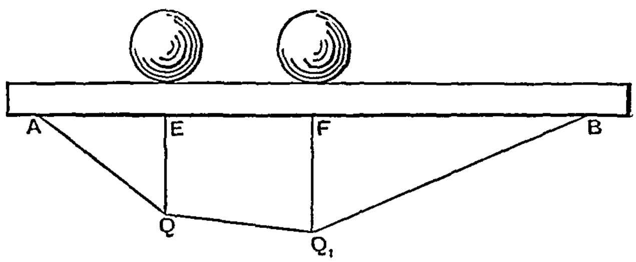

Fig. 58.

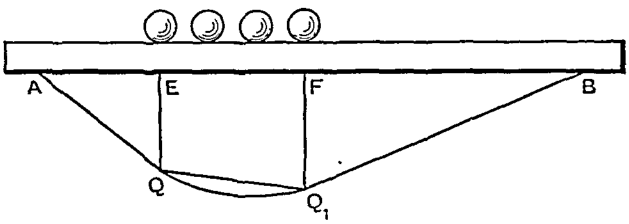

Fig. 59.

Instructive lessons can be learnt from this case by varying the values of m and n.

Supposing, for instance, that $m=n$, then the maximum bending moment will be at the point for which (Equation 25)

$$x=\frac{l^2}{2l}=\frac{l}{2},$$

that is, at the centre of the beam, and the bending moment at that point, from Equation 24, will be

$$\begin{aligned}
M_c&=\frac{l(l-2m)}{2l}\cdot w\cdot\frac{l}{2}-\frac{\left(\frac{l}{2}-m\right)^2}{2}\cdot w,\\
&=\frac{\frac{l}{2}-m}{2}\left(l-\frac{l}{2}+m\right)w,\\
&=\frac{\left(\frac{l}{2}-m\right)\left(\frac{l}{2}+m\right)}{2}\cdot w,\\
&=\frac{\frac{l^2}{4}-m^2}{2}\cdot w \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (26).
\end{aligned}$$

If now $m=0$, so that n is also 0, that is, the load is distributed uniformly over the whole beam, then $M_c=\frac{l^2}{8}\cdot w,$

the result previously obtained when considering Case 7 (Equation 15).

Again, if $m=\frac{l}{4}$, that is, the central half of the beam is uniformly loaded,

$$\begin{aligned}
M_c&=\frac{\frac{l^2}{4}-\frac{l^2}{16}}{2}\cdot w,\\
&=\frac{3}{32}\cdot l^2w\,;
\end{aligned}$$

The moment at any point P distant x from A is (if P be between o_2 and o_3)
M_P = Reaction A × leverage − weights between P and A × leverage,

$$= \left(\frac{W_1(l-m_1)}{l} + \frac{W_2(l-m_2)}{l} + \frac{W_3(l-m_3)}{l}\right) \times x - W_1(x-m_1) - W_2(x-m_2) \quad (27).$$

Graphic Representation[1] (Fig. 60).—In the graphic representation the dotted lines show the triangle of bending moments due to each load separately (see Case 9).

Thus Ar_1B is the triangle for W_1,
Ap_2B is the triangle for W_2,
Aq_3B is the triangle for W_3.

The bending moments caused by all the loads are represented graphically by a polygon, which can be obtained as follows:—

The bending moment caused by W_1 immediately under itself is o_1r_1, the bending moment at this same point caused by the weight W_2 is o_1p_1, and the bending moment caused by W_3 is o_1q_1; the total bending moment at this point caused by W_1, W_2, and W_3 is equal to $o_1r_1 + o_1p_1 + o_1q_1$, and o_1t_1 is set off equal to this.

In the same way the total bending moment at W_2 is found and set off as o_2t_2, and the total bending moment at W_3 as o_3t_3.

Join At_1, t_1t_2, t_2t_3, and t_3B, and the polygon of moments is obtained.

MOMENT OF RESISTANCE.

The application of the preceding rules enables us to ascertain the effect produced by external forces acting transversely upon a beam—that is, in other words, to calculate the *bending moments* produced by loads or other external forces.

The next step is to investigate the nature of the resistance offered to these forces.

This resistance is afforded by the internal structure of the beam, which, for instance, consists in timber and wrought iron of a number of fibres lying closely together and running in the direction of the length of the beam, but in cast iron of closely adhering grains.

There are two methods by which the value of this resistance can be ascertained—

1. By reasoning upon certain assumptions.
2. By experiment.

Of these two methods the first is not practically used in the calculation of beams of rectangular cross section, but its application to them is described somewhat fully in detail, in order to clear the ground for the better understanding of the calculation of beams of more complex section.

Method of Ascertaining the Moment of Resistance by Reasoning.—The effects produced upon the fibres of a beam by external forces acting upon the beam may be shown as follows:—

[1] See Appendix VI. where a similar case is worked entirely by a graphic method.

Let Fig 61 represent a straight *rectangular* beam supported at A and B and Fig 61*a* a cross section of the same beam. Mark upon the beam two vertical straight lines *op*, *qr*, and also a line NL drawn parallel to the length of the beam and midway of its depth, and intersecting *op* in *n* and *qr* in *l*

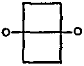

Fig 61*a*

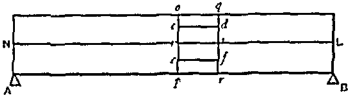

Fig 61

The beam can be considered as made up of *horizontal* layers of fibres, so that *oq* will represent a *portion of the top layer and pr of the bottom layer*, NL will represent the central layer, *nl* a portion of it, and *cd* and *ef* portions of intermediate layers.

If the beam be now slightly bent as in Fig 62 (where, however, the bend

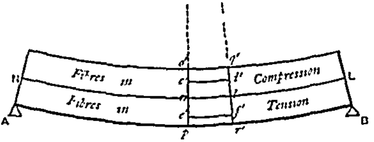

Fig 62

ing is exaggerated to make the case clearer), it will be found that the lines *op* and *qr*, originally vertical, are inclined inwards, and that *nl* remains unchanged in length. It therefore follows at once that $o'q'$ and $c'd'$ are shorter than oq and cd, but that $p'r'$ and $e'f'$ are longer than pr and ef

Fig 63 is an enlargement of the central part of Fig 62, and o_1p_1 has been drawn through *n* parallel to *qr*; clearly o_1o and c_1c are the amounts by which *oq* and *cd* have been shortened by the bending, and similarly ee_1 and pp_1 are the amounts by which *ef* and *pr* have been lengthened

It is evident, therefore, that *all* the fibres above *nl* have been compressed, and those below *nl* extended, by the stress put upon them by the bending action

Only the fibres in the layer represented by NL are uninfluenced by the bending action; they are neutral, having neither compression nor tension to resist. This layer of fibres is called the *neutral layer*

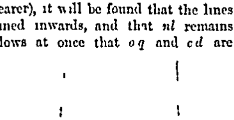

Fig 63.

If a cross section of the beam be made, the neutral layer[1] will be intersected in a straight line (OO, Fig. 61*a*), and this straight line is called the *neutral axis.*

As already mentioned at p. 24, if the bending of the beam were carried far enough to produce cross breaking, it would be found that the fibres at the cross section of maximum stress above the neutral layer were crushed, and the fibres below were torn asunder.

It will therefore be seen that the resistance of the beam to bending is afforded by the resistance of the fibres above NL to compression and of those below NL to tension, and the question is, How can we arrive at the value of this resistance ?

Figs. 64 and 65 may make us understand more clearly exactly what it is we wish to find out.

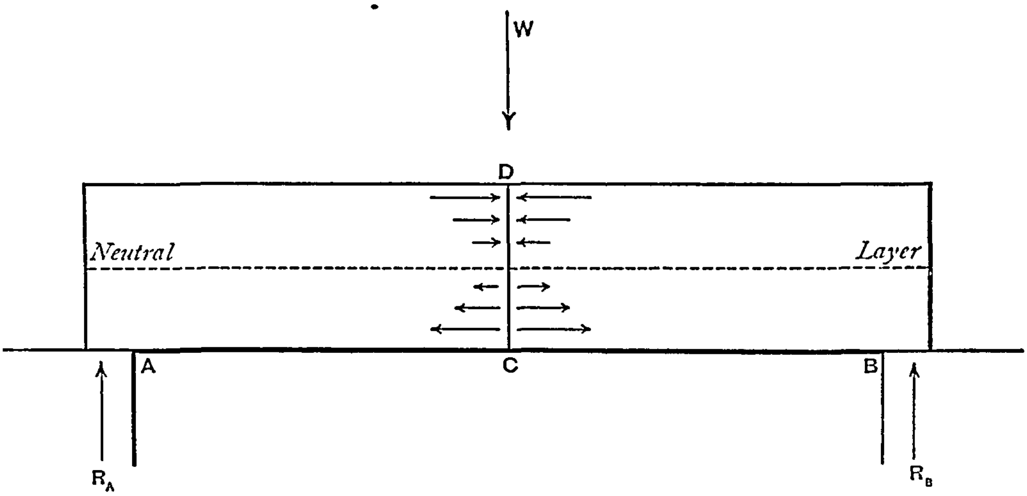

Fig. 64.

Taking a simple case of a beam supported at both ends and loaded in the centre.

Fig. 64 shows the general direction of the forces which produce the bending moment and of those which offer resistance at the cross section CD under the load.

R_A and R_B are the reactions at A and B respectively which act with a leverage equal to half the span of the opening, *i.e.* AC or BC, and

$$\left.\begin{array}{l} R_A \times AC \text{ or} \\ R_B \times BC \end{array}\right\} = \text{the bending moment.}$$

Supposing now that the beam is cut across at CD and separated, the load being also divided equally ; and that by some means the stress on each fibre is maintained exactly at what it was before the beam was cut ; the two halves of the beam will clearly remain in equilibrium, and the state of things is represented in Fig. 65.

Considering the left-hand half of the beam, it will be seen that the forces R_A and $\frac{W}{2}$ tend to turn that portion of the beam in the positive direction, and that the internal stresses tend to resist it in the negative direction. The forces R_A

[1] The neutral layer is sometimes called the neutral surface and in some works the neutral axis.

For example—If a compression of 1 ton per square inch shortens a fibre by $\frac{1}{1000}$th of its length, a compression of 2 tons per square inch on the fibre will shorten it $\frac{2}{1000}$ths of its length, 3 tons $\frac{3}{1000}$ths, and so on, up to the elastic limit; and in the same way, if a tension of 1 ton per square inch stretches a fibre $\frac{1}{1000}$th, 2 tons will stretch it $\frac{2}{1000}$ths, 3 tons $\frac{3}{1000}$ths, and 5 tons $\frac{5}{1000}$ths.

2. That the amount of shortening produced by a given stress causing compression will be equal to the amount of stretching produced by an equal stress causing tension.

Thus if a compression of 1 ton per square inch upon a fibre shortens it $\frac{1}{1000}$th of its length, a tension of 1 ton per square inch will stretch it $\frac{1}{1000}$th of its length, and so on.

3. That the amount of stretching or shortening in a fibre is proportional to its distance from the neutral axis of the section under consideration.[1]

It follows from assumptions 2 and 3 (see Appendix IV.) that the neutral axis passes through the centre of gravity of the section.

Thus if CDEF (Fig. 66) represents the cross section of a beam 12″ deep, then OO, the neutral axis, is to be drawn through the centre of gravity of the cross section, that is in this case midway between CE and DF.

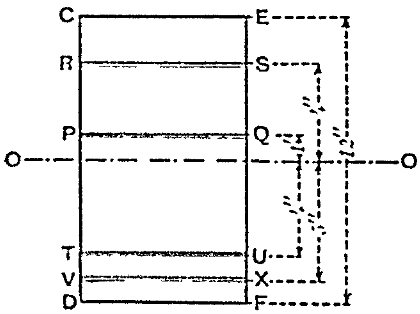

Fig. 66.

If PQ be 1 inch from OO,
RS „ 4 inches „
TU „ 4 inches „
VX „ 5 inches „

then the shortening of each fibre in the layer RS will be four times that of the fibres at PQ; the shortening at CE six times that at PQ. The stretching at VX will be $\frac{5}{4}$ of that at TU, and the stretching at DF = $\frac{6}{4}$ that at TU.

In the case under consideration, it is to be observed that, owing to the neutral axis being midway between CE and DF, the amount of shortening of the fibres at CE is equal to the lengthening of the fibres at DF.

It will be seen that in Fig. 63, o_1np_1 and $o'np'$ are drawn as straight lines,

The mean compression on the fibres above the neutral axis will be that represented by qq' ($=3$ tons) at a point q, half-way between O and the extreme fibre.

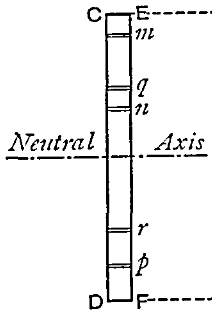

Fig. 69.

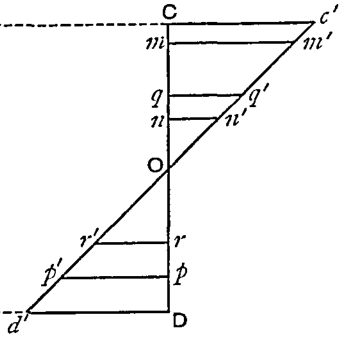

Fig. 69*a*.

In the same way the mean tension on the fibres below the neutral axis is $rr' = 3$ tons.

This mean compression acts over a surface $6'' \times 1''$, the total compression is therefore $6'' \times 1'' \times 3$ tons $= 18$ tons.

In the same way, below the neutral axis the total tension is $= 3$ tons $\times 6'' \times 1'' = 18$ tons.

We have then in Fig. 69 a cross section of the beam, and in Fig. 69*a* a diagram which shows the resistance offered by the fibres (equal to the stress upon them) at any point in the cross section, and also the total resistance of the cross section both in compression and tension. The diagram also shows us how this resistance is distributed, *i.e.* that the fibres at the neutral axis contribute nothing to the resistance, but that the resistance of the fibres increases in proportion to their distance from the neutral axis until it reaches a maximum in the extreme layers of fibres CE and DF, where each undergoes a stress equal to the limiting stress of 6 tons per square inch.

Graphic Method of Finding the Moment of Resistance.[1]—It is, however, convenient to combine the cross section of the beam with the diagram as follows:—

Draw the cross section of a beam CEDF as before (Fig. 70), and make a new scale of tons in which the length CE $=6$ tons, the limiting stress per square inch, and draw the stress diagram (Fig. 71) similar to Fig 69*a* above.

Fig. 72 is merely a modification of Fig. 71 and clearly gives the same results, and can be inserted in Fig. 70 as shown. Now the area of the triangle CEn is half the area of the rectangle CEOO, and, therefore, the total compression is—

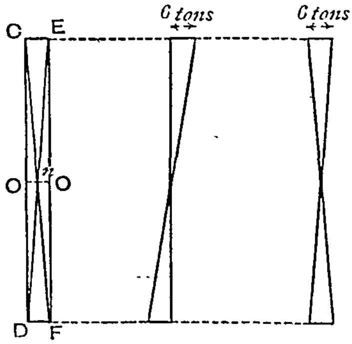

Fig. 70. Fig. 71. Fig. 72.

$$= \text{area of rect. CEOO} \times 3 \text{ tons},$$
$$= \text{area of triangle CE}n \times 6 \text{ tons},$$
$$= \frac{6 \times 1}{2} \times 6 = 18 \text{ tons},$$

the same value as before. In other words, a compression of 6 tons *uniformly* distributed over the triangle CEn is equivalent to the *actual* compression existing at the section of the beam. And in the same way a tension of 6 tons uniformly distributed over the triangle nDF is equivalent to the actual tension. The areas of the triangles CEn and nDF can therefore be called the *equivalent areas of resistance*.

In the case of beams more than 1 inch wide, the application of this method of showing the quantity and distribution of the resistances is very simple.

Stresses in Beam 4 inches wide.—Thus a beam 4 inches wide and 12

[1] This method is described as it illustrates the principles and for the sake of those who prefer drawing to calculations, but it is easier to use the formulæ, p. 54.

inches deep is the same as four beams each an inch wide and 12 inches deep (such as that we have been considering) placed side by side.

The diagram showing the intensity of stress on the fibres of such a beam will be the same as before (Fig. 69*a*).

In the diagram (Fig. 73) showing the quantity and distribution of stress, as there are 4 inches in CF, each acted upon by a stress of 6 tons per square inch, CE will represent 4 × 6 = 24 tons. CF*n* will be the *equivalent area* for compression, and *n*DF the equivalent area for tension.

$$\text{The area } CEn = \tfrac{1}{2} \times 4'' \times 6'',$$
$$= 12 \text{ square inches.}$$
$$\text{The resistance to compression of } CFn = 6 \text{ tons} \times 12 \text{ square inches},$$
$$= 72 \text{ tons.}$$
$$\text{The tensile resistance of } nDF = \tfrac{1}{2} \times 4'' \times 6'' \times 6 \text{ tons},$$
$$= 72 \text{ tons}$$

or in each case four times as much as the resistance (18 tons) of the beam 1" wide and 12" deep.

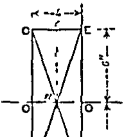

Fig. 73

In the same way the resistance of any beam may be found, whatever its breadth, depth, and strength of fibre.

In all these diagrams it will be noticed that the equivalent area representing tension is equal to that representing compression. It is evident that this must be the case, for if the total compression above the neutral axis were not equal to the tension below, there would not be equilibrium.

Value of Moment of Resistance.—Having thus drawn the triangles showing the amount and distribution of the resistance of the cross section, it will be easy to ascertain the value of the moment of resistance, which, as explained at p. 13, has to be made equal to the bending moment produced by the load.

It will make the general rule more clear if we deal first with a particular case.

Let us take the case we have already been considering, that of a beam 4" wide and 12" deep, having a limiting stress upon the extreme fibres of 6 tons per square inch.

We know from p. 46 that the amount and distribution of the resistance in compression is shown by the triangle CF*n*, that in tension by *n*DF, Fig. 73.

This diagram tells us then that if 6 tons per square inch according to the scale were acting uniformly all over the triangle CE*n*, it would offer in quantity and distribution exactly the same resistance as is offered by the varying stress which actually occurs over the rectangle CFOO. Again, the actual varying tension on OODF is represented in quantity and distribution by 6 tons (on the scale) per square inch acting uniformly all over the triangle *n*DF.

Now if 6 tons were acting on each square inch of CF*n*, we know by the elementary rules of mechanics that it will be the same thing if we consider the total number of tons acting over CE*n*, as being concentrated at the centre of gravity and acting there.

The centre of gravity of CE*n* is at the point *e*, and that of *n*DF at *g*.[1]

It has been shown above that the total compression is 72 tons, and that the total tension is the same.

---

[1] *e* is 2 inches or ⅓ of *en* from *c*, *g* is 2 inches from *f*.

The resistance of the cross section is therefore equivalent to two forces, each equal to 72 tons, and acting at c and g.

On reference to p. 43 it will be seen that these forces form a *couple*, whose moment can be found by multiplying one of the forces by the distance between the forces, or, as it is called, by the *arm of the couple*, in this case cg. The moment of this couple is clearly the moment of resistance required.

The moment of resistance of the cross section under consideration is therefore $= 72 \times cg$.

These centres of gravity are found by bisecting CE at e and DF at f—joining ef and taking ec and gf equal to $\frac{1}{3}$ of en and nf respectively.

Now cg is evidently equal to $cn + ng$ Fig. 73, and

$$cn = \tfrac{2}{3}en = \tfrac{2}{3} \times 6 = 4'',$$
$$ng = \tfrac{2}{3}nf = \tfrac{2}{3} \times 6 = 4'',$$
$$\therefore\ cg = 8'',$$

or the moment of resistance

$$= 72 \text{ tons} \times 8 \text{ inches},$$
$$= 576 \text{ inch-tons.}$$[1]

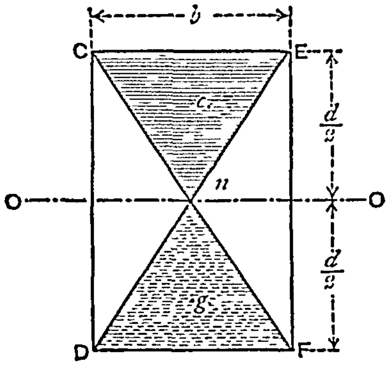

Fig. 74.[2]

Safe resistance Area—General Case.—Now to take a general case (Fig. 74). Suppose the breadth of the beam in inches is called b, the depth in inches d, and the limiting stress to be allowed on the outside fibres r_o tons per square inch. As in the previous case, the triangles CEn and nDF are the equivalent areas of resistance each $\frac{1}{4}bd$, the stress being r_o tons per square inch. The total compression is therefore $r_o \times \frac{1}{4}bd$, and the total tension has the same value.

Thus, as in Fig. 73, $$cn = ng = \tfrac{2}{3} \div \frac{d}{2},$$

so that the distance cg, or the length of the arm of the couple, will be

$$\tfrac{1}{3}d + \tfrac{1}{3}d = \tfrac{2}{3}d.$$

Resulting Formula.—The moment of the couple, *i.e.* the moment of resistance required, is therefore

$$r_o \times \tfrac{1}{4}bd \times \tfrac{2}{3}d = \tfrac{1}{6}\,r_o bd^2.$$

We have now found a general formula for the moment of resistance of a rectangular beam, in terms of its breadth, depth, and of the limiting stress allowed upon the extreme fibres.

The method by which this formula has been arrived at is shown step by step in the preceding paragraphs, and as the formula has been deduced from

---

[1] Another way of looking at this is: The fibres in CEn act with an average leverage equal to the distance from their centre of gravity to the neutral axis $= cn = 4$ inches; in the same way the fibres of nDF act with an average leverage of 4″.

Thus we have total resistance

$$\begin{aligned} &\quad\ \text{tons} \qquad\qquad \text{tons} \\ &\text{CE}n \times 6 \times 4'' + n\text{DF} \times 6 \times 4'', \\ &= 72 \times 4 \quad + 72 \times 4, \\ &= 576 \text{ inch-tons.} \end{aligned}$$

[2] The shaded portions of this figure represent in magnitude and distribution the stresses over the section of the beam—just when the extreme fibres on each side are subjected to the limiting stress.

5 by calculation from the diagrams are the same as the moments of resistance found in column 7 from the formula.

Moreover, we see that the width, etc., being the same, the moment of resistance, *i.e.* the strength of the beams, varies as:

$$16 : 36 : 64, \text{ i.e. as}$$
$$4^2 : 6^2 : 8^2, \text{ i.e. as}$$

the squares of the depths of the beams.

Effect of width upon strength.—Let us see the effect of varying the width upon the strength of a beam.

Take beam C, and instead of 2″, give it first a width of 3″, then of 4″, then of 6″, calculating by the formula we have:

Size of Beam.	Value of Moment of Resistance $r_o \frac{bd^2}{6}$.
8″ × 2″	$3 \times \frac{2 \times 8 \times 8}{6} = 64$
8″ × 3″	$3 \times \frac{3 \times 8 \times 8}{6} = 96$
8″ × 4″	$3 \times \frac{4 \times 8 \times 8}{6} = 128$
8″ × 6″	$3 \times \frac{6 \times 8 \times 8}{6} = 192$

We see from the above that when the depth remains the same the resistance or strength of the beam varies as the width.

Thus the resistance of a beam 6″ wide is one and a half times that of a beam 4″ wide, twice that of a beam 3″ wide, three times that of a beam 2″ wide.

General rule.—We see then, as already mentioned at p. 49, that *the strength of rectangular beams of the same material vary directly as their breadth and as the square of their depth.*

This indeed is apparent from an examination of the formula, for the resistance of the beam is shown by that formula to vary as bd^2.

General Formula.—This formula,

$$\text{Moment of resistance} = r_o \frac{bd^2}{6} \quad . \qquad (28),$$

is therefore, as has been mentioned above, a general expression for the value of the moment of resistance of a beam of rectangular section, when it is subjected to such a load that the limiting stress on the fibres nowhere exceeds the elastic limit of the material.[1]

If the assumptions mentioned at p. 43 were true for all conditions of stress,

---

[1] This formula is practically the same as that given by Rankine and other writers, who have approached the subject from the side of advanced mathematics. They show the moment of resistance to be equal to $\frac{r_o I}{y_o}$, where I is the moment of inertia of

that is, if they were true for every degree of stress from very small ones up to the breaking point, then r_o would be that value of the stress either of tension or compression which was known to cause rupture by tearing or crushing in the fibres of the material, and could be ascertained by direct experiments upon the tensile strength and resistance to compression of the material

Experiment has shown, however, that these assumptions are true only for stresses such as are well within the elastic limit of the material, the shortening or lengthening of the fibres varying regularly with the compressive or tensile stresses *(see p 11) so long as the elastic limit is not exceeded* Beyond that, the ratios of the shortening and lengthening to the corresponding stresses producing them advance more and more rapidly until, on approaching fracture, they become irregular, but always considerably in excess of what they were under the lower stresses.

The effect of stresses in tension often differs also from the effect of the same stresses in compression, and the result of all this is that the position of *the neutral axis is altered—it no longer passes through the centre of gravity* of the cross section, *e g* if the material is at the point of rupture stronger to resist compression than tension, the neutral axis will then be nearer the outermost fibre in compression than to the outermost fibre in tension Thus in a beam of such material, supported at both ends and loaded, the neutral axis will be nearer the top than the bottom edge

This difference between facts and the assumptions, at the time when the *breaking stress is approached, renders the theory founded upon them inapplicable* for the purpose of ascertaining the resistance of beams to actual *breaking*

It should be noticed that in these examples the weight of the beam itself is left out of the consideration When it is large enough to require attention it must of course, be treated as a distributed load, and its effect be added to those of the load proper

METHOD OF ASCERTAINING THE MOMENT OF RESISTANCE BY EXPERIMENT AND CALCULATION

When we have to deal with beams loaded so that they are more than slightly bent, or when the intensity of stress upon their fibres exceeds the elastic limit, the theory and the method just described no longer apply, there is no sufficiently simple theory which is applicable, and we must have recourse to experiment.

Careful experiments have accordingly been made, from which it has been found that the resistance of *rectangular* beams of the same length and of similar section similarly loaded up to the *breaking* point varies directly as the breadth and as the square of the depth, *i e* as bd^2

This seems to agree with the result arrived at by reasoning in the case of beams only slightly bent by a load

the cross section, and y_o is the distance from the neutral axis to the extreme fibre subject to a stress r_o. In the case of a rectangular timber beam $y_o = \frac{d}{2}$ and $I = \frac{bd^3}{12}$, therefore $\frac{r_o I}{y_o} = r_o \frac{bd^2}{6}$, and the formula reduces to that given above

If the agreement were complete, the formula given at p. 50, *i.e.*

$$M = r_o \frac{bd^2}{6},$$

might be applied even to cases of beams loaded up to the breaking point, by taking r_o as the ultimate resistance of the fibres per square inch to tension or compression.

This formula may be and is used for beams so loaded, when an important alteration has been made in it. The coefficient r_o no longer represents the resistance of the material to tension or compression up to the elastic limit; but experiment shows that it has an entirely new value—depending upon the material, the form of beam, and disposition of the load—and that this new value can only be ascertained by experiment, and will be denoted by f_o.

Value of f_o for timber.—The method in which this value was ascertained was as follows:—

Small experimental beams each 1″ square and 12″ long between the supports were supported at the ends and loaded in the centre with a weight W.

From Equation 13 we know that in such beams

$$\text{the bending moment} = \frac{Wl}{4}.$$

Substituting f_o for r_o in Equation 28, we have the moment of resistance

$$= f_o \frac{bd^2}{6}. \quad . \quad . \quad . \quad . \quad (29).$$

Hence
$$\frac{Wl}{4} = f_o \frac{bd^2}{6},$$

or
$$f_o = \frac{Wl}{4} \times \frac{6}{bd^2}.$$

The weight W was gradually increased until the beam broke, and then the value of W, l, b, and d in the above equation being known, f_o could be found.

Modulus of rupture of timber.—The value of f_o thus found is called the *modulus of rupture* [1] of the material. It varies of course for different materials—the stronger the material the higher the modulus. Thus a beam of oak would require a greater weight W to break it than a beam of pine, and f_o would therefore be greater.

It is not accurately known why the *modulus of rupture* thus found by experiment differs so widely, as it does in many instances, from the resistance of the fibres to either tension or compression.

It must be remembered, however, that the fibres of beams are not in-

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[1] The modulus of rupture thus found is always equal to 18 times the weight W that will break the beam across. The reason for this is easily seen if the values of l, b, and d for the experimental beam are substituted in Equation 29 above—

$$l = 12'',$$
$$b = 1'',$$
$$d = 1''.$$

$$f_o = \frac{Wl}{4} \times \frac{6}{bd^2},$$
$$= \frac{W \times 12}{4} \times \frac{6}{1 \times 1^2} = 18W.$$

dependent filaments, but are united together laterally with an adhesion less or greater according to the nature of the material

This lateral adhesion has been shown by Mr Barlow to lead to stresses of a very complicated character, which so far have not been dealt with by any simple mathematical theory, and its effect is practically to alter the value of the coefficient f.

There is also another consideration The strength of the individual fibres cannot be uniformly the same, as it is assumed to be So long as the *limit of elasticity is not passed, this difference of strength is so small as not to* affect the result, but after that point the weaker fibres begin to yield more than the others, and finally, just before rupture, these weaker fibres break first and leave the whole strain to come successively upon fewer and fewer fibres, until complete rupture takes place Under these conditions, f, *cannot* be a constant coefficient.

The value of f, obtained by the experiments above referred to is gener*ally used in practice, though it is in most cases too high, for the following reason*

The small beams (speaking of timber) experimented upon were as a rule selected specimens perfectly dry, free from defects, such as large or dead knots, etc., which tend to interrupt the continuity of the fibres and thus weaken the beam

Large scantlings cannot be obtained so well seasoned or free from defects as the small experimental beams, and therefore in calculating the strength of large beams the value of f, should be taken considerably lower than the value found for the experimental selected beam of the same material

For example, in the case of Red pine, the modulus of rupture found by experiments upon small selected beams, varies from 7100 to 9540 lbs.[1], whereas the average modulus found by breaking large beams is 45 cwts or 5040 lbs per square inch [2]

It would be expensive to ascertain the modulus by breaking very large beams, and after all, even if this were done, the timber of the beams used would probably not be exactly similar to that experimented upon, and would therefore have a different modulus

It will be seen, then, that the method of ascertaining and applying the modulus of rupture is only a rough way of correcting the imperfections of the theory upon which the formula is based

The method answers, however, sufficiently well in practice, and it has the advantage of being very simple An engineer in a distant country having to deal with an entirely new material can at once ascertain its modulus of rupture by experimenting upon small bars as above described, and having found the modulus he can use the ordinary formula given above

It is far easier for him to find the modulus of rupture in this way than to find the amount of extension and compression of the material under different stresses—which would be necessary if the perfect mathematical theory could be worked upon

Modulus of rupture for different sections of beam —In using the *modulus of rupture* thus obtained by experiment, it must be remembered that it is only applicable in the calculation of beams of similar section to the experimental beam from which the modulus was deduced

In cases where the modulus obtained from a rectangular beam of cast iron placed thus ▯ was applied to the calculation of the resistance of a beam with a circular sec

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[1] Rankine's *Useful Rules and Tables*, p 199 [2] Seddon's *Builders Work*

tion, the error caused was found to be 174 per cent, and in a case where it was applied to a square beam placed on edge, thus, ◇, the error was 190 per cent.

In practice, however, beams of these sections are not used for building construction in any material, and they need not be further considered.

Even a difference in the method of loading the beam used, as compared with the method in which the experimental beam was loaded to find the modulus, has been found to cause errors in results; but these errors are comparatively small, and may be neglected in using a method which after all is merely a rough and ready way of arriving at the result.

The moduli of rupture given in Table I. are those obtained by breaking beams by means of weights placed on their centres, but these moduli may, without material error, be used in calculating beams with any distribution of load.[1]

Practical Formula.—In practice, the rough and ready way of using the formula (Equation 29) in conjunction with the modulus of rupture is as follows:—

To find the weight that will break a beam.—Find from pp. 27 to 40 the bending moment, M, for the given conditions of load—equate this with the moment of resistance, $\overline{M}$ (Equation 29, p. 52), and we have

$$M = \frac{f_o b d^2}{6} \qquad . \quad . \quad . \quad . \quad (30).$$

Take the case of a beam supported at the ends and loaded at the centre,[2]

W^1 = breaking weight in centre

$\left.\begin{array}{l} l = \text{length} \\ b = \text{breadth} \\ d = \text{depth} \end{array}\right\}$ all in the same dimensions, either feet or inches,

f_o = modulus of rupture for the material (see Table I.)

$$M = \frac{W^1 l}{4} \text{ (see Equation 13, p. 33)}$$

$$\frac{W^1 l}{4} = \frac{f_o b d^2}{6}$$

$$W^1 = \frac{f_o 4 b d^2}{6l}$$

To find the load that a beam can safely bear.—W^1 is, however, the *breaking* load. To find the *safe* load the factor of safety F must be introduced. Taking W as the *safe* load with that factor.

$$FW = \frac{f_o 4 b d^2}{6l}$$

$$W = \frac{f_o 4 b d^2}{F \times 6l}.$$

To find the section of a beam to bear safely a given weight.—Take the same case as before—a beam supported at ends and loaded in centre.

$$F \times \frac{Wl}{4} = \frac{f_o b d^2}{6}$$

$$bd^2 = \frac{6FWl}{4f_o}.$$

From this find bd^2, and assuming a value for either b or d, find the other, which will give the sectional dimensions of the beam.

Examples of the calculation of beams are given at p. 75 and seq.

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[1] For moduli of rupture of other materials, see Part III.

[2] It must be clearly understood that M will vary according to the arrangement of the beam and loads. Its different values are given at pp. 27 to 40, also in a concise form in the Table with Appendix VII. (see also Appendix XXI.)

SHEARING STRESS

We have hitherto considered only the effect of the direct stresses upon a beam that is, the stresses which eventually act upon the fibres by extending or compressing them in the direction of their length

There are, however, other stresses acting upon a beam the existence of which will be rendered evident upon considering the next three diagrams

Vertical Shearing Stress —If a beam AB, Fig 76, supported at the ends, be bent the fibres above the neutral layer are evidently compressed and those below it extended, but no other stresses in the beam are apparent to the observer

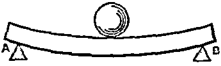


Fig 76

If, however, the continuity of the fibres be interrupted by cutting the beam into vertical slices, these slices, when the beam is loaded, will slip one past the other as shown in Fig 77

In the uncut beam there must be the same *tendency* for each section to slide vertically upon the next, although *actual sliding* is prevented by the continuity of the fibre or cohesion of the

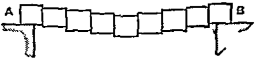


Fig 77

Fig 78

particles The *force* causing this tendency is called the *vertical shearing stress*

Horizontal Shearing Stress —In the same way if the beam be divided into different layers of fibres by cutting it into planks the planks or layers of fibres will slide upon each other as in Fig 78, when loaded

In the uncut beam there must be a *tendency* to slide in the same way, and the force causing this tendency is called the *horizontal shearing stress*

Rules for finding Amount of Vertical Shearing Stress [1]

The amount of the vertical shearing stress is easily ascertained by the application of two simple rules—

[1] *The practical formulæ* for finding the shearing stresses are on pp 57 to 60 numbered (31) to (43C) each being in connection with figures showing the distribution of loads diagrams of shearing stresses etc

1. VERTICAL SHEARING STRESS IN CANTILEVERS.—*The shearing stress at any section is equal to the weight on the beam between that section and the outer end.*

2. VERTICAL SHEARING STRESS IN BEAMS SUPPORTED AT EACH END.—*The shearing stress at any section is equal to the difference between the reaction at either support and the weight between that support and the section in question.*

These rules hardly require any proof, but it may make them more clear to examine a case of each.

Shearing stress in cantilevers.—AB, Fig. 79, is a cantilever, or a beam loaded at the end with a weight W. We may divide the beam into imaginary vertical sections at *ki*, *hg*, *fe*; these are only a few out of an infinite number of sections infinitely near to one another.

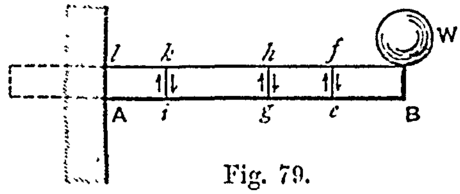


Fig. 79.

Now W acting downwards tends to shear off the piece B*f*, with a force W, acting, as shown by arrow thus, ↓, on the section *fe*; this is resisted by the opposite surface of the section *fe*, the resisting force acting as shown by the arrow ↑ upwards.

Supposing then that the resistance prevents the shearing at surface *fe*, consider the next surface *hg*. Here the load W still acts with a force W downwards, tending to shear off the piece B*h*.

Again in the same way at *ki*, W the load acts downwards, and a resistance = W acts upwards, as shown by the arrows. Nothing has intervened to increase or diminish the force W caused by the load.[1] The same takes place at any intermediate section, right back to A*l*.

The shearing stress at any section from B to A is therefore always the same, *i.e.* = W as shown graphically in Fig. 82.

Shearing stress in supported beams.—As regards the second rule, take the case of a beam (Fig 80) supported at the ends and uniformly loaded.

Supposing the beam to be 8′ span and loaded with *w* lbs. per foot run:

R_A and R_B each $= 4w$.

At the section marked 1↑↓1 the reaction $R_A = 4w$ acts upwards ↑, and is

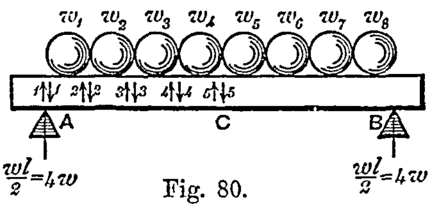


Fig. 80.

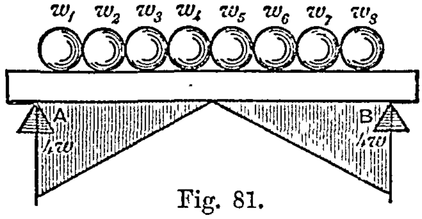


Fig. 81.

resisted by the four weights $4w$ acting downwards, so that the shearing force at section $1 = R_A$.

[1] The weight of the beam itself does really increase it, but we have arranged to neglect it as insignificant. If it is not insignificant it would cause a shearing stress similar to that caused by a distributed load, as described on the next page.

At the section marked $_2\!\!\uparrow\!\downarrow^2$ one fourth of the upward force of the reaction $R_A = 4w$ *is cancelled by the weight* w_1 *acting downwards, so that the* upward force at the section 2 is $3w$, which is resisted by $3w$ acting downwards between it and the centre

Similarly at section 3 the shearing force is $(R_A - 2w) = 4w - 2w = 2w$, and at section 4 it is $1w$

At section 5 (the centre) the upward force of the reaction is quite neutralised by the four weights w_1, w_2, w_3, w_4 acting downwards and the shear*ing force* = 0

Thus at each section the shearing force $= R_A$ − weights between the section and support A, as stated in Rule 2 above The same reasoning applies to the part CB of the beam

The shearing stress is evidently decreasing from $4w$ at the supports to 0 at the centre, and may be graphically shown as in Fig 81

The truth of the rules is so apparent, and their application to different cases so simple, that it will be sufficient to show a few of the most useful cases graphically, giving at the same time the value of the shearing stress in terms of the weights and lengths of the beams

The shearing stress at any point P is denoted by S_P, and at any other points A, B, C by S_A, S_B S_C, etc respectively

Graphic Representations[1] of Vertical Shearing Stress, and Values for the Same

Case 1 (Fig 82)—BEAM FIXED AT ONE END AND LOADED AT THE OTHER

Shearing stress $S = W$ throughout (31)

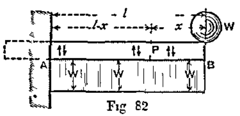


Fig 82

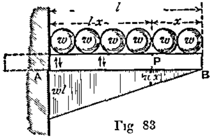


Fig 83

Case 2 (Fig 83)—BEAM FIXED AT ONE END AND LOADED *UNIFORMLY THROUGHOUT ITS LENGTH*

$S_P = wx$. The point P being distant x from B (32)

$S_A = wl$ (32 A)

Case 3 (Fig 84)—BEAM FIXED AT ONE END, LOADED AT THE OTHER END AND ALSO UNIFORMLY

A combination of Cases 1 and 2 The loads are omitted in the figure for clearness, but are as in Figs 82 and 83

[1] These are merely graphic representations of results obtained algebraically, but in Appendix VI will be seen a graphic method of obtaining the shearing stresses

$$S_P = W + wx \quad . \quad . \quad . \quad . \quad . \quad (33).$$
$$S_A = W + wl. \quad . \quad . \quad . \quad . \quad (33\ A).$$

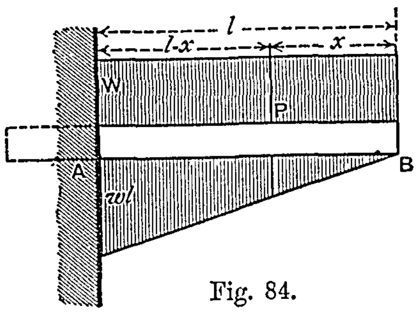


Fig. 84.

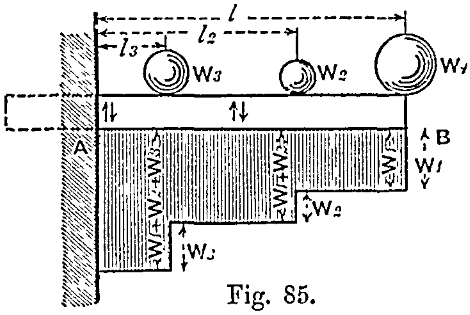


Fig. 85.

Case 4 (Fig. 85).—BEAM FIXED AT ONE END AND LOADED WITH SEVERAL CONCENTRATED WEIGHTS W_1, W_2, W_3.

$$S_A = W_1 + W_2 + W_3 \quad . \quad . \quad . \quad (34).$$

S at any section = the sum of the loads between that section and the outer end of the beam. . . . . (34 A).

Case 5 (Fig. 86).—BEAM FIXED AT ONE END AND LOADED UNIFORMLY OVER A PART OF ITS LENGTH.

The shearing stress at any point P under the load distant x from A is

$$S_P = w(z + y - x), \quad . \quad . \quad . \quad (35),$$

and

$$S_A = wz, \quad . \quad . \quad . \quad . \quad . \quad (35\ A).$$
$$S_B = wz. \quad . \quad . \quad . \quad . \quad . \quad (35\ B).$$

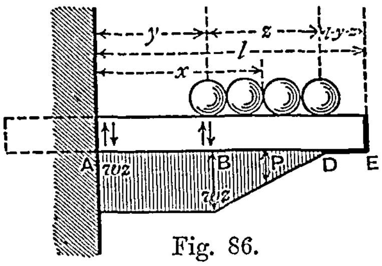


Fig. 86.

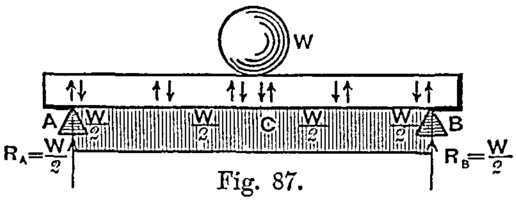


Fig. 87.

Case 6 (Fig. 87).—BEAM SUPPORTED AT BOTH ENDS AND LOADED IN THE CENTRE.[1]

$$S \text{ throughout} = \frac{W}{2}. \quad . \quad . \quad . \quad (36),$$

except at C where the stress changes direction and

$$S_C = 0 \quad . \quad . \quad . \quad . \quad (36\ A).$$

The arrows indicate the different directions of the shearing stress on the opposite side of the centre. Thus anywhere in CA the stress is thus $\uparrow\downarrow$ tending to shear the beam as in Fig. 88, whereas in the half BC the stress is thus $\downarrow\uparrow$ tending to shear the beam as in Fig. 89. At C the stresses in opposite directions cancel one another, and there is no shear.

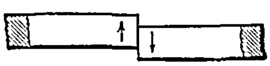


Fig. 88.

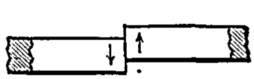


Fig. 89.

[1] See Appendix V.

Case 7 (Fig. 90).—BEAM SUPPORTED AT THE ENDS AND UNIFORMLY LOADED.

See the remarks on p. 56 with regard to this case.

$$S_A = \frac{wl}{2}, \quad . \quad . \quad . \quad . \qquad (37)$$

$$S_B = \frac{wl}{2}, \quad . \quad . \quad . \quad . \qquad (37\ A)$$

$$S_x = \frac{wl}{2} - wx \quad . \qquad (37\ B),$$

$$S_c = 0 \quad . \quad . \quad . \quad . \quad . \quad . \qquad (37\ C)$$

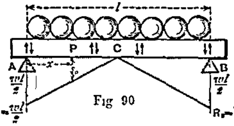


Fig 90

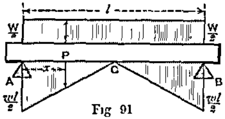


Fig 91

Case 8 (Fig. 91).—BEAM SUPPORTED AT BOTH ENDS, LOADED IN THE CENTRE AND ALSO UNIFORMLY.

This case is a combination of 6 and 7. The load is omitted for clearness

$$S_A = S_B = \frac{wl}{2} + \frac{W}{2}, \qquad (38).$$

$$S_x = \frac{wl}{2} - wx + \frac{W}{2} \qquad (38\ A)$$

$S_c = 0$ (stress changing direction) (38 B)

Case 9 (*Fig. 92*).—*BEAM SUPPORTED AT BOTH ENDS AND LOADED AT ANY POINT.*

Between A and D

$$S_x = R_A = \frac{m}{l} W \quad . \qquad (39)$$

Between B and D

$$S_x = R_B = \frac{l - m}{l} W \quad . \qquad (40)$$

At D $\qquad S_D = 0$ (stresses changing direction) (40 A).

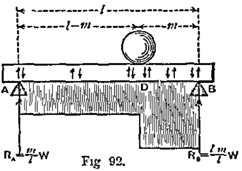


Fig 92.

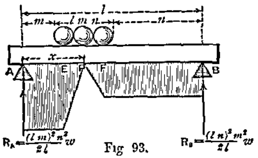


Fig 93.

Case 10 (Fig. 93).—BEAM SUPPORTED AT BOTH ENDS AND PARTIALLY LOADED WITH A UNIFORM LOAD.

Between A and E

$$S = R_A = \frac{(l-m)^2 - n^2}{2l} \cdot w \qquad . \quad . \qquad (41).$$

Between B and F

$$S = R_B = \frac{(l-n)^2 - m^2}{2l} \cdot w \qquad . \quad . \qquad (41\ A).$$

At any section P under the partial load distant x from A

$$S_P = \left\{ \frac{(l-m)^2 - n^2}{2l} - (x-m) \right\} w \qquad . \qquad (42).$$

Referring to p. 38 it will be a good exercise for the student to deduce the previous cases of beams supported at both ends from this case.

Case 11 (Fig. 94).—BEAM SUPPORTED AT BOTH ENDS AND LOADED WITH ANY NUMBER OF CONCENTRATED LOADS.

Between A and D

$$S_A = R_A = \frac{W(l-m_1)}{l} + \frac{W_2(l-m_2)}{l} + \frac{W_3(l-m_3)}{l} \qquad . \qquad (43).$$

Between D and E

$$S = R_A - W \qquad . \quad . \quad . \quad . \qquad (43\ A).$$

Between E and F

$$S = R_A - W - W_2 \qquad . \quad . \quad . \qquad (43\ B).$$

Between F and B

$$S = R_B = \frac{Wm_1}{l} + \frac{W_2m_2}{l} + \frac{W_3m_3}{l} \qquad . \qquad (43\ C).$$

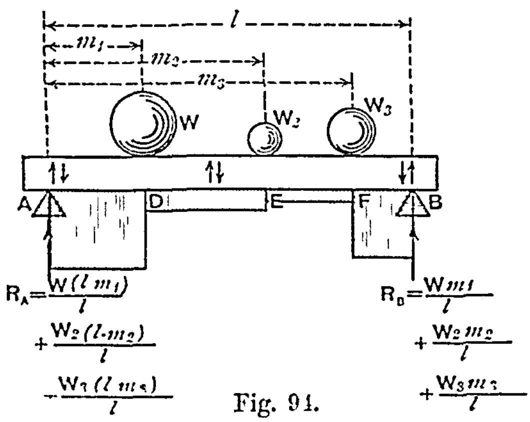


Fig. 94.

Horizontal Shearing Stress.—We have hitherto considered only the vertical shearing stress, which tends to make sections of a loaded beam slide vertically upon one another, as in Fig. 77.

We have now to consider the horizontal shearing stress, which tends to make the layers slide horizontally upon one another, as in Fig. 78.

This horizontal shearing stress is at every point in the beam equal to the vertical shearing stress at that point

This is proved thus —On or near the neutral layer of an unloaded rectangular beam draw a little square, Fig 95 If the beam is then loaded until it bends slightly, this square will be distorted into a rhombus Fig 96 Now we know that at the neutral layer of a rectangular beam there are no direct stresses

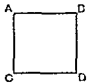


Fig 95

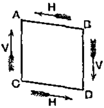


Fig 96

The distortion must therefore have been produced by *the vertical shearing stresses* V, V acting as shown by the arrows, and it is opposed by the horizontal shearing stresses H H Now since there is equilibrium, H and H must evidently be equal to V and V, or the horizontal shearing stress must be equal to the vertical shearing stress at the point.[1]

We know then the total amount of shearing stress at each section—the vertical is found as before explained and the horizontal is equal to it

Distribution of Shearing Stress—It is necessary to know further how these shearing stresses are distributed over the section

It can be proved mathematically that the shearing stress is distributed over the section as shown in Fig 97

The curve is a parabola the relative proportion of whose ordinates is shown in the diagram The area[2] of the parabola represents to scale the total shearing stress at the section

From this the horizontal shearing stress at any point can easily be ascertained Thus suppose the total shearing stress S at any section of a beam 12 deep is 2 tons Then

$$\tfrac{2}{3}\,NL \times CD = S$$

$$\tfrac{2}{3} \times NL \times 12 = 2 \text{ tons}$$

$$NL = 2 \text{ tons} \times \tfrac{3}{2} \times \tfrac{1}{12}$$

$$= \tfrac{6}{24}$$

$$= \tfrac{1}{4} \text{ ton}$$

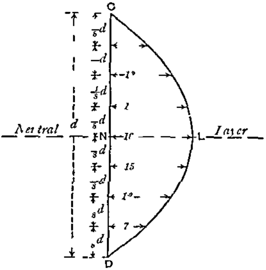


Fig 97

Having thus found the stress on NL to be $\frac{1}{4}$ ton we make a scale on which the length NL $= \frac{1}{4}$ ton, and we can find the stress at any other point by measuring the ordinate of the parabola at that point

It is unimportant in most cases to know the exact distribution of the shearing stress over the section, but we can see from the diagram that it is greatest at the centre and vanishes at the top and bottom layers of the beams

---

[1] Cunningham p 244

[2] The area of a parabola $= \frac{2}{3}$ the circumscribed rectangle The area $= \frac{2}{3} \times CD \times NL$

This is exactly the converse of the distribution of the direct stresses. The difference is clearly shown by the diagrams, Figs. 98 and 99.

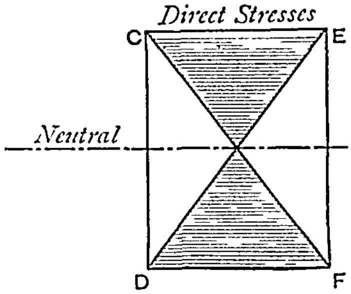


Fig. 98.

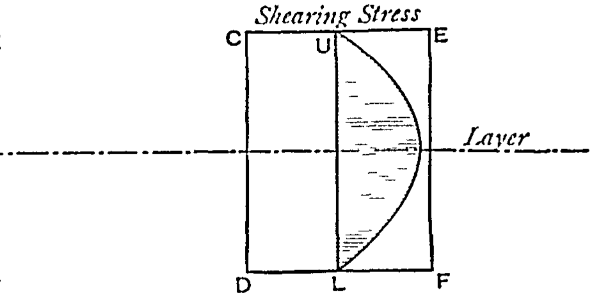


Fig. 99.

We see from these diagrams that the shearing stress is greatest along the neutral layer where the direct stress is the least, and least at the upper and lower fibres where the direct stress is greatest.

In Fig. 99 the amount of the shearing stress is exaggerated in order that it may be clearly shown, but its amount is generally very small in comparison with the direct stresses; so small that in rectangular beams which have so much more substance near the centre than is required to meet the direct stresses, there is sure to be plenty to meet the small shearing stress, and its consideration may therefore safely be neglected.

In built-up iron beams, however, it is different; in those the shearing stress plays an important part (see p. 157).

BEAMS OF UNIFORM STRENGTH.

If we glance at the graphic representation of the bending moments at the different points of a beam supported at the ends and uniformly loaded throughout its length, Fig. 100, we see that the bending moment is greatest at the centre and gradually diminishes towards the ends, and that in fact exactly at the points of support there is no bending stress whatever.

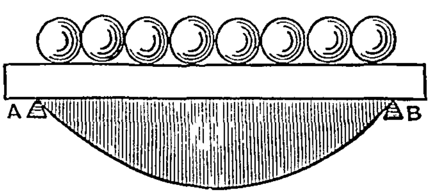


Fig. 100.

If, therefore, the beam be made of the same section throughout, it is evident that it is unnecessarily strong at every point except the centre, and that it may be reduced in section as it approaches the points of support, because the bending moment becomes less and less as those points are approached.

There are three ways in which the beam may be reduced so as to make it of equal strength throughout its length—

1. By varying the depth, keeping the breadth the same; as in Figs. 102-105.

2. By varying the breadth, keeping the depth the same; as in Figs. 106-109.

3 By varying both breadth and depth

The same observations apply to other cases of loading Thus, for example, in a cantilever of rectangular section with either a single or a uniform load, there is superfluous material except near the point of fixing (see Figs 102, 103), and in a supported beam carrying an isolated load there is superfluous material except under the load

In all such beams then a certain portion of the material is superfluous, and can be cut away without injuring their strength

In timber beams, as a rule, it does not pay to cut away this superfluous material, the cost of labour being greater than the value of the material saved, but in some cases for the sake of appearance, or in large beams to reduce the weight it may be desirable

It can, however, never be necessary to cut timbers to the exact curves shown These curves are the forms which are theoretically correct, but practically they are valuable only as dividing the parts of the beam which have no excess of strength from those parts which are unnecessarily strong and can therefore be cut away, thus making a beam of approximate uniform strength

The following figures show the shapes of beams of equal strength throughout their length so far as the *direct* stresses only are concerned, and in each case ignoring the shearing stress and the weight of the beam itself

The *shearing stresses would necessitate some substance being left on at the* ends of the supported beams

It is not difficult to prove mathematically that beams of equal strength throughout their length are of the forms shown but such mathematical proof would here take up more space than can be afforded for it, and the question is not one of much practical importance Case 1 (p 28) is, however, worked out as follows, and the other cases can be worked out in a similar manner

Calculation to ascertain the form of a cantilever of equal strength throughout its length, loaded at one end and of the same breadth throughout

Bending moment at any point P (Fig 101) distant x from the end W = Wx.

Moment of resistance at P = $\frac{1}{6}f_0by^2$

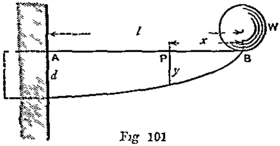


Fig 101

Then since at any point bending moment = moment of resistance, we have—
At P: $Wx = \frac{1}{6}f_o by^2$. At A: $Wl = \frac{1}{6}f_o bd^2$.

$$\therefore \frac{Wx}{Wl} = \frac{\frac{1}{6}f_o by^2}{\frac{1}{6}f_o bd^2},$$

$$\frac{x}{l} = \frac{y^2}{d^2},$$

$$y^2 = \frac{d^2}{l}x,$$

which is the equation to a parabola with its vertex at B; the under side of the beam would therefore be shaped to this curve.

Shapes of Beams of uniform Strength.

a. WHEN THE BREADTH IS CONSTANT THROUGHOUT THE LENGTH OF THE BEAM, BUT THE DEPTH VARIED TO SUIT THE VARYING STRESS.

Figs. 102-105 show the disposition of load and the shape of the beam in elevation, the dotted line being the beam of constant depth and breadth as calculated.

In Fig. 102, AB is a parabola with the vertex at B.
In Fig. 103, AB is a straight line.

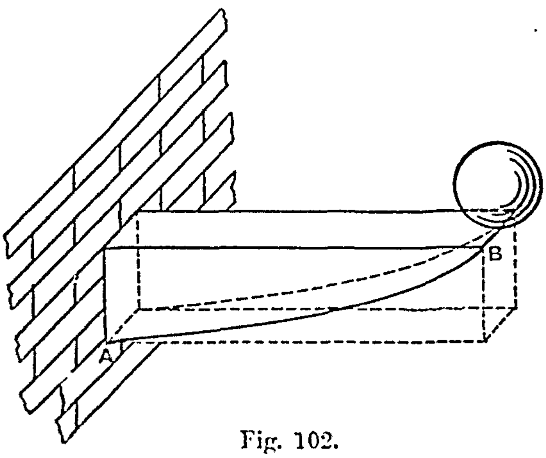


Fig. 102.

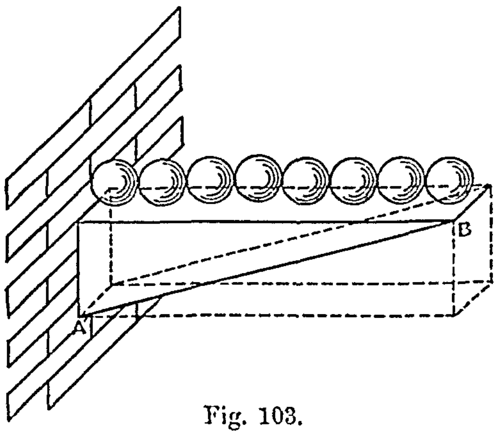


Fig. 103.

In Fig. 104, AP and PB are parabolas with their vertices at A and B.

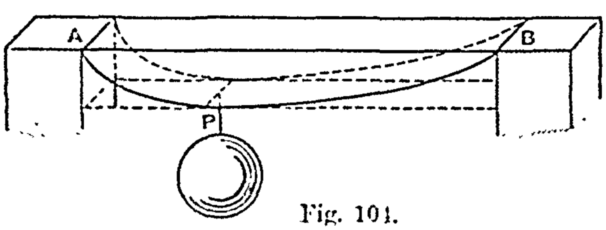


Fig. 104.

In Fig. 105, ABC is a semi-ellipse, AB being the major axis.

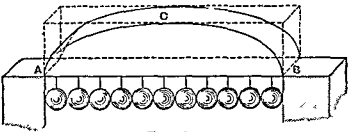


Fig. 105.

b. WHEN THE DEPTH IS KEPT CONSTANT THROUGHOUT THE LENGTH OF THE BEAM, AND THE BREADTH VARIED TO SUIT THE VARYING STRESS.

Figs. 106-109 show the disposition of load and the shape of the beam in plan.

In Fig. 106, ABD is a triangle.

In Fig. 107, AB, DB are both parabolas with vertices at B.

In Fig. 108, DAC and CBD are two triangles with their bases at DC.

In Fig. 109, ACB and ADB are parabolas with their vertices at the centre points C, D.

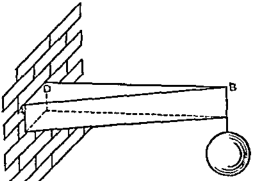


Fig 106.

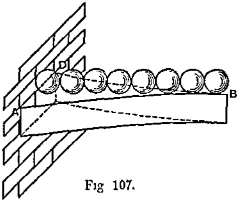


Fig 107.

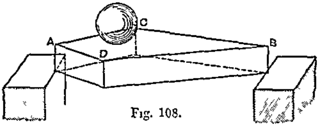


Fig. 108.

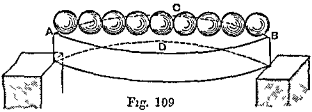


Fig. 109

DEFLECTION.[1]

In many structures it is necessary that the beams, whether of wood or iron, forming part of the structure, should be not only *strong* enough, but *stiff* enough; that is, not only must rupture be prevented, but the beams must not bend too much.

For example, it would never do if the joists supporting a ceiling were to bend beyond a certain small amount under the load, although they might be *strong* enough, because a very slight bending is sufficient to cause cracks in the ceiling below. Such joists must therefore be *stiff* as well as *strong*.

It is necessary, therefore, to be able to calculate how stiff a given beam is, that is, how much it will bend under its load, in order to be able to judge whether the amount of bending is likely to cause inconvenience; and if so, to increase the size of the beam in such a manner as to prevent such excessive and inconvenient bending.

The amount of deflection that may be expected in a beam of known form and material, with a given amount and distribution of load, can easily be ascertained by the use of the formulæ given below. The investigation of the formulæ, however, involves the use of the Calculus; their proof will therefore not be attempted in this work.

The student must be content to accept these formulæ, which are based upon those given by Professor Rankine, after investigations made by himself and other mathematicians.

Formulæ for ascertaining the Deflection of Beams of any Kind.

General Formula.—The general formula applicable to all kinds of beams, of any material and with various distributions of load, is

$$\Delta = \frac{nWl^3}{EI} \quad . \quad . \quad . \quad . \quad (44).$$

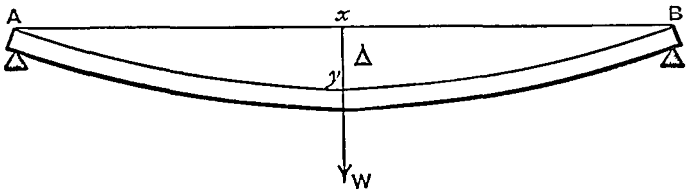


Fig. 110.

In this formula

Δ = the maximum deflection in inches. Thus in Fig. 110, if the beam AB deflect as shown by the arc AyB, the line xy is denoted in the above formula by Δ.

W = the total load on the beam.

---

[1] *The formulæ for practical use* are Equations 44, 45, 46, 47, pp. 66, 67, 68, and in Appendices VIII., IX., XXI. *Examples* are given, pp. 76 to 80.

l = the length of the beam in inches

E = the value of the modulus of elasticity of the material expressed in the same units as W

I = the moment of inertia about the neutral axis of that section of the beam where the greatest stress occurs with the given distribution of load

n is a coefficient, the value of which varies for each class of beam and for each distribution of the load

The following Table shows the different values of n for three classes of beams and for four distributions of load

TABLE C

Arrangement of Beam and Load	Class of Beam: 1. Uniform cross section throughout its length See Figs 61a and 176	2. Uniform strength Same depth throughout its length See Figs 106 to 109	3. Uniform strength Same breadth throughout its length See Figs 102 to 105
Fixed at one end, loaded at the other	$\frac{1}{3}$	$\frac{1}{2}$	$\frac{2}{3}$
" " loaded uniformly	$\frac{1}{8}$	$\frac{1}{4}$	1
Supported at both ends, loaded in the middle	$\frac{1}{48}$	$\frac{1}{32}$	$\frac{1}{24}$
" " loaded uniformly	$\frac{5}{384}$	$\frac{1}{64}$	[illegible]

Maximum deflection under any load—To find the maximum deflection of any beam under a given load it is merely necessary to substitute the values of the different letters, and the result gives the value of Δ.

The values of W and l are of course known, E is given in Table I, and I, the moment of inertia, can be found by means of the information given in Appendix XIV

The deflection under proof load, or under loads which produce a known stress, may be ascertained from the following formulæ—

$$\Delta = \frac{n\, r_o l^2}{E y_o} \tag{45}$$

for all beams, and

$$\Delta = \frac{n\,(r_c + r_t) l^2}{E d_s} \tag{46}$$

for beams with cross sections of equal strength [1]

In these formulæ r_o is the greatest stress on the weakest side of the beam, y_o is the distance of the neutral axis from the extreme fibre of the beam on that weakest side at the section of greatest stress, r_c, r_t are the limiting stresses in compression and tension respectively, d_s the depth of the beam at the section of greatest stress The other letters have the same signification as in Equation 44

The following Table shows the different values of n for three classes of beams and for four distributions of load

[1] These are beams of such a form that the limiting stress is reached on the uppermost and lowermost fibre of the beam at the same instant

TABLE D.

Arrangement of Beam and Load.	Class of Beam.		
	1 Uniform cross section throughout its length. See Figs. 61*a* and 126.	2 Uniform strength and same depth throughout its length. See Figs. 106 to 109.	3 Uniform strength and same breadth throughout its length. See Figs. 102 to 105.
Fixed at one end, loaded at the other . .	$\frac{1}{3}$	$\frac{1}{8}$	$\frac{2}{3}$
,, ,. uniformly loaded . .	$\frac{1}{4}$	$\frac{1}{8}$	1
Supported at both ends, loaded in the middle	$\frac{1}{12}$	$\frac{1}{6}$	$\frac{1}{6}$
,, ,, uniformly loaded .	$\frac{5}{48}$	$\frac{1}{8}$	$\frac{1}{7}$

Simpler Formula.—For ordinary rectangular beams of the same section throughout their length, such as wooden beams nearly always are, the formula 44 can be much simplified by substituting for the moment of inertia I its value for a rectangular section, namely $\frac{bd^3}{12}$. (See App. XIV. and IX.)

Equation 44 becomes $$\Delta = \frac{12nWl^3}{Ebd^3} \quad . \quad . \quad . \quad . \quad . \quad (47).$$

Comparison between strength and stiffness.—We found that the *strength* of rectangular timber beams varied as $f_0\frac{bd^2}{l}$, *i.e.* directly as the breadth, as the square of the depth, and as the modulus of rupture of the material, but inversely as the length.

By examining Equation 47 we see that the amount of deflection of rectangular beams varies as $\frac{l^3}{Ebd^3}$: that is, the greater the length the greater the deflection ; the greater the modulus of elasticity, the breadth, or the depth, the less the deflection.

The deflection arises from want of stiffness, therefore the *stiffness* or resistance to deflection will vary according to an exactly opposite set of conditions—in fact, it varies as $\frac{Ebd^3}{l^3}$. It will be greater directly as E is greater (thus an oak beam, for which E is greater than for a similar fir beam, will be stiffer than the fir beam), also directly as the breadth and cube of the depth are greater ; but, on the other hand, it will be less as the cube of the length is greater : that is, of two beams of the same section and material, the longer will be less stiff than the other, not in proportion to its length, but of the *cube* of its length.

Recapitulating, we see that for beams of rectangular cross section

The strength of beams varies as $\frac{f_o bd^2}{l}$,

,, stiffness ,, ,, $\frac{Ebd^3}{l^3}$.

Further, since the strength varies as bd^2 and the stiffness as bd^3, we see that by reducing the breadth and increasing the depth in such a way that the strength remains constant, we can obtain stiffer beams. Thus a beam 4 inches broad and 6 inches deep has the same strength as one 1 inch broad and 12 inches deep

From A to *f* and from B to *i* the bottom surface of the beam is concave; from *f* to *i* it is convex.

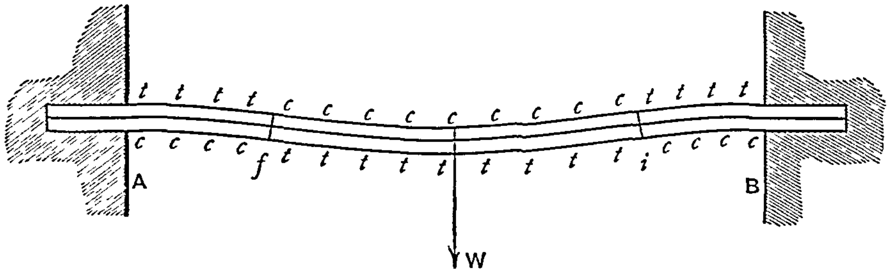


Fig. 112.

Moreover, the stresses upon the beam are different from those of the supported beam. From A to *f* and from B to *i* the upper surface is in tension (*ttt*), the lower surface is in compression (*ccc*). From *f* to *i* the upper surface is in compression (*ccc*), and the lower surface is in tension (*ttt*).

It will be seen that at the points *f* and *i*, where the curvature changes, the nature of the stress also changes; the upper surface, which from A to *f* and from B to *i* was in tension, changes to compression, and the lower surface, which was under compression, to tension.

These points, where the curvature and the stress change, are called the points of *contra-flexure.*

Their distance from A and B varies according to the shape of the beam and the nature of the load. When that distance is known, the calculation of the bending moments, etc., becomes a simple matter.

In fact, the beam thus fixed and loaded is exactly in the condition of two cantilevers A*f* and B*i*, carrying a beam *fi* between them, which is supported at its ends *f* and *i* by hanging from the ends *f* and *i* of the cantilevers.

The cantilevers may be calculated as shown in Case 1, p. 28, and the supported portion as in Case 6, p. 32.

The only difficulty, therefore, is to ascertain the exact distances of the points of *contra-flexure* from A and B.

Position of points of contra-flexure in fixed beams with different distributions of loads.—Without making any attempt to describe the mathematical investigations by which the positions of the points of contra-flexure are ascertained, we will give their positions for different classes of beams with various distributions of loads.

TABLE E.

DISTANCE OF POINTS OF CONTRA-FLEXURE.

Arrangement of Beam and Load.		Class of Beam. Uniform cross section.	Class of Beam. Uniform strength, with uniform depth.	No. of Fig.
		Distance of points of contra-flexure.		
Fixed at both ends.	Load in centre .	0·25 l from A and B	0·25 l from A and B	Fig. 113
" "	Loaded uniformly	0·211 l " "	0·25 l " "	Fig. 115
Fixed at one end and supported at the other	Load in centre .	0·273 l from A	0·33 l from A	..
Fixed at one end and supported at the other	Loaded uniformly	0·267 l from A	0·33 l from A	Fig. 117

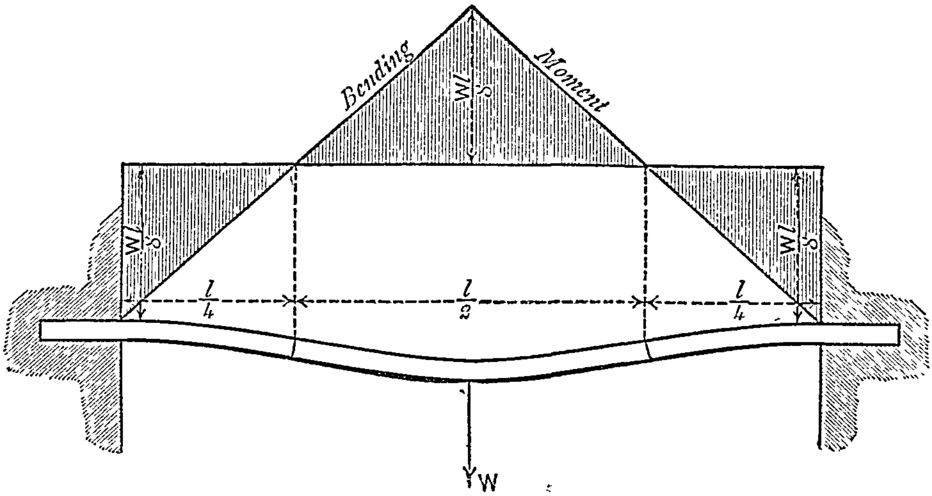


Fig. 114.

Shearing stresses.—In this case the shearing stresses are the same as in a beam supported at both ends and loaded in the centre (see Appendix VIII.)

Deflection.—It can be shown that the maximum deflection is $\frac{1}{4}$th of the maximum of a similar beam supported at both ends (see Appendix VIII.)

Case 2.—BEAM OF UNIFORM CROSS SECTION FIXED AT BOTH ENDS AND UNIFORMLY LOADED.

In this case the points of contra-flexure are distant $\cdot 211l$ from A and B, Fig. 115.

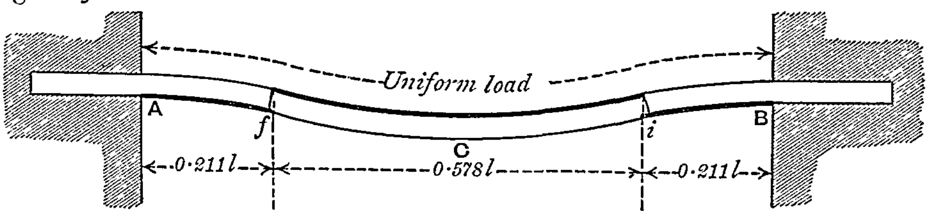


Fig. 115.

M_A = weight distributed on cantilever × leverage + weight at end × length of cantilever,

$$= w \times \cdot 211l \times \frac{\cdot 211l}{2} + \frac{w \times \cdot 578l}{2} \times \cdot 211l,$$

$$= 0\cdot 022wl^2 + 0\cdot 061wl^2,$$

$$= 0\cdot 083wl^2 = \frac{wl^2}{12} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (51).$$

M_C = Reaction × leverage − weight between C and f × leverage,

$$= \frac{\cdot 578wl}{2} \times \frac{\cdot 578l}{2} - \frac{\cdot 578wl}{2} \times \frac{\cdot 578l}{4},$$

$$= \frac{0\cdot 334wl^2}{8} = \frac{wl^2}{24} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (52).$$

$\frac{wl^2}{12}$ and $\frac{wl^2}{24}$ are the exact values of M_A and M_C, as can be proved by another method; the fraction 0·211 is only approximate.

TABLE F.

Arrangement of Beam and Load.		Class of Beam. Uniform cross section. Bending moment		Class of Beam. Uniform strength, with uniform depth. Bending moment	
		At fixed ends.	In centre.	At fixed ends.	In centre.
Fixed both ends.	Load in centre	$\frac{1}{8}$ Wl	$\frac{1}{8}$ Wl	$\frac{1}{8}$ Wl	$\frac{1}{8}$ Wl
" "	Loaded uniformly	$\frac{1}{12}$ wl^2	$\frac{1}{24}$ wl^2	$\frac{1}{16}$ wl^2	$\frac{1}{16}$ wl^2
" "	Loaded uniformly and in centre	$\frac{1}{8}$ Wl+$\frac{1}{12}$ wl^2	$\frac{1}{8}$ Wl+$\frac{1}{24}$ wl^2	$\frac{1}{8}$ Wl+$\frac{1}{16}$ wl^2	$\frac{1}{8}$ Wl+$\frac{1}{16}$ wl^2
Fixed one end and supported at other	Load in centre	$\frac{3}{16}$ Wl	$\frac{5}{32}$ Wl	$\frac{1}{4}$ Wl	$\frac{1}{8}$ Wl
" "	Loaded uniformly	$\frac{1}{8}$ wl^2	$\frac{9}{128}$ wl^2 In centre of supported portion	$\frac{wl^2}{6}$	$\frac{wl^2}{18}$ In centre of supported portion

Shearing Stress and Deflection.—For these see Appendix VIII.

Objections to fixed beams in practice.—Fixing the ends of beams is generally objectionable in practice. In the case of timber, confining the ends keeps them from the air and causes them to rot; in the case of iron beams it prevents them from expanding and contracting freely.

There are other reasons against fixing special forms of beams, which will be mentioned in treating of those forms.

As fixed beams are rarely required in practice, it is not necessary to go farther into the subject to consider their deflection, shearing stresses, etc.

Enough has been said to show the student that the stresses in a beam fixed at the ends are very different from those in a supported beam, not only in amount but in distribution.

Continuous Girders.[1]—When a beam or girder extends without break in itself over two spans or more it is said to be continuous.

Fig. 118 shows to an exaggerated degree the curves formed by

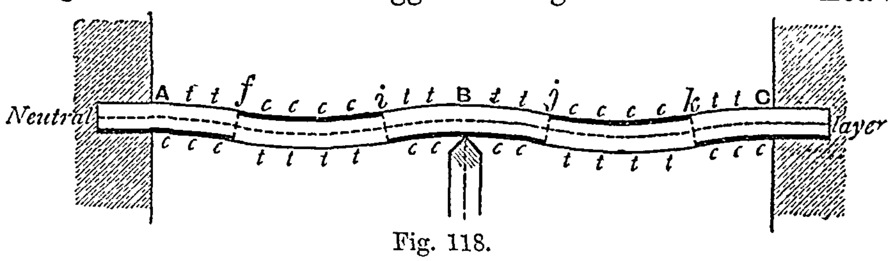


Fig. 118.

a uniformly loaded girder extending over two spans, AB, BC, and fixed at the ends.

It will be seen that the general arrangement resembles two pairs of cantilevers, *Af*, B*i* and B*j*, C*k*, and between them two beams, *fi*, *jk*, supported at the ends.

[1] *The formulæ for practical use* are given in Appendix XI.

The upper portions of Af, Bi, Bj, and Cl are evidently in tension the lower in compression, whereas in fi, jl the upper portions are in compression and the lower in tension

The points of contra flexure are at f, i, j, and l

Again, if the ends of the girder are not fixed, but merely supported, the curves will be as shown in an exaggerated form in Fig 119, and the portions in compression and tension respectively are shown by the thick and thin lines as before

Fig 119

It will be seen that the general curves of each span in Fig 118 resemble those of a beam fixed at both ends (see Fig 112) and the curves of each span in Fig 119 resemble those of a beam fixed at one end and loaded at the other, as shown in Fig 117

The distances of the points of contra flexure from the abutments and the value of the bending moments vary according to the section of the girder, the distribution of the load, etc

The calculations connected with continuous girders are very complicated, and not suited for a work of this kind Enough information has been given above to indicate which portions are in compression and which in tension, and that is all that is required by the syllabus, but as cases occur of bressummers continuous over two spans and of rafters continuous over many spans some further information on the subject is given in Appendix XI

EXAMPLES

In order to illustrate the application of the rules given in the previous pages, it will be well to give in some detail the calculations necessary for a few examples of such timber beams as are used in practice

Before making calculations it is always desirable to state the preliminaries with great care These include all known particulars relating to the case, such as the span, the nature and distribution of the load, also the moduli of rupture and elasticity of the materials, etc etc

Great care must be taken also that the dimensions weights, etc, are all expressed in the same units, i e that feet and inches

Referring to Fig. 103 we see that the theoretical form of the beam would be as shown in Fig. 120.

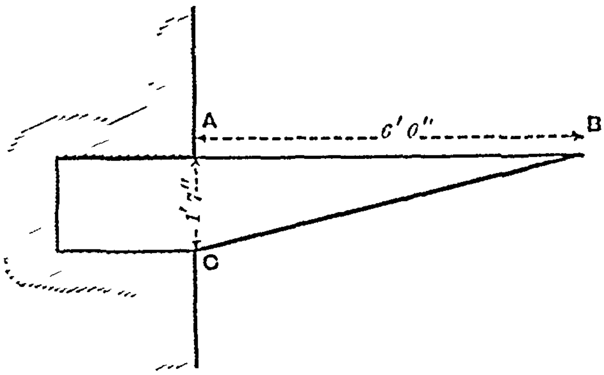


Fig. 120.

The calculated depth is given at AC, where the beam is built into the wall, and the line joining CB is the under side of the beam.

In practice the end *would not be brought to a* feather edge as shown at B, but would have a thickness there of an inch or two, as shown in Fig. 121. It will be seen that in most cases there would be no advantage in using a cantilever of uniform strength, for though the theoretical form would contain less timber, still, practically, it would contain nearly as much, and would involve more labour than the beam of uniform depth throughout.

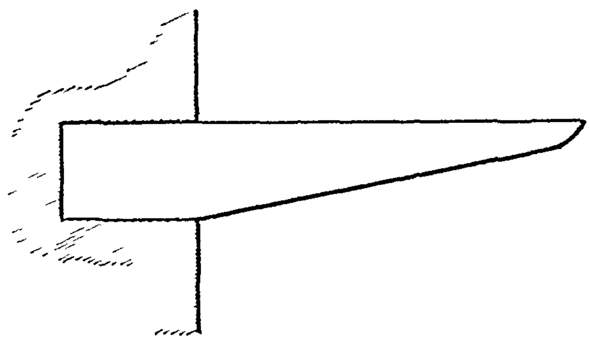


Fig. 121.

It would be better, therefore, to use the latter in ordinary cases.

Balcony with live load.—It will be noticed that the load on the balcony has been taken as a dead load.

If, however, the balcony is liable to be occupied by a crowd of excited people moving about, it would be safer to consider them as causing a live load. This live load should be reduced to a corresponding dead load by doubling it. The total weight would then be as follows :—

Weight of platform as before = 360 lbs.
Weight of people 2 × 1800 = 3600 „
Total weight = 3960 lbs.

Taking this value for the weight instead of 2160 lbs., the calculation would proceed as before with the following results :—

Uniform cross section

(1) For strength, 4″ × 10·4″,
(2) For stiffness, 4″ × 14·5″.

Uniform strength

(1) For strength, 4″ × 10·4″,
(2) For stiffness, 4″ × 23·1″.

In practice it would be preferable to adopt a framed cantilever either of wood or iron. See also Examples 2 and 11.

Example 6—Timber Beam loaded in Centre.—*To find the load a given baulk will carry.* A pitch-pine baulk of 20 feet span 12″ wide 12″ deep is

to be loaded in the centre with a dead load It is to be used for temporary purposes What weight will it safely carry?

Preliminaries —

$$l = 240 \quad b = 12 \quad d = 12\ ,$$
$$f_o = 11000,$$
$$F = 4 \quad \text{Factor of safety}$$

Introducing F we have from Equation 13, p. 33, and Equation 29, p 52, or from Appendix VII

$$F \times \frac{Wl}{4} = f_o \frac{bd^2}{6},$$
$$W = f_o \frac{bd^2}{6} \times \frac{4}{l} \times \frac{1}{4},$$
$$= \frac{11{,}000 \times 12 \times 12^2 \times 4}{6 \times 240 \times 4},$$
$$= 6 \text{ tons (about)}$$

Fir Joist to carry given Load.

Example 7 —*To find the scantling for a fir joist in a single floor*

Conditions —A joist of Baltic fir has a span of 14 feet and is to carry a uniformly distributed dead load of 1400 lbs Find the scantling it should have for (1) strength and (2) stiffness

This case will not be worked out in full, as it is a very simple one

Preliminaries —

$$l = 168 \text{ inches}$$
$$f_o \text{ for Baltic fir} = 5760 \text{ lbs}$$
$$E \quad ,, \quad = 1{,}440{,}000 \text{ lbs}$$
$$w = \frac{1400 \text{ lbs}}{168} = 8\tfrac{1}{3} \text{ lbs per inch.}$$
$$F = \text{factor of safety} = 4$$

Calculation for strength —Calculating for a beam of uniform rectangular section throughout its length

$$\frac{(Fw)l^2}{8} = f_o \frac{bd^2}{6},$$
$$bd^2 = \frac{6 \times (Fw)l^2}{8 f_o},$$
$$= 122$$
$$\text{Taking } b = 2\tfrac{1}{2}'' \quad d^2 = 49, \text{ and } d = 7$$

Calculation for stiffness —To calculate deflection of the beam if $2\frac{1}{2}''$ wide and 7″ deep By Equation 47, p 68

we have $$\Delta = \frac{12nWl^3}{Ebd^3}$$

Here $n = \frac{5}{384}$ (see Table C, col 1, line 4)

$$\Delta = \frac{12 \times 5 \times 1400 \times 168^3}{384 \times 1{,}440{,}000 \times 2\frac{1}{2} \times 7^3}$$
$$= \tfrac{7}{8} \text{ inch nearly}$$

If, however, the beam is to carry a ceiling below, the deflection should not exceed $\frac{1}{40}''$ per foot of span, or $\frac{14}{40}''=\frac{3}{8}''$ altogether.

To find the depth that the beam should have, so that the deflection shall not exceed $\frac{3}{8}''$, we have by Equation 47

$$\frac{3}{8}''=\frac{12nWl^3}{Ebd^3},$$

$$d^3=\frac{12nWl^3\times 8}{Eb\times 3},$$

$$d^3=768,$$

$$d=9\tfrac{1}{2} \text{ inches (about).}$$

Useful Notes.

Strongest rectangular beam that can be cut from a round log (Fig. 122).

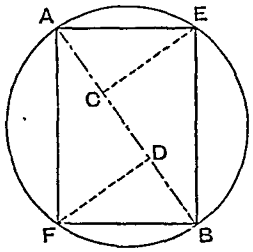


Fig. 122.

Trisect a diameter AB of the log. From the points of trisection C, D raise perpendiculars CE and DF, cutting the circumference at E and F. Join AE, AF, FB, and BE; then AEBF is the strongest beam that can be cut out of the log. It can be shown that

$$\frac{FB}{AF}=\frac{1}{\sqrt{2}}=0{\cdot}7 \text{ nearly.}$$

Stiffest rectangular beam that can be cut out of a round log (Fig. 123).

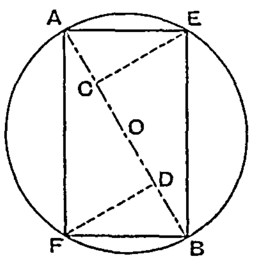


Fig. 123.

Divide a diameter AB of the log into four equal parts at the points C, O, and D. From C and D draw perpendiculars cutting the circumference at E and F. Join AE, AF, FB, and BE. Then AEBF is the stiffest beam that can be cut out of the log, and it can be shown that

$$\frac{FB}{AF}=\frac{1}{\sqrt{3}}=\frac{4}{7}.$$

Proportion of breadth to depth in a beam.—We have seen above that the ratio of breadth to depth is:

For the strongest rectangular beam cut out of a round log, 1 to $\sqrt{2}$.

For the stiffest rectangular beam cut out of a round log, 1 to $\sqrt{3}$.

When, therefore, a single rectangular beam has to be cut from a round log, its proportions will be regulated by one of the above rules, according to whether its strength or stiffness is more important.

In practice, however, the beams used are not generally cut simply out of round logs, and the proportions are governed by other considerations.

The width is made sufficient for lateral stiffness and for fixing the superstructure, and the depth as great as convenient under the circumstances, taking care not to exceed the market size of the timber.

CHAPTER IV

ROLLED BEAMS OF IRON OR STEEL.[1]

THE best section for a beam made of iron or steel is very different from that for a timber beam

We have seen that the direct stresses produced upon the cross section of a timber beam may be graphically shown as in Fig. 124 They are greatest on the layers most remote from the neutral axis, gradually decrease as they approach the axis, and dwindle to nothing at all at the axis itself.

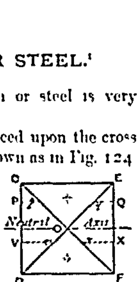


Fig. 124.

In a rectangular solid section, however, just as much material is provided to resist the stress at PQ or VX, where this stress is small comparatively, as at CE or DF where it is largest, and indeed just as much at the neutral axis where there is no direct stress at all, but merely the maximum part of a comparatively small shearing stress (see p. 61).

It is evident then that, so far as the direct stresses are concerned, the portions COD and EOF are useless, and might be removed. To resist the shearing stress a small portion must be left near the neutral axis.

Nevertheless it would be undesirable to cut away these portions in the case of timber beams, as in most cases it would destroy the continuity of the fibres, and would lessen the resistance of the beam to lateral forces, that is, to those forces acting horizontally, which, although not as a rule calculated for (their amount being uncertain), should be allowed for in practice. Moreover, the wood obtained by thus cutting the beam would not pay for the labour of cutting. But in iron, which has in any case to be formed to a particular shape, it is evidently desirable for the sake of economy of material and weight to make beams of the shape best adapted to resist the stresses that will come upon them.

[1] *The formulæ for practical use* are Equations 54, 55, 56, 56*a*, pp. 87, 88, 93, and in Appendix XXI. *Examples* are given at pp. 88 to 91.

Iron or steel beams are therefore rolled of the form shown in Fig. 125.

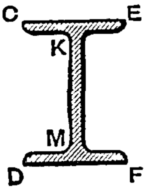


Fig. 125.

CE being called the upper flange
DF „ lower „
KM „ web „

By this arrangement, as will be seen, the bulk of the metal is placed in the flanges where the greatest direct stresses exist, and these flanges are connected by the web KM. If the thickness of the web were regulated by the shearing stress only, the amount of which is comparatively small, it would be made very thin.

Practically, however, for reasons which will soon be mentioned, the web is made thicker than is necessary to resist the shearing stress.

Let us now take a section of an ordinary rolled iron beam (Fig. 126), such as is kept in stock by merchants, and ascertain the nature and distribution of the direct stress upon it.

Resistance of Wrought Iron.

Ultimate Resistance.—We know from Parts II. and III. that the breaking stresses for average wrought iron may be taken as follows :—

Tension . . .	25	tons per square inch.
Compression . . .	16 to 20	„ „
Shearing . . .	20	„ „

But as rolled beams are often of inferior iron the breaking stress in tension should not be taken higher than 20 tons.

Working Resistance.—The working or limiting stresses that may with safety be allowed in the case of rolled beams[1] are

In Tension . . .	5	tons per square inch.
Compression . . .	4	„ „
Shearing . . .	4	„ „

These are lower than those sometimes allowed in the case of iron for roof trusses.

To find the Moment of Resistance of an I Beam.

There are several different ways, more or less accurate, of finding the moment of resistance of a rolled iron beam.

A simple, and at the same time accurate method, is founded upon exactly

[1] In consequence of the iron in rolled beams being free from joints, welds, and other sources of defective workmanship, it is the practice of some engineers to take the working stresses upon them as follows :—

In Tension	6	tons per square inch.
Compression . . .	5	,, ,,
Shearing . . .	5	,, ,,

As such beams are, however, often made from inferior iron, it is better and safer to adhere to the limiting stresses given above.

the same principles, and worked out in almost the same manner, as the calculation described at p 43 *et seq* for a timber beam

Graphic Method, *including Web.*—The internal direct stresses produced in a wrought iron beam, slightly bent by a central load, are similar to those produced in a timber beam.

The fibres above the neutral axis are in compression, those below it in tension

The intensity of stress in the different layers of fibres is greatest in those which are most remote from the neutral axis, and becomes smaller and smaller as they approach the neutral axis, until, at the axis itself, there is no direct stress at all

The amount and distribution of the stresses may be graphically shown, just as they are for a timber beam in Fig 74

Fig 126 is the section of a rolled iron beam 10" deep, with flanges 4" wide and ½" thick, the web being also ½" thick

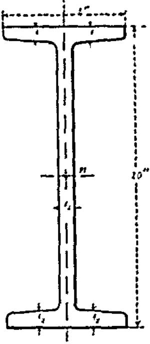


Fig 126
Linear scale
3 inches = 1 foot

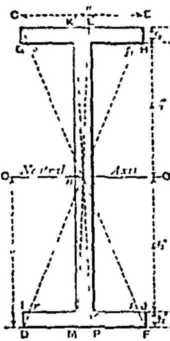


Fig 128
Linear scale 3 *inches* = 1 *foot.*
Scale of stresses 1 *inch* = 16 *tons*

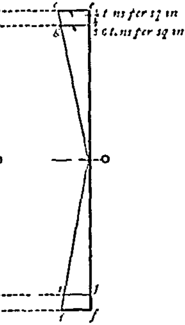


Fig 127
Scale of stresses
1 *inch* = 16 *tons*

The centre of gravity of this section will be at the central point *n*, and the neutral axis will pass through it (see p 44)

The limiting or working stress for wrought iron in compression being 4 tons, and in tension 5 tons per square inch, it is evident that the limit will be first reached in the flange which is under compression, *i e* in the present case (*a beam supported at the ends and loaded*) the upper flange

To put this in other words When the stress upon each flange is 4 tons per square inch, *the compression flange* will be undergoing the limiting stress allowed for it, but the tension flange will be undergoing only four fifths of the stress that it could rightly bear

Fig 127 is a diagram showing the intensity of stress on the different layers

of fibres at the moment when the extréme fibres under compression at CE are subjected to the limiting stress of 4 tons per square inch at *ce*.

When this intensity is called out, a similar stress *df* of 4 tons per square inch occurs on the lowermost layer DF. This layer would safely withstand 5 tons per inch, but that stress cannot be called out without exciting a similar stress of 5 tons per square inch on the uppermost compression layer CE, *i.e.* a stress in excess of the limit of 4 tons per square inch.

A cross section of the rolled iron beam is shown in Fig. 128, with the angles square for the sake of simplicity, and having marked upon it the amount and distribution of the stress upon the section.

At CE the compression flange is 4″ wide, the stress at CE is therefore four times $ce = 4 \times 4$ tons $= 16$ tons.

At GH the flange is also 4″ wide, and the stress four times $gh = 4'' \times 3{\cdot}6$ tons $= 14{\cdot}4$ tons.

At KL the web is $\frac{1}{2}$ inch wide and the stress upon it at that point is $= \frac{1}{2} \times gh = \frac{1}{2} \times 3{\cdot}6 = 1{\cdot}8$ ton. Two points *k* and *l* are thus obtained on the diagram of stress, as shown in Fig. 129, which is an enlargement of part of Fig. 128.

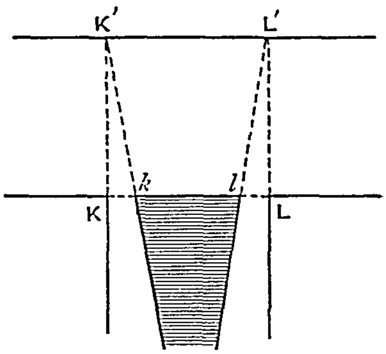


Fig. 129.

The diagram of stress is completed (the web being rectangular) by joining *kn* and *ln* (Fig. 128). A moment's consideration will show that the same two points can be obtained by dropping perpendiculars KK′ and LL′ on to CE, and joining K′*n* and L′*n*, the intersections with KL are the points required.

Thus the hatched portion above the neutral axis represents the amount and distribution of the compression on the fibres of the beam.

In the same manner may be constructed the hatched diagram of tensile stresses below the neutral axis.

It will be seen at once that the result we have obtained amounts to drawing C*n*, E*n* as far as their intersection with GH, and D*n*, F*n* as far as their intersection with I, J, and then completing the diagram by joining K′*n*, L′*n*, M′*n*, and P′*n*. So that in practice Fig. 127 is not required.

Thus C*oknlp*E is the equivalent area of compressive stress. This area, multiplied by 4 tons per square inch, will give the actual amount of resistance to compressive stress.

Similarly the hatched figure D*rmnps*F below the neutral axis, multiplied by the intensity of stress 4 tons per square inch, gives the amount of resistance to tension.

Now we know that in the case of the timber beam the moment of resistance is equal to the stress of either kind multiplied by the distance between the centres of gravity of the equivalent areas shown on the diagram.

It is, however, simpler in this case, and quicker, to take the parts of the rolled beam separately.

Thus, in Fig. 128, the moment of resistance of the flanges is equal to stress area C*oklp*E × distance between the centre of gravity of C*oklp*E and of D*rmnps*F × limiting stress.

The moment of resistance of the web = stress area *kln* × distance between the centres of gravity of *kln* and *mpn* × limiting stress.

Putting this into figures we have—

$$\overline{M} \text{ of Flanges} = \underset{CoklpE}{\text{Stress Area}} \times \underset{\text{Stress}}{\text{Limiting}} \times \underset{\text{and } DrmpsF}{\text{Distance between Centres of Gravity of Areas } CoklpE}$$

$$= \left\{ \tfrac{1}{2}\,(4'' + \tfrac{9}{10}4'') \times \tfrac{1}{2} \right\} \times \underset{4}{\text{tons}} \times 9\ 53 \text{ inches},$$

$$= \tfrac{1}{2} \times \tfrac{38}{10} \times 4 \times 9\ 53,$$

$$= 72\ 43$$

$$\overline{M} \text{ of Web} = \underset{lli}{\text{Stress Area}} \times \underset{\text{Stress}}{\text{Limiting}} \times \text{Distance between Centres of Gravity of Areas } lln \text{ and } mpn$$

$$= \left\{ \tfrac{9}{20} \times 4\tfrac{1}{2}'' \times \tfrac{1}{2} \right\} \times 4 \text{ tons} \times 6 \text{ inches},$$

$$= \tfrac{81}{80} \times 4 \times 6,$$

$$= 24\ 3$$

Therefore, $\overline{M}$ of Flanges and Web $= 72\ 4 + 24\ 3$,
$= 96\ 7$ inch tons

STRESS DIAGRAMS

In cases where the flanges of the beam differ very greatly from rect-

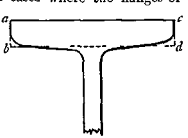


Fig 130

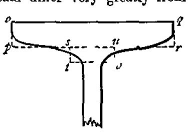


Fig 131

angles, the nearest equivalent rectangles may be sketched in and the cal culations carried out as if the flanges were of this shape

Thus in Fig 130 *abcd* may be taken as the shape of the flange, and in Fig 131 the flange may be considered as made up of *opqr* and *stuv* The moment of resistance of each rectangle should then be found, as previously explained, and the sum of these moments will be the moment of resistance required

Alternative Method.—The following is another method, which may occasionally be employed with advantage

In Fig 132 the stress diagram for the shallow rectangle *sv* is obtained as explained for Fig 128 Now it is evident that the depth *su* of the rectangle may be diminished until it becomes nothing, without altering the position of the points u_1 and v_1 The rectangle *su* would then coincide with the line *uv* It thus appears that u_1v_1 represents the stress in the layer of fibres *uv*, and in the same way x_1y_1 represents the stress in the layer xy Hence u_1, x_1, y_1, and v_1 are points in the stress diagram of the flange

And similarly any number of points can be found on the stress diagram, as shown in Fig 132

Area and centre of gravity of stress.—The area of the stress diagram and the distance of its centre of gravity from *n* must now be found The former can be obtained by reducing in the usual way to a triangle, or better still, by means of a planimeter, should one be available, the latter can also be determined by a graphic method, which is too complicated for these Notes.

Both, however, can be found by the following simple artifice with ample accuracy for all practical purposes.

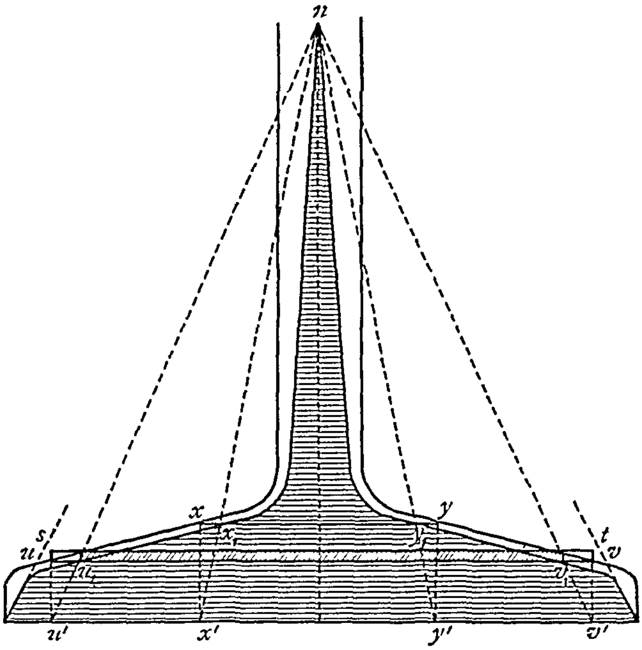


Fig. 132.

Draw the stress diagram on a thick piece of Bristol board, and cut it carefully out with a sharp penknife.

To find the area weigh the cut-out diagram, and then weigh one square inch of the same Bristol board ; the first weight divided by the second will give the area required in square inches.

To find the centre of gravity.—Make a small hole at each of the corners D and F, so that when a needle is passed through, the piece of cardboard will swing perfectly freely. Attach a small weight to a fine thread and secure the thread to the needle as shown in Fig. 133. Mark the position of the thread on the diagram, both when suspended from D and from F ; the intersection of these two lines is the centre of gravity required. In the particular case under consideration the centre of gravity lies on the centre line through n, owing to the symmetry of the diagram.

Mathematical Method of finding the moment of resistance of a wrought iron I beam.—The graphic method described, pp. 83 to 85, gives exactly the same result as the accurate mathematical formula given in Rankine's and other works, and is simpler to understand.

The mathematical formula is—

$$\overline{M}=\frac{r_o I}{y_o} \quad . \quad . \quad . \quad (53),$$

where r_o = limiting stress per square inch,

I = moment of inertia (see Appendix XIV.),

$y_o=\frac{d}{2}$.

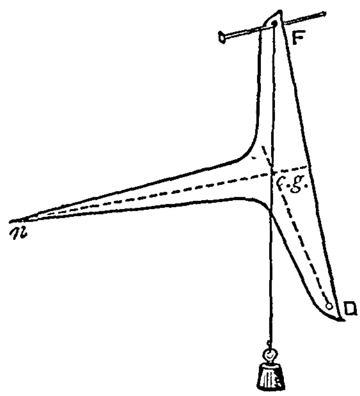


Fig. 133.

I is worked out for the present example in App. XIV., and found to be 120·7. Substituting in Equation 53, we have

$$r_o = 4 \text{ tons for compression,}$$
$$y_o = 5 \text{ inches,}$$
$$M=\frac{r_o I}{y_o}$$
$$=\frac{4\times 120{\cdot}7}{5}.$$
$$\overline{M}=96{\cdot}7 \text{ as before.}$$

When the beam with its load is in equilibrium, the moment of resistance thus found is of course equal to the bending moment found in the manner already explained.

The calculations necessary to ascertain the weight a given rolled iron beam will carry, or to find the dimensions of such a beam to carry a load of particular weight and distribution are similar to those already described for timber beams.

Examples of some of the cases likely to occur in practice are given as illustrations at p. 85 *et seq*

Approximate and Practical Formulæ for Rolled Iron Beams.

Approximate Method, No 1 (*ignoring web*).—In most books of formulæ, and in many other works describing the calculation of rolled beams, the *resistance to direct stresses afforded by the web is ignored*, *the resistance of the flanges alone is calculated*

Thus in Fig 128, p 83, the resistance indicated by the triangle Lln would be omitted.

Instead, however, of taking the stress area $CoHhE$ as representing the varying resistance of the flange, the whole area of the flange is taken

Again, instead of taking the distance between the centres of gravity of the stress areas as the length of the arm of the couple, the full depth of the beam is taken

The resulting formula stands thus—

$$\overline{M} = \text{area of flange} \times \text{limiting stress} \times \text{depth},$$

$$= A \times r_s \times d \qquad (54)$$

This formula gives defective results, and the thicker the web (in proportion) the greater the error The error will therefore be greater in small rolled beams (*as in the above instance*) *than in large ones* *Taking the beam* just dealt with, and $r_s = 4$ tons

$$\overline{M} = 4 \times \tfrac{1}{2} \times 4 \times 10,$$

$$= 60 \text{ inch tons}$$

We know from p 85 that its actual value, as there calculated, is 96 7 inch tons.

This formula should therefore be employed only as a trial method to pick out a suitable section, as shown in Example 9, and to be followed up by the accurate method already explained

Approximate Method, No 2 (*including web*) —In the method just described the flanges only are considered, the web ignored

From a glance at Fig 128, p 83, it will be seen that the moment of the *triangle of stress thus ignored, i.e. Lln, is approximately*

$$= \text{area } Lln \times \tfrac{2}{3}d \times r_s,$$

$$= \tfrac{1}{4} \text{ area of web} \times \tfrac{2}{3}d \times r_s,$$

$$= \tfrac{1}{6} \text{ area of web} \times d \times r_s.$$

If, therefore, we add $\frac{1}{6}$ area of web to the area of the flange and multiply by d, we shall have another formula for the moment of resistance, namely—

$$\overline{M} = (A + \tfrac{1}{6} \text{ area web}) \times r_s \times d \qquad (55)$$

Taking the same beam as before,

$$\overline{M} = (4 \times \tfrac{1}{2} + \tfrac{1}{6} \times 10 \times \tfrac{1}{2}) \times 4 \times 10,$$

$$= 113 \text{ inch tons.}$$

This formula gives too large a result, and is therefore not a safe one to use, the true value of $\overline{M}$ lies between the values found by the two approximate formulæ

Practical Formula for Rolled Iron Beams, whose Length, Breadth, Depth, and Weight are given.

The student should observe that in the case of rolled iron beams the choice is practically limited to the sizes which are usually rolled by manufacturers, and in many cases to what happens to be in stock. Most manufacturers publish a list of sections, and many also supply illustrations of full-sized sections. The process is therefore to select from such a list that section which best fulfils the requirements of the case. For such a purpose the approximate formulæ given above are very useful, but it sometimes happens that only the depth, the width, and the *weight* per foot are obtainable, and it is to be observed that this is the information usually given when specifying for a rolled iron beam. The following formula based on these data is therefore useful, namely—

Approximate Method, No. 3.—Safe *distributed* load in tons

$$= 0{\cdot}52(w - 0{\cdot}3bd)\frac{d}{\mathrm{L}} \quad . \quad . \quad . \quad \textbf{(56)},$$

where w = weight of rolled iron beam in lbs. per foot,
b = breadth in inches,
d = depth in inches,
L = span in feet.

This formula is constructed on the supposition that the maximum safe stress is 5 tons per square inch, and it will be found on trial to give fairly good results.

Distributed Load that can be safely borne by a Rolled Iron Beam of given Dimensions.

Example 8.—*Conditions.*—Suppose an I rolled iron beam to be of the section shown in Fig. 126 and to be 12 feet between supports, what load uniformly distributed over its length will it safely carry?

Calculation by mathematical Method.—*Preliminaries.*—Taking the safe limit of stress in compression at 4 tons per square inch, we know from p. 86 that the moment of resistance of this beam = 96·7 inch-tons.

The maximum bending moment (see Case 7, p. 33) $\mathrm{M}_c = \dfrac{wl^2}{8}$.

But $\mathrm{M}_c = \overline{\mathrm{M}}$,

$$\therefore \frac{wl^2}{8} = 96{\cdot}7 \text{ inch-tons},$$

or since $l = 12 \text{ feet} = 144''$,

$$\therefore \mathrm{W} = wl = \frac{96{\cdot}7 \times 8}{144},$$
$$= 5{\cdot}37 \text{ tons}.$$

Calculation by approximate Method No. 1.—The value of $\overline{\mathrm{M}}$ found by the first approximate method which ignores the web was 80 inch-tons (see p. 87).

Hence
$$\frac{wl^2}{8}=80,$$
$$wl=\frac{80\times 8}{144},$$
$$=4\cdot 44 \text{ tons.}$$

Thus this method gives as the distributed weight that can safely be carried by the beam 4·44 tons, whereas by the more accurate method it was found to be 5·37 tons.

Distributed Load that can safely be borne by a Rolled Iron Beam of given Length, Width, Depth, and Weight.

Example 9.—A rolled iron beam 12 inches deep and 6 inches broad is found from a Table of Sections to weigh 56 lbs. to the foot. Hence, from the *approximate formula No. 3* (56), for a span of 18 feet we have

$$\text{Safe distributed load}=0\cdot 52(56-0\cdot 3\times 6\times 12)\tfrac{12}{18},$$
$$=11\cdot 9 \text{ tons.}$$

The average thickness of the flange of such an iron beam is 1 inch. Hence, using *approximate formula No. 1*,

$$\frac{wl^2}{8}=A\times r_e\times d$$
$$\text{Safe distributed load } wl=\frac{8\times 6\times 1\times 5\times 12}{18\times 12},$$
$$=13\cdot 3 \text{ tons.}$$

The moment of inertia of this section will be found to be 400. Hence by the *mathematical formula* (see Equation 53, p. 86)

$$\frac{wl^2}{8}=\frac{r_e I}{y_e}.$$
$$\text{Safe distributed load } wl=\frac{5\times 400\times 8}{6\times 18\times 12},$$
$$=12\cdot 3 \text{ tons.}$$

Example 9A.—Again, take the case of the rolled iron beam $16''\times 6''$ of which the moment of inertia is 730, and let us find the safe distributed load for a span of 20 feet, when the beam is supported at the ends.

On referring to a Table of Sections such as is published by manufacturers it will be seen that this section weighs 63 lbs. to the foot. Hence by *approximate formula No. 3*

$$\text{Safe distributed load}=0\cdot 52\ (63-0\cdot 3\times 6\times 16)\ \tfrac{16}{20},$$
$$=13\cdot 8 \text{ tons},$$

and by the *accurate mathematical formula*

$$\text{Safe distributed load}=\frac{5\times 730\times 8}{8\times 20\times 12},$$
$$=15\cdot 2 \text{ tons.}$$

Section for a Joist of a specified Span to carry a given Distributed Load.

Example 10.—*Conditions.*—A rolled iron floor girder of 20 feet span and fixed at the ends has to carry a uniformly distributed dead load of 1 ton per foot run of its length.

Find the dimensions required

(1) For strength,
(2) For stiffness.

Preliminaries.—

$w = 1$ ton per foot run $= \frac{1}{12}$ ton per inch $= 186$ lbs.

$l = 20$ feet $= 240$ inches.

$E = 29,000,000$ lbs. for wrought iron bars.

Resistance to direct Stresses.—This being a case of a beam *fixed* at the ends, the greatest bending moment is at the point of fixing (see Case 2, p. 72),

and is (Equation 51, p. 72) $\qquad \frac{wl^2}{12}.$

Therefore $\qquad M_A = \frac{\frac{1}{12} \cdot (240)^2}{12} = 400$ inch-tons,

and hence the rolled beam must have a moment of resistance of 400 inch-tons.

It is practically necessary to select a section from those kept by manufacturers. Supposing, therefore, we have a sheet of sections before us, and, as a first trial, select a section 12″ deep, with flanges 6″ wide, and 1″ thick. Using the *approximate formula No 1* given at p. 87 (Equation 54) we find

$$\overline{M} = 6 \times 1 \times 4 \times 12,$$
$$= 288 \text{ inch-tons},$$

which is a great deal too small.

Trying next a section $17'' \times 6'' \times 1\frac{1}{4}''$ we find

$$\overline{M} = 6 \times 1\tfrac{1}{4} \times 4 \times 17,$$
$$= 510 \text{ inch-tons},$$

which *is more than necessary. A section* $16'' \times 6'' \times 1''$ gives $\overline{M} = 384$ inch-tons. This section will therefore probably do, since the value of $\overline{M}$ found by the approximate formula is too low.

The moment of resistance of this section may be found by the *graphic method* thus:

The upper and lower flanges may be taken as having a mean thickness of 1 inch, and the web is $\frac{9}{16}$ inch thick.

The equivalent areas are drawn as described at p. 83, and we have the following moments of resistance (Fig. 134):—

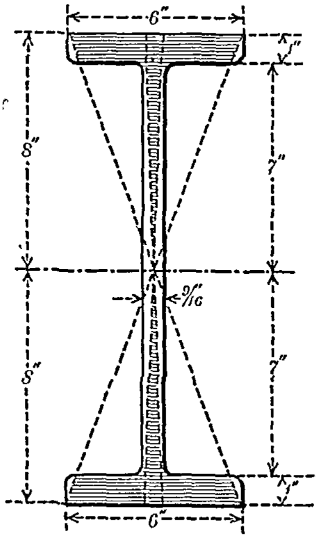


Fig. 134.

For flanges[1]

	tons	inch-tons
$\frac{1}{2}(6'' + \frac{7}{8}6'') \times 1'' \times 15'' \times 4$	=	337·5

For web

$\frac{9}{16}'' \times \frac{7}{8} \times \frac{1}{2} \times 7'' \times \frac{2}{3} \times 14'' \times 4$	=	64·3
Total moment of resistance	=	401·8

This section will therefore be suitable in point of strength.

[1] 15″, the distance taken between the centres of gravity of the stress diagrams for the flanges, is a little less than the true distance, because the stress diagram for the flange is a trapezium, not a rectangle. The true distance is

$$2\left(7 + \tfrac{1}{3} \cdot \frac{2 \times 6 + \frac{7}{8}6}{6 + \frac{7}{8}6} 1\right),$$
$$= 2(7 + 0{\cdot}51),$$
$$= 15{\cdot}02,$$

and the moment of resistance for the flanges becomes 342 inch-tons.

It must of course be understood that no holes are to be made in either flange. If any are necessary, the breadth or thickness of the top and bottom flanges must be increased accordingly.

Resistance to Shearing.—We know then that the beam selected is able to bear the direct stresses that will come upon it; the next question is whether the web will bear the shearing stresses.

The greatest shearing stress is at the points of fixing and it there amounts to $\frac{wl}{2}$ (see Case 7, p. 59) $= \frac{20 \text{ tons}}{2} = 10$ tons.

This has to be resisted by the sectional area of the web. This area is

$$14'' \times \tfrac{9}{16}'' = \text{nearly } 8 \text{ square inches,}$$

the shearing stress will therefore be $\frac{10 \text{ tons}}{8 \text{ sq. in.}} = 1\frac{1}{4}$ ton per square inch.

Now the safe working shearing stress may be taken as 4 tons per square inch, so that the web is amply strong enough.

Deflection.—From Case 2, p. 72, and Appendix VIII., we see that the maximum deflection of a beam of uniform section fixed at both ends and uniformly loaded throughout its length is at the centre, and is

$$\Delta = \frac{Wl^3}{EI} \times \frac{1}{384}.$$

We know (Equation 53, p. 86) that

$$I = \overline{M} \times \frac{d}{2r_c},$$

$$= 402 \times \frac{16}{2 \times 4},$$

$$= 804.$$

$$\Delta = \frac{20 \times 12 \times 186 \times 240^3}{29{,}000{,}000 \times 804} \times \frac{1}{384},$$

$$= 0.06 \text{ inch},$$

or much less than the specified limit for floor girders (see p. 69) of $\frac{1}{40}''$ per foot of span, viz. in this case, $\frac{20}{40} = \frac{1}{2}''$.

The section of girder chosen is therefore both strong enough and stiff enough for the purpose, if care is taken in setting it that the ends are properly *fixed*.

Remarks on Rolled Beams

Large rolled beams not theoretically economical. Rolled beams much used because cheap and useful. Can be fixed at ends. Rolled iron beams when more than 12 or 14 inches deep are not theoretically economical, because their section is uniform throughout their length, and their flanges are equal to one another. They are therefore not of uniform strength throughout their length, and thus more material is used in them than is theoretically required.

A glance at pp. 34 and 59 will show that in a section of uniform strength both web and flanges would vary in dimensions at different parts according to the stress that comes upon them.

Thus for a beam with a uniformly distributed load and supported at the ends the area of the flanges required at the centre to meet the maximum direct stress becomes unnecessarily great as the ends are approached, because the direct stress diminishes (see Fig. 50) towards the ends.

This is still more the case with a load at the centre (Fig. 48).

The web, on the contrary, would theoretically require to be greater at the supported ends to resist the shearing stress, and would gradually diminish towards the centre.

The flange in tension might be smaller than that in compression, as it can be safely subjected to a greater stress per square inch.

In the same way, beams of uniform strength for other dispositions of load would theoretically require to have flanges and web with varying sections to suit the stresses at different points.

The method in which rolled beams are manufactured (see Part III.) renders it impossible that their section can be varied to suit the stress that comes upon them at different points.

It results, therefore, that these beams rolled of uniform section throughout must contain more material than is theoretically required. This is not of much importance in small beams; when, however, the beam is large, the waste of material becomes important, and it is better to build up such a beam, or girder, with plates and angle irons so disposed as to have at each point, as nearly as may be, the exact amount of material which is required to meet the stress at that point.

The method of doing this is described in Chapter VIII.

Rolled beams are cheap and convenient, and are therefore used for a great many useful purposes in spite of the fact that they contain more metal than is theoretically necessary to meet the stresses upon them.

One advantage of rolled iron beams is that they can be placed with either flange uppermost, thus preventing all chance of mistakes which occur with other girders put up by ignorant men. Moreover, as the flanges are equally strong, these beams can be *fixed* at the ends, thus greatly adding to their strength—giving them a great advantage over cast iron girders, which cannot so readily be fixed.

Of course as compared with cast iron girders they possess a still greater advantage in being made of a sound and reliable material which will not give way suddenly.

Market Sections.—Rolled I joists can readily be obtained in either iron or steel, from 3 to 20 inches in depth, with flanges up to 8″ wide. The largest sizes are not economical to use, as explained above.

Girders built up with the aid of I *iron beams* are sometimes used. The following are some of the forms:—

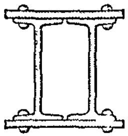


Fig 13

Fig 136

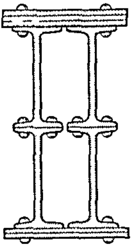


Fig 137

Fig 135 may be useful in some cases, but a beam built up with plate and angle iron would probably be better

Figs 136 and 137 are distinctly faulty owing to the waste of material at the centre of the section

Steel rolled joists or beams are similar in form to those of wrought iron and can be calculated in exactly the same way, using the following working stresses tension $6\frac{1}{2}$ tons compression $6\frac{1}{2}$ tons per square inch

Adopting these stresses Formula 56 p 88 may be modified for steel as follows —

Safe distributed load in tons

$$= 0.67\ (w - 0.3bd)\frac{d}{L} \qquad (56a)$$

With regard to the limiting stresses in rolled joists of iron or steel it should be remembered that while a working compressive stress in the upper flange equal in intensity to that of tension in the lower flange *may be permissible in beams well supported* laterally, yet that in the case of unsupported beams liable to lateral flexure these stresses should be modified in accordance with the proportions of the top flange considered as a column but deriving assistance from the stiffness of the solid web

CHAPTER V.

CAST IRON GIRDERS.[1]

THE calculation for cast iron girders may be based upon the same simple methods that have already been explained for timber beams and wrought iron I girders.

There is, however, one principal point of difference, which makes the calculation at first sight not quite so straightforward and simple as in the cases previously explained.

This difference is caused by the nature of the material. In the timber and wrought iron beams the resistance to tension per square inch was the same or not very different from the resistance to compression; but in cast iron the resistance to tension is very small compared with the resistance to compression, as will be seen at once by a glance at the following figures.

Resistance of Cast Iron.

Ultimate Resistance.—The ultimate resistance, *i.e.* the resistance to actual rupture of cast iron obtained *under proper specifications* for girder work, may be taken as follows:—

Tension . . .	9 tons per square inch.
Compression . . .	48 " "
Shearing . . .	$8\frac{1}{2}$ " "

Working Resistance.—Working or safe limiting stress to be put upon cast iron per square inch:—

Tension . . .	$1\frac{1}{2}$ tons per square inch.
Compression . . .	8 " "
Shearing . . .	2·4 " "

It will be seen from the above that the ultimate resistance to tension is about one-sixth of the resistance to compression. The limiting stress to be allowed in tension is therefore one-sixth of that allowed in compression.

Flanges.—If the tension flange is not made equal in strength to the compression flange it will tear across before the compressive

[1] *The formula for practical use* will be found at p. 95, Equation 57, and in App. XXI.

stress upon the other flange is equal to what the flange is able to bear It will be seen that approximately, if the web be ignored altogether, the tension flange should, in order to be of equal strength with the compression flange, have an area six times as large

Approximate Formula for Calculation of Cast Iron Girders.

The formulæ ordinarily used for the calculation of the strength of cast iron girders are based upon the valuable experiments of the late Mr Eaton Hodgkinson

Those experiments were made with girders having webs so thin that the webs may practically be ignored, because they afforded very little assistance to the flanges in resisting the direct stress

Mr Hodgkinson arrived at the conclusion that Fig 138 is the best form of section for a beam required to bear an ultimate or breaking strain —

"The section of the flanges being in the ratio of 6 to 1, or nearly in that of the mean crushing and tensile strength of cast iron '

He also inferred from his experiments that "when the length depth and top flange in different cast iron beams with very large flanges are the same, and *the thickness of the vertical part between the flanges is small and invariable* the strength is nearly in proportion to the size of the bottom flange Also that in beams which vary only in depth every other dimension being the same, the strength is nearly as the depth

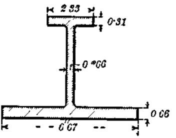


Fig 138

From these data[1] he found the moment of resistance $= C \times a \times d$

a being the area of the tension flange in inches

d being the depth of the girder

C being a constant or modulus found, by breaking several experimental beams, to be $6\frac{1}{2}$ tons

This value of C gives the resistance to actual breaking, the working modulus may be taken at one-fifth of this, i e $1\frac{1}{4}$ ton

As the bending moment equals the moment of resistance we have

$$M = C \times a \times d \qquad (57)$$

For instance, in the case of a beam supported at the ends and loaded in the centre

$$\frac{Wl}{4} = Cad,$$

$$W = \frac{4Cad}{l},$$

$$= \frac{4 \times 6\frac{1}{2} \text{ tons} \times ad}{l},$$

$$= \frac{26 \text{ tons} \times ad}{l}$$

[1] These results are not here stated in the exact form given by Mr Hodgkinson, but are modified to make them in accordance with the formulæ given above for timber and wrought iron beams

If l be reduced from inches to feet $W = \frac{2 \cdot 16ad}{L}$; and this is the shape in which the formula is generally given in books, where W is the *breaking* weight in tons.

It will be seen that this formula roughly gives the moment of resistance of the tension flange.

In practice it is found that when cast iron girders fail, it is generally by the rupture of the flange which is in tension—the flange in compression is generally more than strong enough.

If, therefore, the tension flange is strong enough, the girder is likely to be safe.

The strength of the compression flange may, however, be approximately found by the same formula by substituting 48 tons, the breaking stress in compression, for the value of C.

In recent practice the area of the top flange is raised from one-sixth of the bottom flange to one-third or one-fourth, so that by increased breadth sufficient resistance shall be offered to lateral bending.

This formula is very simply applied as shown in Examples 11 and 11A, and though it is only approximate, it is quite accurate enough for the calculation of cast iron girders. The material of which they are made is so treacherous and uncertain that it is always necessary to use a large factor of safety, or (what amounts to the same thing) a small limiting stress in calculating them. Any minute accuracy in the method used for ascertaining their resistance would therefore be useless.

Graphic Method of ascertaining the Section for a Cast Iron Girder.[1]

The distribution of stress over a cast iron girder under a safe load, and subject only to a small limiting stress, can be shown in a manner similar to that already described for timber and for wrought iron beams, and its strength determined from the diagram thus obtained.

Such a diagram will show the actual distribution of the stresses over both flanges and the web at the moment when the extreme fibres under tension are subjected to the working or limiting stress, and can easily be constructed as follows.

[1] This is described as it illustrates principles, and for the sake of those who prefer drawing to calculations.

Thus at CD there are 4″ of fibres, each acted upon by a stress equal to $d'a'$ or $\frac{13}{7} \times$ the limiting stress, so that $C^1D^1 = \frac{13}{7} \times 4 = 7\frac{3}{7}$ inches. At *gh* there is 1 inch of fibres acted upon by $\frac{12}{7}$ of the limiting stress, so that the width of *the stress area at this point is* $= \frac{12}{7}$ *inch and reduces gradually until at* the neutral axis the direct stress is nothing.

The equivalent shaded area in tension below the neutral axis is of course equal to the equivalent shaded area in compression above the neutral axis, being in each case about $17\frac{3}{7}$ square inches.

The centres of gravity of these two equivalent areas are 15·97 inches apart, and the moment of the couple, *i.e.* the moment of resistance of the girder, is equal to the equivalent stress area of either kind in square inches × the limiting stress per square inch × the distance between centres of gravity = $17\frac{3}{7}$ square inches × $1\frac{1}{2}$ ton × 15·97 inches = 417·5 inch-tons.

COMPARISON WITH RESULT FROM APPROXIMATE FORMULA.—Let us compare this result with that obtained from the approximate formula given at p. 95, namely—

$$\overline{M} = Cad.$$

Taking $C = 1\frac{1}{4}$ ton, as at p. 95, we find

$$\overline{M} = 1 \times (16 \times 1\tfrac{1}{4}) \times 20,$$
$$= 400 \text{ inch-tons.}$$

But the limiting stress has been taken above at $1\frac{1}{2}$ ton per square inch. If C be taken $= 1\frac{1}{2}$ ton, $\overline{M} = 480$ tons.

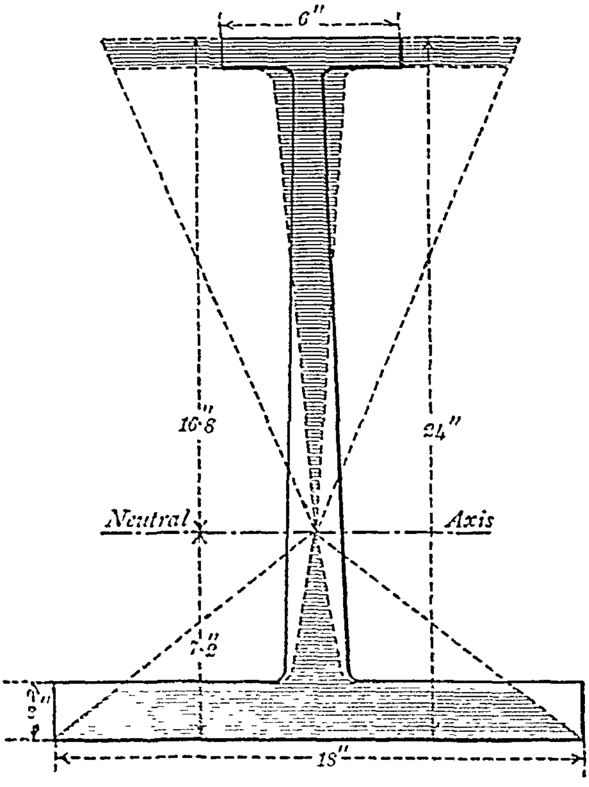


Fig. 142.
Linear scale $1\frac{1}{2}''=1$ *foot.*
Scale of stresses 1 *inch* = 12 *tons.*

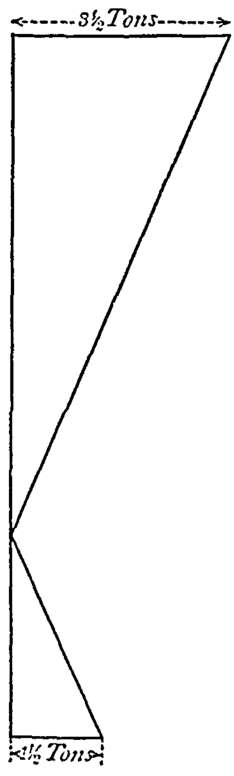


Fig. 143.
Scale of stresses
1 *inch* = 4 *tons.*

Section with tapering Web.—In a girder of the section shown in Fig. 142, and at p. 109, Part I., where the web tapers in thickness from the upper to the lower flange, supported at the ends and loaded, the stresses would be as shown in Fig. 143.

It will also be noticed that the maximum stress in compression, *i.e.* that on the extreme upper fibre, is only $3\frac{1}{2}$ tons.

Mathematical Method for calculating a Cast Iron Girder.

The mathematical method may be described, though the graphic method just described is equally accurate, and safer to use.

The formula used for M is $\frac{f_c}{y_c}I$, or $\frac{f_t}{y_t}I$, in which

f_c = limiting stress in compression
f_t = " " tension
y_c = distance from N.A. to extreme fibre of compression flange
y_t = " " " tension flange
I = moment of inertia.

The former of these two formulæ is used when the compression flange is weaker than the other, the latter formula when (as is usually the case) the tension flange is the weaker.

A case worked out by this method is shown at p. 101.

Figs. 141 and 142 give a clear idea of the nature and distribution of the direct stresses on a cast iron girder, but for the reasons given at p. 96 it is seldom necessary to go into any accurate calculations for such girders.

The *shearing stress* is distributed in the same way as described at p. 61 for timber beams. It is nearly all borne by the web, being a maximum at the neutral axis and diminishing as the flanges are approached, until at the flanges themselves the shearing stress is almost nothing.

Practical Points connected with the Form of Cast Iron Girders.[1]

Proportions of Flanges.—In both the sections given it will be seen that the compression flange is never called upon to undergo anything like the working stress that it can safely bear.

Thus in Fig. 140 the stress on the extreme fibre in compression is only $2\frac{1}{4}$ tons instead of 8 tons per square inch, and in Fig. 143 it is only $3\frac{1}{2}$ tons per square inch.

It appears, therefore, that so far as resistance to crushing is concerned, the compression flange might be made smaller; practically, however, other points have to be considered.

The flange which undergoes compression must not be made too narrow, or (though safe against crushing) it will fail by buckling sideways.

The greater the span the wider the flange should be. No rule has been laid down for this, but it is well not to make the compression flange narrower than one-sixtieth of the span.

[1] See also Part II.

It will be further seen that in some cases, when from construction of the whole building it is impossible for the cast girder to buckle, the compression flange need not have an area exceeding one-sixth of the area of the tension flange, and this proportion is sometimes taken as the universal rule.

When, however, the compression flange has to be widened to resist cross-buckling, or when it is required to be wide to support a wide form of load such as a wall above, then it is often necessary to make it of greater area than one-sixth of that of the tension flange. The consequence is that some writers propose one-fourth and others one-third as the proportion (see p. 96).

Each case must, however, depend upon circumstances. As a rule a compression flange having one-sixth the area of the tension flange is large enough, if it is wide enough to resist cross-buckling, and to carry the load upon it.

The web must of course contain sufficient area to withstand the shearing stress that comes upon it.

In order to prevent it from "buckling" sideways it is customary to strengthen it by feathers or stiffeners, as shown at *f* in Fig. 144 and in Part I. Figs. 245, 246.

These stiffeners should, however, be avoided as far as possible, because the angles formed by their junction with the web and flanges tend to produce weak points in the castings, for the reasons explained in Part III.

The depth of a cast iron girder must of course in most cases be governed by circumstances, but when possible it is desirable to give it a depth of at least one-twelfth the span, or one-tenth when considerable stiffness is required. The depth is, however, very often made much less than this in practice.

Camber.—Cast iron girders should be constructed with a rise in the centre equal to $\frac{1}{150}$ to $\frac{1}{200}$ of their span, so that they may not when loaded sag or droop below the horizontal line.

Points connected with casting.—So far as the requirements of good casting go,[1] for flanges 2′ wide it is not wise to have a less thickness than $1\frac{1}{2}''$, and for flanges 18″ wide not less than 1″, but as a rule[2] the thickness of any part should not exceed $\frac{1}{12}$ of the width of the part.

When one flange is to be thinner than the other, as in Fig. 142, the web may taper from one to the other, each end being equal in thickness to the flange it joins.

Some founders prefer that the thickness of the metal should be the same throughout web and flanges, as in Fig. 141.

In any case, there should be no sudden variation in the thickness at any part, and no sharp angles.

Girders of uniform Strength.—The method by which cast iron girders are made is such that they can without any difficulty be cast so as to be of uniform strength, the form varying according to the load they have to carry.

Thus for a girder with a uniformly distributed load the shape would be

---

[1] Wray. [2] Adams.

approximately that shown in Figs 144 or 145, the material being reduced where the stress is less, so as to form curves.[1]

The alteration in section may be obtained by altering the depth, as in

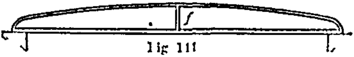


Fig 144

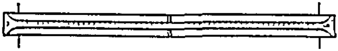


Fig 145.

Fig 144, and leaving the flanges of uniform width throughout, or by altering the width of the flanges at different points according to the stress upon them, keeping the depth of the girder constant throughout, as in Figs 146, 147

ELEVATION

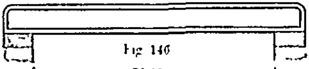


Fig 146

PLAN

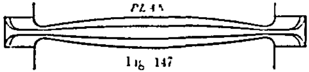


Fig 147

A cast iron girder should never be fixed at the ends, because of the change from compression to tension which takes place (as shown in Fig 112) in the top flange as the fixed end is approached This is not desirable even when the girder is designed accordingly, and is unsafe if the ordinary design is followed, for then the top flange at the point of fixture will be unduly strained in tension

EXAMPLES OF CAST IRON BEAMS

Cast Iron Cantilever

Example 11—*Conditions*—A cast iron cantilever for a balcony (Fig 148) has a projection of 6 feet, and has to carry a distributed dead load (including its own weight) of $\frac{1}{2}$ ton per foot run The depth at the wall must not exceed 3 feet Find the dimensions for the cantilever

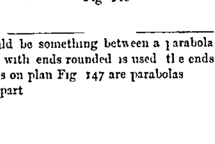


Fig 148

Preliminaries

$w = \frac{1}{2}$ ton = weight per foot run on the cantilever including its own weight,
$= \frac{1}{24}$ ton per inch run[2]

[1] Theoretically the curve for Fig 144 should be something between a parabola and an ellipse In practice an arc of a circle with ends rounded is used the ends being half the depth of the centre The curves on plan Fig 147 are parabolas

[2] The cantilevers are assumed to be 5 feet apart

l = length = 72 inches.
C = 1 ton per square inch = coefficient for Hodgkinson's formula.
a = area of top (or tension) flange in inches (to be found).
d = depth of cantilever at wall = 36 inches.
E = modulus elasticity of cast iron = 17,000,000 lbs. per square inch.

Now since $$M = \overline{M},$$

hence (Case 2, p. 28) $$wl \cdot \frac{l}{2} = Cad,$$

that is, $$\tfrac{1}{24} \times 72 \times 36 = 1 \times a \times 36.$$

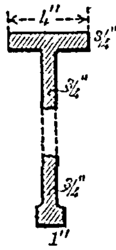


Fig. 149.

Hence area of tension flange is $a = 3$ square inches, and the flange may be $4'' \times \frac{3}{4}''$

area of compression flange $= \frac{1}{6} \times 3,$
$= \frac{1}{2}$ square inch.

Thus the web is thick enough to act as a compression flange, or an enlargement may be made for appearance, as shown in Fig. 149.[1]

The shearing stress at the wall will be $= wl = 3$ tons (Fig. 83, p. 57).

The thickness of web t required to resist shearing

$$= \frac{\text{shearing stress}}{\text{depth of web} \times \text{working resistance to shearing per square inch}},$$

$$t = \frac{3 \text{ tons}}{36 \times 2},$$
$$= \tfrac{1}{24} \text{ inch},$$

the actual thickness $\frac{3}{4}''$ is therefore 18 times as strong as it need be.

Great waste of metal and sacrifice of appearance would result from making the cantilever of uniform depth throughout. The depth required by theory at different points may be found by equating the moment of flexure at each point with the moment of resistance.

At any point P distant x from the wall $M_P = \dfrac{w(l-x)^2}{2}$.

We have then $\dfrac{w(l-x)^2}{2} = Cad, \qquad d = \dfrac{w(l-x)^2}{2} \times \dfrac{1}{Ca}.$

Taking $l = 6$ feet, When $x = 1$ $\quad d = \dfrac{\frac{1}{2}(6-x)^2}{2} \times \dfrac{1}{1 \times 3},$

$$
\begin{array}{lll}
 & = \frac{5^2}{4} \times \frac{1}{3} = \frac{25}{12} = 25 & \text{inches} \\
x = 2 & d = \frac{4^2}{4} \times \frac{1}{3} = \frac{16}{12} = 16 & ,, \\
x = 3 & d = \frac{3^2}{4} \times \frac{1}{3} = \frac{9}{12} = 9 & ,, \\
x = 4 & d = \frac{2^2}{4} \times \frac{1}{3} = \frac{4}{12} = 4 & ,, \\
x = 5 & d = \frac{1^2}{4} \times \frac{1}{3} = \frac{1}{12} = 1 & \text{inch} \\
x = 6 & d = 0. &
\end{array}
$$

Setting off these values as ordinates at the different points, and drawing the curve of the bottom flange through the extremities of the ordinates, we have Fig. 150.

[1] It should be remembered that this proportion of compression to tension area applies to girders with parallel flanges. If the cantilever be designed as shown in Fig. 150 the compressive stress may be ascertained by the method of moments round the point A, and the flange area must then be proportioned to the stress so found.

The curve might have been ascertained by drawing a parabola with its apex at B through CB.

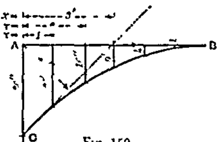


Fig. 150.

It has also been shown that area of the web is largely in excess of the requirements. It is evident, therefore, that large holes may be made in the web, as in Fig. 148, which will lighten the cantilever and improve its appearance without making it too weak. The end of the cantilever may with advantage be thickened as shown by the dotted line, Fig. 148.

The deflection of the cantilever cannot be accurately ascertained, because it has been designed by an approximate formula, in which the resistance afforded by the web has not been taken into account.

Assuming, however, that the stress on the tension flange will not exceed the limit of 1 ton per square inch, the probable deflection may be ascertained by Equation 45, p. 67.

$$\Delta = \frac{n r_t l^2}{E y_t},$$

in which $n = 1$ (for a cantilever of uniform strength and breadth),

$r_t = 1$ ton,

$l = 72$,

$y_t = 16$ inches nearly.

Hence $$\Delta = \frac{1 \times 2240 \times 72^2}{17{,}000{,}000 \times 16},$$

$= 0{\cdot}04$ inch $= \frac{1}{25}$ inch.

The allowable deflection, even if there were to be a ceiling below is $\frac{1}{40}$ per foot run of cantilever $= \frac{6}{40}$, so that this cantilever is amply stiff enough.

If any holes are made in the upper flange to aid in securing the platform, a sufficient width must be added to the upper flange to make up for the loss in area caused by the holes.

Clearly, also, it is not strictly accurate to take the load (including weight of cantilever) uniformly at $\frac{1}{2}$ ton per foot, as the weight of the cantilever itself varies from 8 lbs. per inch run at the wall to almost nil at the end. In this small cantilever this is of no consequence, but in larger work it would become important.

Cast Iron Girder.

Example 11A.—*To find the weight that a given girder will bear.*

Conditions.—A number of old cast iron girders in stock have their cross section throughout of the form and dimensions shown in Fig. 139. Span = 20 feet. What distributed load per foot run will they safely bear when supported at the ends?

1. *By the approximate rule* (Equation 57), taking limiting stress in tension at $1\frac{1}{2}$ ton per square inch,

$$\frac{wl^2}{8} = C a l,$$

$$w = \frac{C a l \times 8}{l^2},$$

$$=\frac{1\frac{1}{2}\times 16\times 20\times 8}{240^2},$$

$$=0{\cdot}067 \text{ ton per inch,}$$

$$=0{\cdot}8 \text{ ton per foot run.}$$

If C is taken at $1\frac{1}{4}$ ton, as recommended at p. 95, the safe distributed load would be $\frac{5}{6}$ $0{\cdot}8=0{\cdot}66$ ton per foot run.

2. *By the graphic method*, taking the limiting stress in tension as $1\frac{1}{2}$ ton. From p. 98 we know $\overline{\mathrm{M}}$ to be 417·5 inch-tons. Hence

$$\frac{wl^2}{8}=417{\cdot}5.$$

$$w=0{\cdot}058 \text{ ton per inch,}$$

$$=0{\cdot}70 \text{ ton per foot.}$$

3. *By the mathematical method.*—Finding I as in Appendix XIV. to be 1950, and knowing the tension flange to be the weakest, we have, since

$$\frac{wl^2}{8}=\frac{r_t\mathrm{I}}{y_t},$$

$$\frac{wl^2}{8}=\frac{1\frac{1}{2}\times 1950}{7},$$

$$=417{\cdot}8 \text{ inch-tons,}$$

and $w=0{\cdot}058$ ton per inch, as by the graphic method.

Deflection.—To find the deflection of this girder when loaded with a distributed load of ·7 ton per foot as found above by the graphic and mathematical methods.

From Equation 44, p. 66, and Table C, we have $\Delta=\frac{n\mathrm{W}l^3}{\mathrm{EI}}$,

$n=\frac{5}{384}$ for a girder of uniform cross section supported at the ends and loaded uniformly.

I = 1950, see Appendix XIV. E = 17,000,000 lbs. per square inch.

$$\Delta=\frac{5}{384}\times\frac{14\times 2240\times(20\times 12)^3}{17{,}000{,}000\times 1950}=\frac{1}{6}\text{ inch.}$$

The deflection allowable if there is to be a ceiling below would be $\frac{1}{40}''$ per foot of span, or $\frac{20}{40}=\frac{1}{2}''$. The girders would therefore be amply stiff enough.

CHAPTER VI.

TENSION AND COMPRESSION BARS.

IN the previous chapters we have considered various methods of calculating the strength of beams made of one piece of material—either wood, wrought or cast iron. Such beams can only be made of comparatively small dimensions, and in the case of wood or wrought iron it is, as already pointed out, uneconomical to make them beyond a certain size, because, in a uniform section, which does not accommodate itself to the varying stresses, much material is thrown away.

Large beams or girders are therefore built up of smaller pieces suitably connected together, thus forming either plate girders or open-work girders in wrought iron, or trusses in timber. Moreover, roof trusses are composed of a number of pieces or members jointed together (see Parts I.-II.). The various members of such structures will be subject to tension or compression or shear according to the position of the member in the girder or roof truss. The first step, after designing such a structure, is to ascertain the amount and nature of the stress each member has to bear; and the next to design each member so as to safely resist the stress in it, as will be explained in the present chapter. Lastly, the various connections must be of adequate strength to transmit the stress from one member to another. This forms the subject of "*joints*," and will be considered in the next chapter.

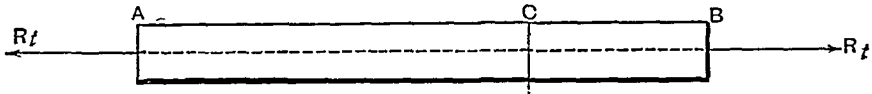
It will, however, be best to leave for the present the question of finding the stresses in the various members; we will therefore deal first with the design of separate members, assuming the stress upon each as known.

Tension Bars.

Symmetrical Stress.—Let AB, Fig. 151, represent a tension bar stretched by the forces R_t, R_t. The dotted line shows the line of action of these forces, and we will suppose that it is exactly in the centre of the bar. By cutting the bar across at C, as

shown in Fig. 152, the tension forces excited in the fibres of the bar are, so to speak, brought to view; and since the line of

Fig. 151.



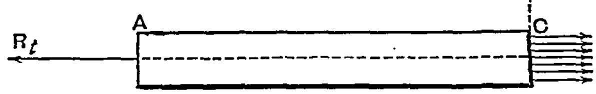


Fig. 152.

action of the force R_t is central, these forces are equal, or, in other words, the stress is *uniformly distributed* over the cross section of the bar. Clearly therefore: *Safe resistance to tension of the bar = safe resistance to tension of the material per unit of area × area of cross section.*

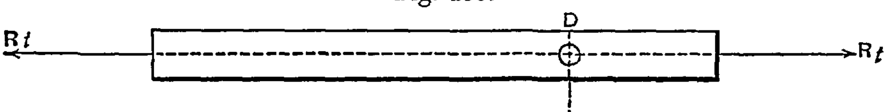
Thus a wrought iron bar $2{\cdot}5'' \times 1''$ could resist safely 5 tons per square inch or $5 \times 2{\cdot}5 = 12{\cdot}5$ tons.
Or, again, a round iron bar 1″ diameter could resist safely

$$5 \times 0{\cdot}78 = 3{\cdot}9 \text{ tons.}$$

Effective Area.—Let us now inquire into the effect of making holes in the bar, such as would be made for rivets.

In the first place let there be one hole at the *centre line* of the bar, as at D, Fig. 153. The cross section at D is clearly the

Fig. 153.



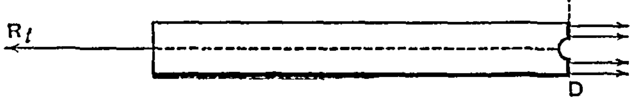


Fig. 154.

weakest in the bar, and it is there where rupture would take place. The stress at the section is shown in Fig. 154, and it will be noticed that, as before, the stress is uniformly distributed, but that the area over which it acts is reduced. This reduced area is called the *effective area* of the tension bar and: *Safe resistance to tension of bar = safe resistance to tension of the material × effective area.* Or in symbols

$$R_t = r_t \times A \quad . \quad . \quad . \quad (58).$$

The same formula would clearly be applicable if two holes

of elasticity which may exist in different parts of the column; for instance, a variation of 2 per cent in the modulus of elasticity in a solid cylindrical column will reduce the strength by 26 per cent.

Struts of rectangular section.—It can be shown, by an investigation too difficult for these Notes, that the *average* ultimate stress per square inch for an ideal column of rectangular section, in which there is no variation of the modulus of elasticity, is given by the formula

$$r_c = \pi^2 \mathrm{E} \cdot \frac{1}{12} \frac{d^2}{l^2} \quad . \quad . \quad . \quad . \quad (59),$$

where d is the shortest side of the rectangular cross section and l the length of

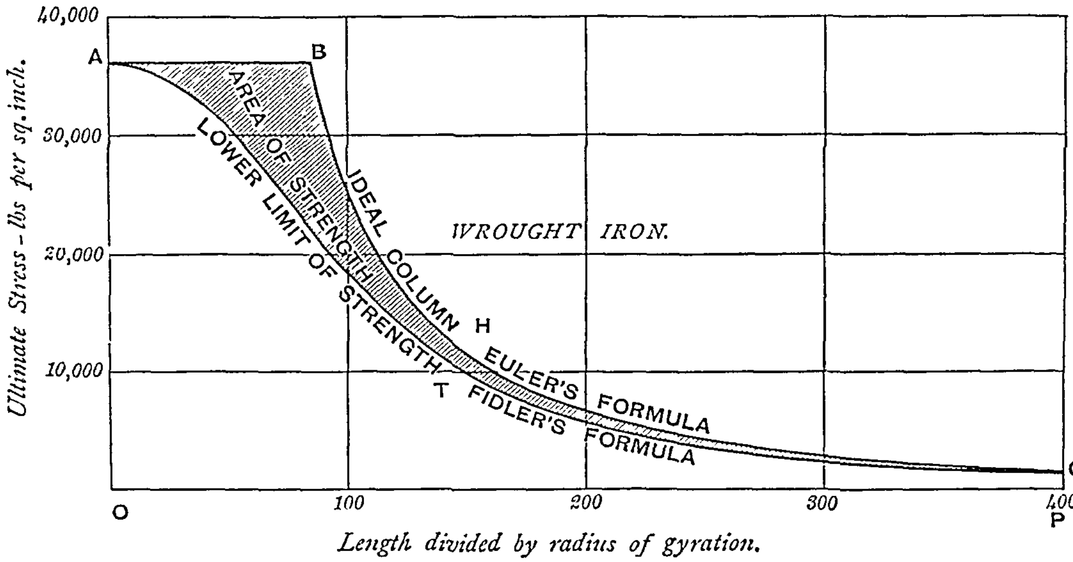


Fig. 166.

the column. This formula is given in *Der Constructeur*, by Rouleaux; it is based on Euler's theory, and is plotted graphically in Fig. 166 in the curve BHC, on the assumption that the material is wrought iron, that is, taking E = 1200 tons. In this figure the abscissæ measured along OP are the ratios[1] $\sqrt{12} \cdot \frac{l}{d}$, and the ordinates are the corresponding breaking weights in lbs. per square inch. AB is drawn at the ultimate resistance to crushing of the material.

Now Prof. Fidler shows in the paper above alluded to, that under the most unfavourable conditions which are all likely to occur in practice, the column may be weakened to the extent shown by the curve ATC. This curve therefore gives the minimum strength of wrought iron columns of rectangular cross-section for varying ratios of $\sqrt{12} \cdot \frac{l}{d}$. The *actual* strength of any particular column may therefore lie anywhere between these two curves, and Prof. Fidler comes to this conclusion: "that the strength of columns *cannot be defined by any*

[1] This ratio is length divided by radius of gyration (see top of next page). If κ be the radius of gyration of a cross section then $\kappa^2 = \frac{\text{moment of inertia of the cross section}}{\text{area of the cross section}}$ = for rectangular sections (see Appendix XIV.) $\frac{bd^3}{12} \times \frac{1}{bd} = \frac{d^2}{12} \therefore \kappa = \frac{d}{\sqrt{12}}$.

hard and fast line, even when the modulus for the whole column and the ultimate strength of the material are accurately known; but, on the contrary, the strength may have any value less than that of the ideal column within certain limits. The strength of columns must, therefore, be represented by *an area*, as shown in Fig 166, within which the results of individual experiments may be expected to place themselves at haphazard"

The advanced student is strongly recommended to read Prof Fidler's paper,[1] in which will be found the formula he arrives at—a formula which is, however, too complicated for these Notes.

Struts of any cross section—We have confined ourselves so far to columns of rectangular cross section; the same reasoning, however, applies, whatever may be the cross section, but we must substitute for $\frac{1}{\sqrt{12}}d$ (which is the radius of gyration for a rectangular section) (see Equation 59) the expression for the radius of gyration of any cross section, which can be found from

$$(\text{Radius of gyration})^2 = \frac{\text{Moment of inertia of the cross section}}{\text{Area of cross section}}$$

The value of the moment of inertia of the cross section varies according to the axis about which it is taken, but when taken about an axis through the CG of the cross section, perpendicular to the direction in which the column tends to bend, it is a minimum, and the radius of gyration found from this moment of inertia is the one to insert in the formula.

Practical Value of d—It is, however, somewhat laborious to obtain this radius of gyration, and for practical purposes, at any rate for building con

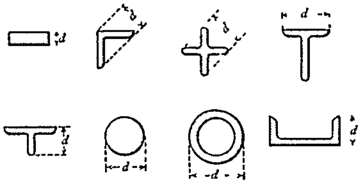


Fig 167

struction, the value of d can be taken as shown in Fig 167, multiplying by a factor n as shown in Table V, which has been compiled from Fidler's formula and from similar tables given in the paper, and is recommended for use

Many other formulæ have been proposed for finding the strength of columns, and some of them will be mentioned later on.

Short and Long Compression Bars.

It is often the practice to divide compression bars into two classes, viz short compression bars and long compression bars. Short compression bars are those which fail by *direct* crushing, and their strengths are calculated as if the pressure were uniformly distributed over the cross section. Long compression bars are those

[1] Also *Bridge Construction*, by T Claxton Fidler

ends fixed is considerably stronger than one with the ends rounded. Again, one end may be rounded and the other fixed, in which case the strength will have an intermediate value (see Fig 171) In any particular case, before the ends of a compression bar are assumed to be fixed, the matter should be carefully considered, because it often happens that what may appear at first sight to be *fixed* is not fixed at all. The criterion is whether the ends are so connected as to *constrain* the bar to bend as shown in Fig 169. Thus the end of a bar fixed to a T iron as in Fig 172 is *not* "fixed," because the bar can bend as shown in Fig. 173 by twisting the T iron. There is, however, a certain amount of constraint due to the resistance of the T iron to bending, so that the bar is stronger than if its ends were purely hinged. These last remarks apply to the case when the ends are flat (see Fig. 174)


Fig 172

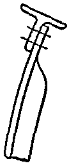


Fig 173


Fig 171

Gordon's Formula. — The formula principally used in this country for long compression bars is that known as Gordon's formula, namely —

$$R_c = \frac{A r_c}{1 + a\frac{l^2}{d^2}} \quad . \quad . \quad (60),$$

where a is a constant determined by experiment, d is the least dimension of

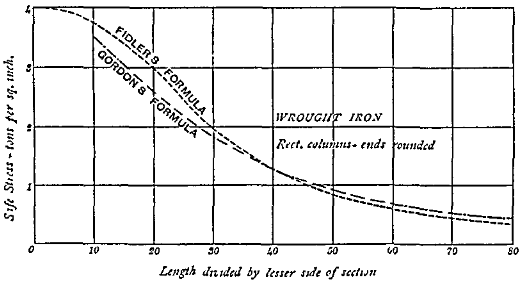


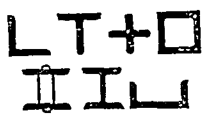
Fig 175.

the cross section, as indicated in Fig. 167, A the area of the cross section, and r_c the safe resistance to compression of the material per square inch.

The value of the constant a depends not only on the material but also on the form of the compression bar. The following Table (G) gives some of these values.

To compare this formula with Fidler's formula, when the material is wrought iron, the curves shown in Fig. 175 have been drawn. It will be seen that the agreement is fairly good, but that Gordon's formula gives lower results for short columns and higher results for the longer columns.

TABLE G.

Material.	Form of Cross-section.	Values of a when Both ends are rounded or hinged.	Both ends are fixed.	One end is rounded, the other fixed.	r_c
Timber	Rectangular	$\frac{4}{250}$	$\frac{1}{250}$	$\frac{1}{100}$	13 cwts.
,,	Circular				
Wrought iron	Rectangular	$\frac{4}{2500}$	$\frac{1}{2500}$	$\frac{1}{1000}$	4 tons.
,,	Circular (solid)				
,,	Circular (hollow)				
,,		$\frac{4}{900}$	$\frac{1}{900}$	$\frac{1}{360}$	
,,					
Cast iron	Circular (solid)	$\frac{1}{100}$	$\frac{1}{400}$	$\frac{1}{160}$	8 tons.
,,	Circular (hollow)	$\frac{1}{200}$	$\frac{1}{800}$	$\frac{1}{320}$	
,,	Rectangular	$\frac{3}{400}$	$\frac{3}{1600}$	$\frac{3}{640}$	
,,	Cross-shaped	$\frac{3}{200}$	$\frac{3}{800}$	$\frac{3}{320}$	

N.B.—In using this Table it must be remembered that it is calculated with a factor of safety of 4. In many structures, such as those of cast iron or timber, a factor of safety of 6 to 8 or 10 is necessary for safety, and care must be taken in such cases to modify the value of r_c accordingly.

Formulæ for Wooden Struts.—The formulæ in use for wooden struts are more at variance than those for iron, and it will be useful to compare some of them. For instance:—

Professor Rouleaux gives this formula in *Der Constructeur* [1]—

$$R_c = \pi^2 \frac{IE}{l^2} \div \text{factor of safety} \quad . \quad . \quad . \quad (61),$$

in which I is the moment of inertia, E the modulus of elasticity which reduces to the following, taking E = 1,300,000 lbs.[2] and a factor of safety between 4 and 5,

$$R_c = 2000 \frac{bd^3}{l^2} \text{ cwts.}$$

[1] The letters have been changed to agree with the notation in this book. See also Equation 59.

[2] This modulus must be converted into cwts. before reducing the formula, because the safe load is to be expressed in cwts.

Professor Ritter gives the following—

$$R_c = \frac{Ar_c}{1 + 0.0027\left(\frac{l}{d}\right)^2} \quad . \qquad (62),$$

which is the same as Gordon's, but with a different constant. In both these formulæ the ends are supposed to be rounded

Example 14—To compare the formulæ let us find by each of them the safe resistance to compression of a wooden strut 10 feet long and 4′ × 3″ cross section, ends rounded

By Gordon's formula

$$R_c = \frac{4 \times 3 \times 13}{1 + \frac{4}{250}\left(\frac{120}{3}\right)^2},$$

$$= 5.8 \text{ cwts}$$

By Rouleaux's formula

$$R_c = 2000 \frac{4 \times 3^3}{120^2},$$

$$= 15 \text{ cwts}$$

By Ritter's formula

$$R_c = \frac{4 \times 3 \times 13}{1 + 0.0027 \times 40^2},$$

$$29.3 \text{ cwts}$$

These results vary so, that it is difficult to make use of them, and we must resort to some other method

Table for Wooden Struts.—Table VI was deduced from some experiments by Mr Kirkaldy,[1] but modified to agree with Mr. Fidler's views (see p 110)

The above example works out as follows by means of this Table —

$$\text{Ratio} = \frac{l}{d} = 40 \text{ (col R in Table)},$$

Safe intensity of compression per square inch = 1.06 cwt.,

Total safe resistance = 4 × 3 × 1.06 = 12.7 cwts

Again, from Table VII (Stoney's Table) we find for a ratio of 40, 80 tons as the breaking load per square foot, or taking a factor of safety of 4 the safe load = 20 tons per square foot Hence in the case of the above example

$$R_c = \frac{20 \times 20 \times 4 \times 3}{12 \times 12},$$

$$= 33.3 \text{ cwts.}$$

This Table is, however, deduced from experiments with large timbers with flat ends adjusted as in ordinary practice, so that the ends might be considered as partially fixed Working out the example again for fixed ends by means of Table VI we find

Safe intensity of compression for ratio 40

= 3.60 cwts. per square inch,

$R_c = 12 \times 3.60 = 43.2$ cwts.

Table for practical use—It is clear from the above that the subject of

[1] Corps Papers, Royal Engineers (Vols XXII and XXIII)

$$d = 4 \text{ inches},$$
$$t = 0.5 \text{ inch},$$
$$r_c = 8 \text{ tons}.$$
$$A = \frac{\pi}{4} d^2 - \frac{\pi}{4}(d - 2t)^2,$$
$$= \pi t (d - t) = \pi \tfrac{1}{2}(4 - \tfrac{1}{2}),$$
$$= 5.5 \text{ square inches}$$

Or using a table of areas of circles

$$A = \text{area of circle 4" diameter} - \text{area of circle 3" diameter},$$
$$= 12.57 - 7.07 = 5.5$$

Gordon's Formula.—Hence, using Gordon's formula, since

$$a = \tfrac{1}{800} \text{ (for fixed ends)},$$
$$R_c = \frac{8 \times 5.5}{1 + \frac{1}{800}\left(\frac{120}{4}\right)^2} = 20.7 \text{ tons}$$

Fidler's Formula.—Again, referring to Table V, p 332, we find the value of n for a circular hollow column to be 3.1. The ratio of l to d is $\frac{120}{4}$, therefore $\frac{l}{\kappa} = 3.1 \times \frac{120}{4} = 93$ (see p 333). From Table V we find the safe stress in tons per square inch. When $\frac{l}{\kappa} = 95$ (the nearest to 93) the safe stress is 3.66 tons per square inch.

Hence

$$\text{Safe load} = 5.5 \times 3.66,$$
$$= 20 \text{ tons}.$$

This is rather more strength than required, but looking to the uncertain nature of cast iron, and the possibility of flaws, this section is *not* too large.

Since the actual load is 16.6, and according to Gordon's formula the safe load is 20.7 tons with a maximum intensity of stress of 3.76 tons per square inch, the actual maximum intensity of stress will be

$$\frac{16.6}{20.7} \times 3.76 = 3.0 \text{ tons per square inch}$$

Cast Iron Column

Example 16.—Find the safe load that can be placed on a cast iron column of ⊢⊣ section of the following dimensions. Length 15 feet, width of flanges 6", width across the flanges 8", thickness of metal (flanges and web) 1".

It will be found that the area of the web is half the area of the flanges together. Hence, referring to Table V and p 333, the proper value to take for n is 4.2, so that

$$\text{Ratio } \frac{l}{\kappa} = \frac{15 \times 12}{6} \times 4.2 = 126$$

If, therefore, the ends are supposed to be *rounded*, the safe stress per square inch is 0.86 ton, and the safe load

$$= (2 \times 6 + 6 \times 1) \times 0.86 = 15.5 \text{ tons}$$

But if the ends are supposed to be *fixed*, the safe stress would be 2.19 tons per square inch, Table V, and the safe load 39.4 tons. In practice it would not, however, be safe to rely on the ends being fixed unless special precautions are taken.

Wrought Iron Strut.

Example 19.—Design a wrought iron strut of the form shown in Fig 177 to bear a stress of 1.69 ton, the length being 2′ 11″

In struts of this form it can always be arranged to make the tendency of the strut to bend as a whole less than the tendency of the sides to bend between the distance pieces,[1] or in other words, such a strut will tend to fail as indicated in Fig 178. The strength of such struts will therefore to a large

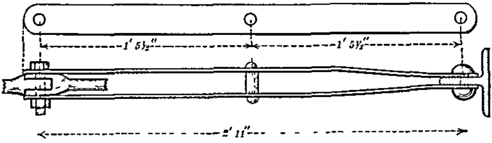


Fig. 177

extent depend on the number of distance pieces used[2] Clearly the portions of each side bar between two distance pieces (such as *ab*, Fig 178) can be looked upon as a long compression bar hinged at the ends

A simple way of designing such struts is to assume some convenient cross section for the flat bar iron forming the sides, and then to calculate how far apart the distance pieces can safely be placed

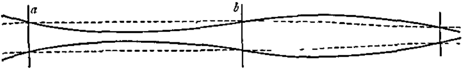


Fig 178

In this example the stress is small, and therefore very small side bars will probably suffice

For instance, assume $1\frac{3}{4}'' \times \frac{3}{8}''$ as the cross section of each side bar Then, as each has to bear half the stress or 0.85 ton, the intensity of stress will be

$$=\frac{0.85}{1\frac{3}{4}\times\frac{3}{8}}=1.3 \text{ ton per square inch}$$

On referring to Table V. it will be seen that this intensity of stress corresponds to a ratio of 135 Hence, since $n = 3.5$ for a rectangular cross section, maximum safe distance between distance pieces

$$=\frac{3}{8}\times\frac{135}{3.5}=14.5''$$

One distance piece is therefore hardly sufficient, and it would be better to use two, as shown in Fig 179

---

[1] The student is recommended to test this statement in the present case by taking the value of $n = 3.0$ from Table V.

[2] Within the limits of the strength of the strut as a whole, which it will be safer to consider as hinged at the ends in either plane of flexure

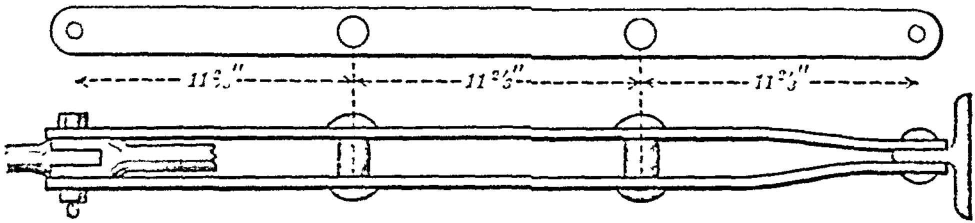


Fig. 179.

will be found that the section of the principal is given as $3\frac{1}{2}'' \times 3 \times \frac{1}{2}''$. The roofs referred to in the above Table are of the same kind and shape as the one in which the stresses are found in Example 42, and the rise ($\frac{1}{5}$ span) and camber of tie-rod are the same as assumed in the example On the other hand, however, as stated in Molesworth's *Pocket-Book*, in the Table the trusses are supposed to be 6 feet 8 inches apart, whereas in the example they were taken as being 8 feet apart, and although not stated, it is probable that the covering on the roofs in the Table is corrugated iron or zinc, whereas in the example the roof-covering is countess slates on $\frac{3}{4}''$ boarding

It will be interesting to investigate this a little more closely

Now, on referring to Example 42, Table II, it will be seen that the stress in the principal (AD) due to roof covering, etc., and snow is 4 45 tons, and on referring to p 209 it will be further seen that this stress is due to a load of 20 2 lbs per square foot of roof surface If the roof-covering is altered *to corrugated iron the load will be reduced by about 8 lbs per square foot* (see Table XIII), or say to 12 2 lbs per square foot The stress of 4 45 tons will therefore be reduced to

$$4\ 45 \times \frac{12\ 2}{20\ 2} = 2\ 68 \text{ tons},$$

and accordingly the total stress in the principal will be reduced from 7 83 tons to

$$7\ 83 - 4\ 45 + 2\ 68 = 6\ 06 \text{ tons},$$

and diminishing the distance apart of the trusses from 8 feet to 6 feet 8 inches will further reduce the stress to

$$6{\cdot}06 \times \frac{6\ 6}{8\ 0} = 5\ 05 \text{ tons}$$

We have now to find what stress a T iron principal 10 8 feet long and $3\frac{1}{2} \times 3 \times \frac{1}{2}$ cross section can bear

As before,

$$\text{Ratio} = \frac{10\ 8 \times 12}{3} \times 4\ 9 = 212$$

$$\text{Safe stress per square inch} = 1\ 01 \text{ ton}$$

$$\text{Total safe stress} = 1\ 01 \times (3 + 3)\tfrac{1}{2},$$

$$= 3 \text{ tons}$$

From the above it appears that it would be wiser to use the stronger section obtained by the calculations on p 120, at any rate for important roofs and in exposed situations, where full allowance for wind pressure ought to be made

Comparison with Hurst's Table—In Hurst's *Pocket Book* $4\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{1}{2}''$ are given as the dimensions for the principals for a trussed rafter roof 40 feet span, rise $\frac{1}{4}$ span, and camber of tie rod $\frac{1}{30}$ span, covering countess slates on boards Distance apart of trusses, 6 feet

Now in this case the length of the principal rafter from A to D (Fig 360) is 11 7 feet, so that

$$\text{Ratio} = \frac{11\ 7 \times 12}{3\ 5} \times 4\ 9 = 196$$

$$\text{Safe stress per square inch} = 1\ 12$$

$$\text{Total safe stress} = (4\ 5 + 3)\tfrac{1}{2} \times 1\ 12,$$

$$= 4\ 8 \text{ tons nearly},$$

which would be almost enough for a corrugated iron covering

CHAPTER VII.

JOINTS AND CONNECTIONS.

A GREAT variety of joints for connecting the various members of wooden and iron structures have been described in Parts I. and II. It is proposed to show in this Part how to determine the dimensions of the various portions of a joint when the stress to be borne is known.

Riveted joints are principally used for built-up wrought iron beams and roofs, but occasionally pin joints are employed. In roofs, screw connections, cotter joints, and pin connections are used, and these will also be described in this chapter.

Points to be observed in making Joints.

These are as follows :—

1. The members of the structure connected together should be weakened as little as possible. They are liable to be weakened, as will be seen in the sequel, either by cutting away material (for instance, a minus screw thread), or by the design of the joint being such as not to transmit the stress uniformly over the cross section.
2. Each part of the joint should be proportioned to the stress it has to bear.
3. Abutting surfaces in compression should be placed as nearly as possible perpendicular to the stress.

RIVETED JOINTS.

The manner in which these joints are made and the technical names given to them were dealt with in Part I., and it therefore only remains to show how the strength of such joints can be calculated.

We will first consider riveted joints in tension.

Riveted Joints in Tension.

Lap Joint.—Starting with a single lap joint with only one

rivet, as shown in Fig. 180, it is clear that this joint is liable to fail—

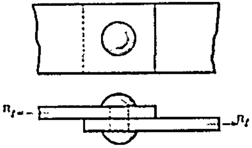


Fig. 180.

1. By one or both bars tearing across, Fig. 181.

2. By the rivet cutting into the metal of the bar, or by the bar cutting into the rivet, according to which metal is the softer, Fig. 182.

3. By the rivet being sheared, Fig. 183 (single shear in this case).

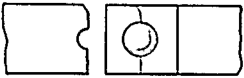


Fig. 181.

4. By the rivet shearing a piece out of the bar, Fig. 184.

Let t be the thickness, b the breadth of the bar, and d the diameter of the clenched rivet and of the rivet-hole,[1] also R_t the tension in the bar, r_t the working resistance per square inch in tension = 5 tons.

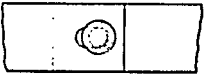


Fig. 182.

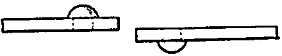


Fig. 183.

1. To guard against the first manner of failing
$R_t = (b - d)tr_t$. (63).

2. The resistance to bearing T_b has been found by experiment to vary as the thickness and as the diameter of the rivet, that is, as td; so that if r_b is the safe resistance to bearing per square inch


Fig. 184.

$$T_b = tdr_b \quad . \quad . \quad . \quad (64).$$

The value of r_b of course depends on the quality of the iron employed, but it is usually safe to take it as 8 tons per square inch.

3. The resistance of the rivet to shearing T_s varies as the area of the cross section of the rivet, namely $0{\cdot}78d^2$, therefore

$$T_s = 0{\cdot}78d^2 \times r_s \quad . \quad . \quad . \quad (65).$$

r_s can be taken at 4 tons per square inch.

4. To guard against the fourth manner of failing, the lap should be sufficient to prevent the piece of the bar shearing out.

[1] See p. 136.

Two surfaces have to be sheared, each having an area of $t \times \frac{l}{2}$ (where l is the lap) and the resistance to shearing is therefore

$$T'_s = 2t \times \frac{l}{2} \times r_s,$$

$$= tlr_s \quad . \quad . \quad . \quad . \quad (66).$$

The joint should be so arranged that T_b, T_s, and T_s' are each greater than T, so that the metal of the tension bar may be subjected to the full, safe, tensile stress it is capable of resisting; that is, the joint itself must not be less strong but a little stronger than the tension bar to be connected.

Example 21.—As an example assume

$$b = 1{\cdot}4'', \ t = \tfrac{1}{2}'', \ d = \tfrac{3}{4}''.$$

Then

$$R_t = (1{\cdot}4 - 0{\cdot}75)\tfrac{1}{2} \times 5 = 1{\cdot}62 \text{ ton.}$$
$$T_b = \tfrac{1}{2} \times \tfrac{3}{4} \times 8 = 3{\cdot}0 \text{ tons.}$$
$$T_s = 0{\cdot}78 \times (\tfrac{3}{4})^2 \times 4 = 1{\cdot}76 \text{ ton.}$$

By putting $T_s' = 1{\cdot}62$ ton we get the *least* value of l, namely

$$l = \frac{1{\cdot}62}{4t} = \frac{1{\cdot}62}{2},$$
$$= 0{\cdot}81 \text{ inch.}$$

Practically, however, a lap of only 0·81 inch is quite insufficient, because the rivet holes would be much too close to the end of the bar, from a manufacturing point of view. In the present case the lap should be

$$2 \times 1\tfrac{1}{2} \times d = 2\tfrac{1}{4} \text{ inches}$$

(see p. 136).

Single riveted lap joint.—The equations already deduced can be easily applied to the case of the joint shown in Fig. 185, thus

$$R_t = (b - 3d)tr_t,$$
$$T_b = t(3d)r_b,$$
$$T_s = 3(0{\cdot}78d^2)r_s.$$

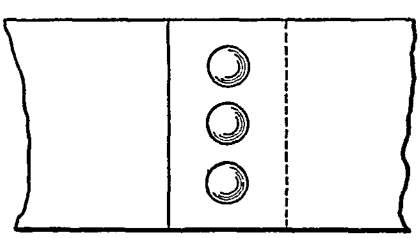


Fig. 185.

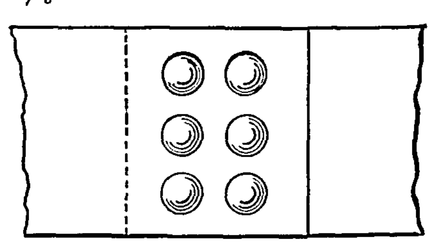


Fig. 186.

Double riveted lap joint.—And similarly for the joint shown in Fig. 186.

$$R_t = (b - 3d)tr_t,$$
$$T_b = t(6d)r_b,$$
$$T_s = 6(0{\cdot}78d^2)r_s.$$

Butt joint with single cover plate.—The joint shown in Fig. 187, with a single cover plate, can be looked upon as two joints like that shown in Fig. 185 put close together, and the equations

for that joint will therefore be applicable. Clearly the cover plate should have at least the same thickness as the plates to be joined.[1]

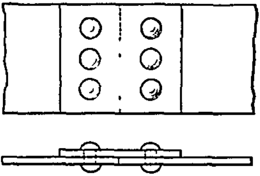


Fig 187

The double riveted single cover joint shown in Fig 188 can be calculated from the equations for the joint shown in Fig 186.

In the above it has been assumed, in finding the resistance to tearing of the bars, that the tensile stress was uniformly distributed over the cross section. A glance at Figs

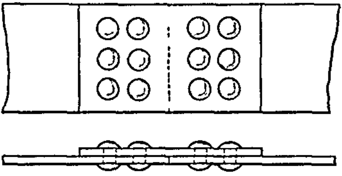


Fig 188

189 and 190 will, however, show that this assumption is not

Fig 189

Fig 190

warranted, and that the strength of the bar is less than calculated, the tendency of the stress being to draw the bars into the position shown. This is one of the cases mentioned at p 108, in which the stress is not transmitted down the centre of the bar, and an allowance should be made, but the values of T_b and T_s are not affected.

Butt joint with double cover plate.—This reduction in the strength of a tie bar can be avoided by using two cover plates as shown in Fig 191, which arrangement clearly allows of the stress being uniformly distributed. As regards the thickness of the cover plates, each should have at least half[2] the thickness of the plates

[1] Prof Unwin says that a single cover plate should be $1\frac{1}{8}$ the thickness of the plates joined at least in boiler work.—*Machine Design*

[2] Prof Unwin says $\frac{5}{8}$, at least in boiler work.

to be joined; the resistances of the covers to tearing and to bearing will then be at least equal to those of the plates. It is to be observed that the shearing resistance of the rivets is doubled by the use of two cover plates, because each rivet has to shear along *two* sections instead of only along *one*, as shown in Fig. 192. These rivets are said to be in "double shear."[1]

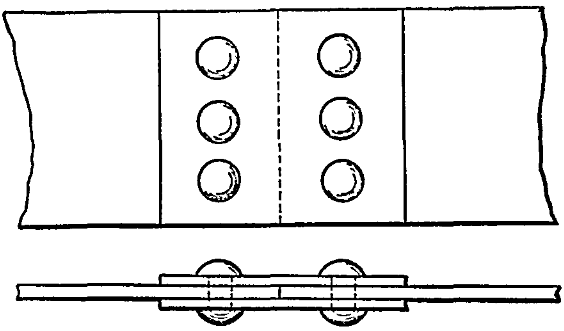


Fig. 191.

Fig. 192.

Joint, Fig. 191.—The equations to find the strength of this joint are therefore

$$R_t = (b - 3d)tr_t,$$
$$T_b = t(3d)r_b,$$
$$T_s = 6(0{\cdot}78d^2)r_s.$$

Comparing the joint shown in Fig. 193 with that shown in Fig. 194 it will be seen that, other things being equal, the first weakens the plates more than the second does, but that they have the same bearing resistance and the same resistance to shearing.

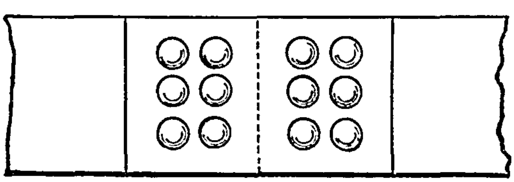


Fig. 193.

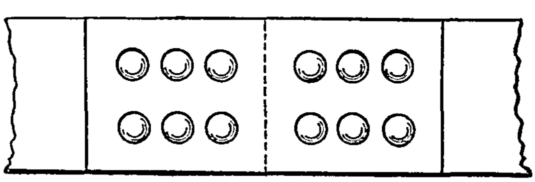


Fig. 194.

The joint shown in Fig. 195 has the same bearing and shearing resistance as the two above, but weakens the plates by only one rivet hole.

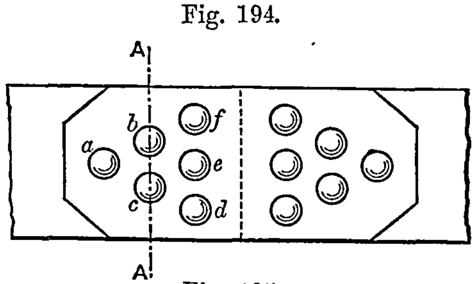


Fig. 195.

Considering a section AA, the plate is weakened by two rivet holes *bc*, but before it can tear across this section, the first or leading rivet *a* must fail by shearing or bearing, which practically makes up the difference: so that this

[1] Some experiments show that the resistance to shearing on two sections is about ⅛ less than double the resistance on one section.

section is as strong as the one through the leading rivet. Similarly, the section through the three rivets *def* is stronger than the two first, there being three rivets *abc* to fail as above, to set against the weakening due to two rivet holes. It should, however, be noticed that the cover plates should be made thicker than half the thickness of the plates, since *their* section is reduced by three rivet holes.

Joint, Fig. 195.—The equations for this joint are therefore

$$R_t = (b - d)tr_t,$$
$$T_b = 6tdr_b,$$
$$T_s = 2 \times 6 \times 0.785d^2 r_s,$$

and the thickness t of each of the cover plates can be found from

$$R_t = 2(b - 3d)t r_t$$

or

$$t = \frac{R_t}{2(b - 3d)r_t}$$

Joint, Fig. 196.—These equations also apply to the joint shown in Fig.

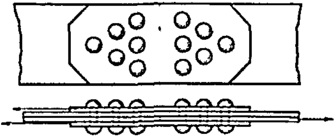


Fig. 196.

196 except as regards the cover plates. The upper plate, in which there is no joint, acts simply as a distance piece, and does not affect the resistance to shearing, or to bearing, of the rivets. But the lower cover plate now transmits more stress than the upper one, as shown by the arrows, and should therefore be made thicker: in the present case very nearly in the proportion $\frac{1}{1} = \frac{2}{1}$, so that $\frac{1}{3}t$ and $\frac{2}{3}t$ are the proper thicknesses to give to the top and bottom cover plates respectively. There is, however, a danger when carrying out the work, of transposing the cover plates, so that in practice both are made of equal thickness, and usually of the same thickness as the plates to be jointed, so as to be on the side of safety.

General Formulæ for riveted Joints other than grouped Joints.

The student ought to have no difficulty in deducing from the foregoing the following general formulæ for riveted joints (other than grouped joints).

Let R_T be the tensile stress to be borne by the tension bar (denoted by R_t in the previous equations).

α the number of rivets required for resistance to *bearing*.

β the number of rivets required for resistance to *shearing*.

s the number of sections at which each rivet tends to shear (*i.e.* $s=1$ for lap and single cover joints, and $s=2$ for double cover joints).

k the number of rivets in the first or outermost row, that is, the number of rivet holes by which the tension bar is weakened.

b the breadth of the bar.

t the thickness of the bar and also of the cover plates.

d the diameter of rivet holes.

Then

$$R_T = r_t . (b - kd)t \quad . \quad . \quad . \quad . \quad . \quad (67).$$

$$\alpha = \frac{R_T}{r_b td},$$

$$= \frac{r_t}{r_b} \cdot \frac{b - kd}{d} \quad . \quad . \quad . \quad . \quad . \quad (68).$$

$$\beta = \frac{R_T}{r_s \times s \times 0{\cdot}78 d^2},$$

$$= \frac{r_t}{r_s} \cdot \frac{(b - kd)\, t}{0{\cdot}78 s d^2} \quad . \quad . \quad . \quad . \quad . \quad (69).$$

Whichever is greater, α or β, must be adopted as the number of rivets in the joint.

The calculations connected with the above are much shortened by the use of Table VIII.

Double-cover Riveted Joint.

Example 22.—Design a double-cover riveted joint similar to Fig. 195, the plates to be joined being 9″ broad and $\frac{5}{8}''$ thick.

We have

$$s = 2,$$
$$k = 1,$$
$$b = 9,$$
$$t = \tfrac{5}{8}'',$$

and from the practical rules for Riveted Joints, p. 136,

$$d = \tfrac{15}{16}.$$

Hence from (68)

$$\alpha = \frac{5}{5} \cdot \frac{9 - 1 \times \frac{15}{16}}{\frac{15}{16}},$$
$$= 6 \text{ nearly}.$$
$$\beta = \frac{5}{4} \cdot \frac{(9 - \frac{15}{16})\frac{5}{8}}{(0{\cdot}78 \times 2 \times (\frac{15}{16})^2},$$
$$= 4 \text{ nearly}.$$

Each side of the joint must therefore contain six rivets. On reference to p. 136, it will be seen that the minimum pitch ought to be $2\frac{1}{2}''$; thus only three rivets can be placed in one row across the width of the bar. Hence the required joint will be as shown in Fig. 197.

With the aid of Table VIII. the result can be obtained more quickly as follows:—

The resistance to bearing of one $\frac{15}{16}''$ rivet in $\frac{5}{8}''$ plates is 4·7 tons, and

the resistance to double shear is $2 \times 2.76 = 5.52$ tons. The number required for bearing must therefore be adopted. This number is

$$= \frac{R_t}{4.7} = \frac{5(9 - 1 \times \frac{15}{16})\frac{5}{8}}{4.7},$$

$= 6$, nearly, as before

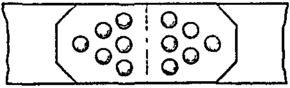


Fig 197

Formulæ for Grouped Joints.

We have next to consider "grouped" joints (see p 93, Part I)

EQUATIONS FOR JOINTS WITH THREE PLATES *For Shearing*—Taking a

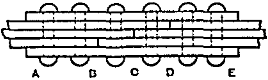


Fig 198

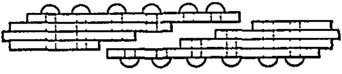


Fig 199

simple case of three plates joined together as shown in Fig 198, we see that the joint can fail by the shearing of all the rivets, as indicated in Fig 199

Let n_1 be the number of rivets in each end group (*i e* between A and B and D and E), n_2 be the number of rivets between the joints (*i e* between B and C and C and D), t the thickness of the plates, and t_c the thickness of the covers.

Then it is clear that the number of rivets sheared through, if the joint were to fail as above, is

$$2n_1 + 2n_2$$

and therefore

$$R_t = (2n_1 + 2n_2)0.78d^2r_s \qquad (70)$$

For bearing—The joint might also fail by the rivets cutting into the plates, as shown in Fig 200 From A to B the resistance to bearing is

$$n_1 \times 3tdr_b,$$

and from B to C it is

$$n_2 \times 2tdr_b,$$

and from C to D

$$n_2 \times tdr_b$$

So that altogether

$$R_t = (3n_1 + 3n_2)dtr_b \qquad (71)$$

From Equations 70 and 71 the *total* number of rivets required can be obtained. Two values will be found for $2(n_1+n_2)$, one being the number of

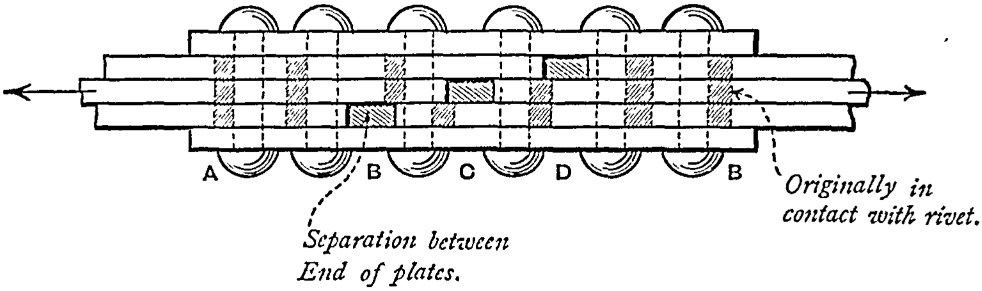


Fig. 200.

rivets required for shearing, and the other the number required for bearing; the larger value is to be taken.

So far we have only found the sum of $2n_1+2n_2$, and to obtain n_1 and n_2 separately we must consider two other ways in which the joint may fail.

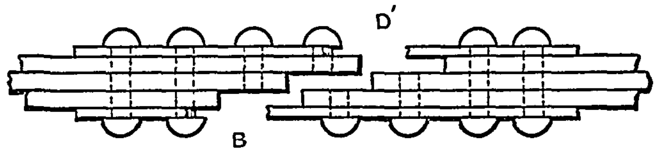


Fig. 201.

Failing as in Fig. 201.—In the first place it may fail as shown in Fig. 201, by the rivets between B and D shearing and the cover plates tearing across at B and D′.

The resistance to tearing of the cover plates is, if n_3 is the number of rivets to be deducted,

$$2\times(b-n_3d)t_2r_t.$$

The value of n_3 depends on the width of the plates and on the pitch of the rivets; for instance, with 10″ plates and 3″ pitch, n_3 would be 3, as shown in Fig. 202.

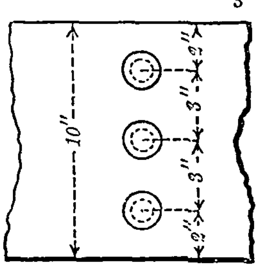


Fig. 202.

The resistance to shearing of the rivets between B and D′ is

$$2n_2\times 0{\cdot}78d^2r_s,$$

so that

$$R_T=2(b-n_3d)t_2r_t+2n_2\times 0{\cdot}78d^2r_s \quad . \quad (72).$$

It must now be determined what value should be given to t_2. On referring to p. 127 it will be seen that when two plates have to be joined, the cover plate *nearest* the joint should be made $\frac{5}{8}t$ thick, and by the same reasoning it will appear that for three plates

$$t_2=\tfrac{3}{4}t.$$

We can therefore find the value of n_2 from equation 72, and hence the value of n_1.

Practically, as already mentioned, it is usual to make $t_2=t$, but this will not affect the above calculations.

Cutting into covers and plates.—The joint can also fail by the rivets cutting into the covers between A and B, and into the plates between B and D.

The resistance to bearing of the covers between A and B is

$$2n_1t_2dr_b,$$

and the resistance of the plates to bearing between B and D is

$$2n_2 t' l r_b + n_2 t d r_b \text{ (see p 129)}$$

So that, assuming as before $t_2 = t$,

$$R_r = (2n_1 + 3n_2) t d r_b \qquad (73)$$

It is only necessary to see that the values already obtained for n_1 and n_2 are sufficient to give the bearing resistance required by this equation

On the whole, therefore, the strength of a grouped joint with three plates can be calculated from the four equations 70, 71, 72, and 73

Similar equations would be obtained whatever the number of plates to be joined, and the following are the general equations which the student should check by working out several cases

General Equations—Let N be the number of plates on *each side* of the joint which are to be connected, then

for shearing (see 70)

$$R_r = \left\{2n_1 + (N-1)n_2\right\} 0\ 78 d^2 r_s \qquad (74),$$

for bearing (see 71)

$$R_r = \frac{N}{2}\left\{2n_1 + (N-1)n_2\right\} d t r_b \qquad (75),$$

for tearing of covers and shearing (see 72)

$$R_r = 2(b - n_3 d) t_c r_t + (N-1)n_2 \times 0\ 78 d^2 r_s \qquad (76),$$

for bearing on plates and covers (see 73)

$$R_r = \left\{2n_1 + \frac{N}{2}(N-1)n_2\right\} d t r_b \qquad (77)$$

The total number of rivets is evidently

$$2n_1 + (N-1)n_2. \qquad (77a)$$

These equations can be solved very simply by the use of Table VIII, as will appear from the following example

Double-cover Grouped Joint

Example 23—Design a double cover grouped joint to connect four plates, each 9" × ¾", the end group of rivets being so arranged as to only weaken the plates by one rivet hole

Preliminaries—Let $d = 1\frac{1}{8}$ inch, which is the usual size of rivet for ¾ plates, and let $r_t = 5$ tons, $r_b = 8$ tons, and $r_s = 4$ tons

Then

$$R_r = 4(9 - 1\tfrac{1}{8})\tfrac{3}{4} \times 5,$$
$$= 118 \text{ tons}$$

Now from Table VIII

$$0\ 78 d^2 r_s = 3\ 98 \text{ tons},$$
$$d t r_b = 6\ 75 \quad ,,$$
$$\frac{N}{2} d t r_b = 13\ 50 \quad ,,$$

Clearly, therefore, a greater number of rivets are required for shearing than for bearing

Hence, from Equation 74

$$2n_1 + 3n_2 = \frac{118}{3\ 98} = 29\ 6 \qquad (78)$$

We must now apply Equation 76. Following the same reasoning as at p. 127 it will be seen that for four plates

$$t_2 = \frac{4}{5}t.$$

Hence assuming $n_3 = 3$ we have (76)

$$118 = 2\ (9 - 3 \times 1\tfrac{1}{8})\ \tfrac{4}{5} \times \tfrac{3}{4} \times 5 + 3n_2 \times 4,$$

whence

$$n_2 = \frac{118 - 34}{12} = 7.$$

The arrangement shown in Fig. 203 only gives $n_2 = 6$, but is near enough,

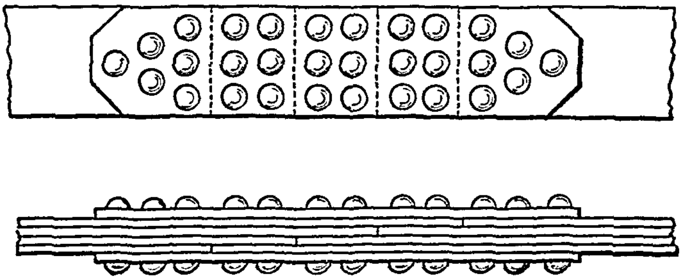


Fig. 203.

considering that the safe resistances assumed for bearing and shearing are rather low. Moreover, in practice t_2 would be made equal to t. Substituting this value (6) for n_2 in equation 78,

$$2n_1 = 29{\cdot}6 - 3 \times 6,$$
$$= 11{\cdot}6.$$
$$n_1 = 5{\cdot}8.$$

Six rivets therefore will do.

Substituting these values of n_1 and n_2 in Equation 77 we find

$$R_T = (2 \times 6 + \tfrac{4}{2} \times 3 \times 6)6{\cdot}74,$$
$$= 323{\cdot}5 \text{ tons},$$

so that there is no fear of the joint failing in the fourth manner.

The joint is shown in Fig. 203.

Double-cover Grouped Joint for Plate Girder.

Example 24.—Design the right-hand joint for the lower boom of the plate girder shown in Fig. 259 and Plate A.

On referring to Example 36 it will be seen that the plates to be joined are two in number, and that they are 9″ wide and $\frac{3}{8}$″ thick; and further that $\frac{3}{4}$″ rivets are used.

Owing to the angle irons connecting the booms to the web, only two rows of rivets can be used, so that $n_3 = 2$.

Take

$$r_t = 5 \text{ tons per square inch,}$$
$$r_b = 8 \quad ,, \quad ,,$$
$$r_s = 4 \quad ,, \quad ,,$$

From Table VIII. it appears that for $\frac{3}{4}$″ rivets

resistance to shearing = 1·77 ton,

resistance to bearing in $\frac{3}{8}$″ plate = 2·2 tons,

so that

$$\underset{2}{\Sigma}\, dtr_b = 2{\cdot}2 \text{ tons}.$$

Hence a less number of rivets are required for bearing than for shearing. Now since two $\frac{3}{4}''$ rivets have to be deducted

$$R_t = 2 \times (9 - 2 \times \tfrac{3}{4})\tfrac{3}{8} \times 5,$$
$$= 28{\cdot}2 \text{ tons}$$

Hence total number of rivets required for shearing

$$= \frac{28{\cdot}2}{1{\cdot}7} = 16$$

Applying Equation 76 we find, remembering that for two plates $t_2 = \frac{2}{3}t$,

$$28{\cdot}2 = 2(9 - 2 \times \tfrac{3}{4})\tfrac{3}{8} \times \tfrac{2}{3} \times 5 + n_2 \times 1{\cdot}77,$$
$$\therefore n_2 = \frac{28{\cdot}2 - 18{\cdot}75}{1{\cdot}77} = 5{\cdot}3 \text{ nearly}.$$

And since $n_3 = 2$, three rows of rivets will be required in the length of half the joint.

Lastly, since from (77*a*)

$$2n_1 + n_2 = 16,$$
$$n_1 = 5{\cdot}35,$$

so that three rows must be used.

In practice, however, t_2 would be made equal to t, and the above calculations would be modified as follows—

$$28{\cdot}2 = 2(9 - 2 \times \tfrac{3}{4})\tfrac{3}{8} \times 5 + n_2 \times 1{\cdot}77,$$
$$\therefore n_2 = \frac{28{\cdot}2 - 28{\cdot}2}{1{\cdot}77} = 0.$$

This appears to be an absurd result, but since there are only two plates to be joined, and the cover plates are made of equal thickness to the plates,

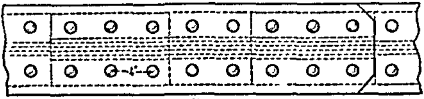


Fig. 204.

Fig. 205.

it follows that the covers are able to transmit the stress without the help of any rivets between the joints. In practice, however, this would be a bad construction, and a couple of rows of rivets would probably be used, as shown in Figs. 204 and 205.

On referring to Example 36 it will be seen that one of the covers can be

formed by extending one of the plates of the booms; the other cover can be added in the usual way, and the angle irons can be cranked over it as shown in Fig. 259 (an objectionable arrangement), or the upper cover can be formed by angle wrappers as shown in Fig. 206.

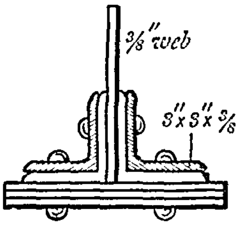


Fig. 206.

Oblique riveted joints.—So far we have only considered riveted joints connecting plates placed in prolongation one of another, but clearly the same rules are applicable when one plate, or set of plates, is placed at an angle to the others as shown in Fig. 207.

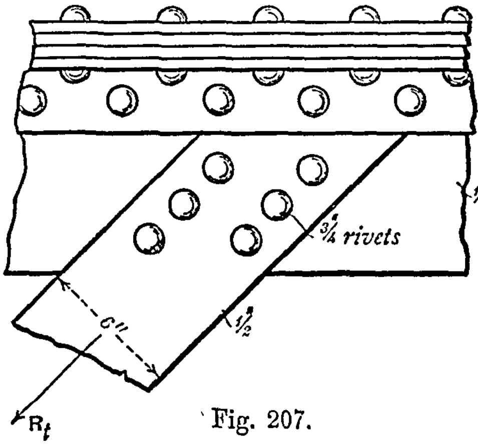


Fig. 207.

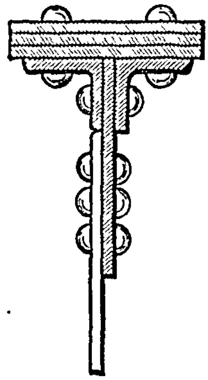


Fig. 208.

Example 25.—Thus, taking the dimensions given in Fig. 207 and deducting two rivets, we have, assuming $r_t = 5$ tons—$r_b = 8$ tons—and $r_s = 4$ tons,

$$R_T = (6 - 2 \times \tfrac{3}{4})\tfrac{1}{2} \times 5,$$
$$= 11{\cdot}25 \text{ tons.}$$

If α is the number of rivets for bearing—

$$\alpha \times \tfrac{1}{2} \times \tfrac{3}{4} \times 8 = 11{\cdot}25,$$
$$\alpha = 4 \text{ nearly.}$$

And if β is the number of rivets for shearing—

$$\beta \times 0{\cdot}78(\tfrac{3}{4})^2 \times 4 = 11{\cdot}25,$$
$$\beta = 6{\cdot}3 \text{ nearly.}$$

So that the joint is weaker in shearing than in bearing.

Using Table VIII. we find resistance to bearing of one rivet

$$= 3{\cdot}0 \text{ tons.}$$

Resistance to shearing of one rivet

$$= 1{\cdot}77 \text{ ton.}$$

Therefore number of rivets required

$$= \frac{11{\cdot}25}{1{\cdot}77} = 6{\cdot}3$$

as before, and six rivets will do.

It might be objected that, in calculating R_T, only one rivet should be deducted to obtain the effective cross section of the tension bar, but it will

be observed that the leading rivet is much nearer to one edge of the bar than to the other, and therefore weakens the bar by more than one rivet hole, as explained at p 126 Professor Reilly has treated this subject at some length in a paper published in Vol XXIX. *Minutes Proceedings of the Institution of Civil Engineers*, and shows that a similar faulty arrangement of rivets, though of frequent occurrence in practice, will increase the intensity of stress on one side of the bar by as much as 26 per cent Professor Reilly not only recommends that the leading rivet should be placed on the centre line of the bar, but also that all the rivets be placed symmetrically about the central line, or at any *rate that the centre of gravity of the rivets lie on the central line*, which he calls the mean fibre Thus arranged, the joint we have just calculated would be as shown in Fig 209 Only one rivet need now be deducted, so that

$$R_T = (6 - \tfrac{3}{4})\tfrac{1}{2} \times 5,$$
$$= 13.1 \text{ tons},$$

and

$$\beta = \frac{13.1}{1.77} = 7 \text{ nearly}$$

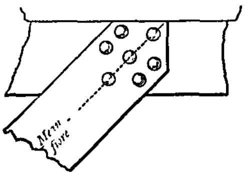


Fig 209.

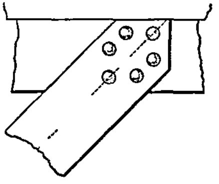


Fig 210

The rivets are rather numerous, and it would be better therefore to use larger ones. Re-calculating with $\frac{7}{8}''$ rivets we find

$$R_T = (6 - \tfrac{7}{8})\tfrac{1}{2} \times 5,$$
$$= 12.8 \text{ tons}$$

And from Table VIII

Resistance to bearing $= 3.50$ tons
Resistance to shearing $= 2.4$ „

Therefore

$$\alpha = \frac{12.8}{2.4} = 5.3$$

Five or six rivets ought therefore to be used, and with six rivets the arrangement of the joint would be as shown in Fig 210

Riveted Joints in Compression

At one time it was considered that the compression stress was transmitted across the joint by the plates abutting against each other, and such would be the case if the joints were perfectly

fitted; the rivets would then transmit no stress, and their only function would be to hold the plates together. Such fitting cannot, however, be ensured in practice, and it is therefore assumed that the whole stress is transmitted by the rivets, and the calculations are in no wise different from those required for tension joints. The following points, however, should be observed—

1. No deduction is to be made for rivet holes, as the metal of rivet completely fills up the hole and passes on the stress.

2. The longitudinal pitch of the rivets sometimes requires to be less than in tension joints, or the plates might buckle between the rivets as shown in Fig. 211.

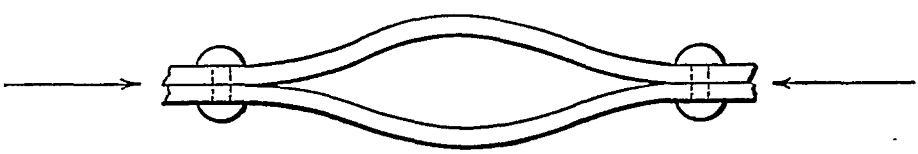


Fig. 211.

Dimensions for Riveted Joints.

Diameter of rivet holes.—If t is the thickness of the thickest plate to be jointed—

Fairbairn's rule: $d = 2t$ for plates under $\frac{1}{2}''$ thick,
$d = 1\frac{1}{2}t$ „ $\frac{1}{2}''$ thick and over.

Unwin's rule: $d = 1{\cdot}2\sqrt{t}$.

Pitch of rivets.—*Minimum* pitch of rivets $= 2 \times d$; *minimum* distance[1] of *centre* of rivet hole from edge of plate $= 1\frac{1}{2}\ d$.

The pitch of rivets in girder work usually varies from 3 to 5 inches, but it should not exceed 10 to 12 times the thickness of a single plate, as otherwise damp may get in between the plates and cause rust, which in time swells and bursts them asunder[2]; 4″ is a common pitch for girder work. The pitch is sometimes made greater in the tension than in the compression flange.

Note upon the Difference between the Figured and Actual Sizes of Rivets and Rivet Holes.

In the calculations given above, the rivets and the rivet holes are assumed to be of exactly the sizes shown or figured upon the drawings.

In the actual work, however, neither the rivets nor the rivet holes are of the sizes shown upon the drawings.

Iron rivets.—To begin with, the rivet as purchased is generally $\frac{1}{32}$ inch smaller diameter than its nominal size—that is, a so-called $\frac{1}{2}$ inch rivet would be $\frac{15}{32}$ inch in diameter.

When, however, the rivet is heated and clenched it will, if the work be

[1] $\frac{1}{16}''$ to $\frac{1}{4}''$ should be added in the case of thick plates and rivets. [2] Stoney.

properly done, very nearly[1] fill the hole, and the hole will be found to be considerably larger than the nominal size of the rivet, especially in the case of punched holes

Punched holes in iron plates—When a hole is punched in a plate the diameter of the punch is generally $\frac{1}{16}$ inch larger than that of the rivet that is to go into the hole, and then again the die is larger, in proportion to the thickness of the plate, than the punch

This results in the punched holes being larger at the lower surface of the plate than at the upper, and the actual diameter of the holes is from 10 to 20 per cent larger than the nominal diameter of the rivets[2]

Strength of iron rivets in punched holes in iron plates—It will be seen, therefore, that the rivets filling punched holes will have a considerably larger shearing area than the rivets shown in the drawings

If rivets of the figured sizes are strong enough to withstand the calculated stress, then the actual rivets in the work will evidently be unnecessarily strong, and their number might, so far as this point alone is concerned, be reduced

On the other hand, however, it must be borne in mind that the stress upon a riveted joint is not divided equally among all the rivets, as is assumed in the calculations, but that in consequence of bad workmanship, and for other reasons (see p 135), increased stress may come upon some of the rivets while others do not take their share

For this reason engineers sometimes add 10 to 20 per cent to the calculated number of rivets to make up for uneven stress

Thus on the one hand, though fewer rivets would be required in the work than those calculated, because the rivets as clenched are actually of larger sectional area, yet on the other hand more rivets would be required in order to make up for defective workmanship and consequent inequality of stress

Strength of actual riveted joint may be practically taken the same as figured—The result is that these two causes affecting the number of rivets actually required as compared with those calculated may be considered to cancel one another,[3] and there is no practical error involved in calculating the rivets according to their figured dimensions This is the usual practice, and it is followed in these Notes

Punched holes in iron tension joints—With regard to the deduction to be made for rivet holes in tension joints the case is somewhat different The diameters of punched holes are, as mentioned above, from 10 to 15 per cent larger than the nominal size of the rivet, and in addition to this, in roughly punched work, the edges of the holes are somewhat torn, so that the effective

---

[1] "In order to get good riveted joints the rivet must be properly put in and must fill the hole when cold, but as a matter of fact this is really never the case, in consequence of the rivets contracting laterally when cooling This is not so much the case with steel rivets, as they are or should be worked at a dull red heat The most perfect joint would be one in which drilled holes were used and the rivets turned and closed quite cold —Moberley, quoted in Stoney on the *Theory of Stresses*, p 684

[2] Stoney

[3] There are other minor circumstances affecting both sides, *e g* rivet iron is generally better than other iron and might be subjected to a higher shearing stress The friction between the plates assists the rivets, on the other hand, an iron rivet sheared on two surfaces has not twice the resistance of one surface, but only $1\frac{3}{4}$ times that resistance

area of the plate is reduced by a width greater than that of the nominal diameter of the rivet.

Mr. Stoney says: "Probably the most accurate method for making an allowance for the injurious effect of punching would be to add a certain percentage, say $\frac{1}{10}$ to $\frac{1}{5}$ of its diameter, to each hole when calculating the effective net area of a punched plate. Mr. White (Director of Naval Construction) states that they were accustomed to allow 4 tons off the very best iron for punching—4 tons off 22 tons. Civil engineers, however, rarely make any similar allowance in riveted girder work, probably through inadvertence, or because the rivet holes are generally pitched farther apart in girders than in ships."[1]

It will be seen, therefore, that in the case of the holes as well as that of the rivets it is the usual practice to base the calculations upon the figured dimensions of the rivets, it being generally considered that the working stresses taken in practice are so low as to leave a sufficient margin to cover all extra stresses caused by enlarged holes, rough punching, injured plates, and other defects in workmanship.

In some cases, however, engineers allow for the damage done to plates by punching, making a rough addition to the diameter deducted to cover this, *e.g.* in the case of a $\frac{3}{4}$ inch rivet they deduct an inch from the effective width for the plate. In all important tension joints this question should be carefully considered and some allowance of the kind made if considered necessary.

Rimering out.—If the hole is punched about $\frac{1}{4}$ inch less than the required diameter, and this margin is afterwards bored out to the required diameter, the damaged metal is removed and no harm has been done by the punching.

Drilled holes.—In the case of drilled holes their diameter is only about $\frac{1}{32}$ inch larger than the nominal diameter of the rivet. The edges round the hole are not damaged, so that there is no practical difference between the actual sizes of the holes and those figured for the rivets.

Steel rivets fill the holes better than iron ones, because they are not so hot when clenched and contract less in cooling.

PIN JOINTS.

These joints are much used for connecting the various tie-bars of roofs to the other members, and examples can be seen on reference to Parts I. and II. Such joints are also occasionally used for open girder work.

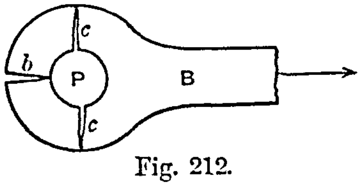


Fig. 212.

These joints are formed with links somewhat of the shape shown in Fig. 212. A little consideration will show that this link is liable to fail by tearing, as shown in the figure. The continuity in the fibres of B is interrupted by the hole for the

[1] Stoney in *Theory of Stresses*, p. 650.

pin P, moreover *b* is further liable to be split from the inside by P (if it is too small) bearing upon it, so that the intensity of stress over the cross sections *cc* is by no means uniformly distributed However, the calculations that would be required on

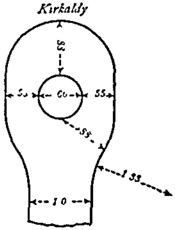


Fig 213

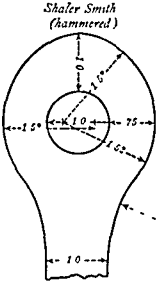


Fig 216

Fig 215

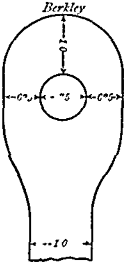


Fig 214

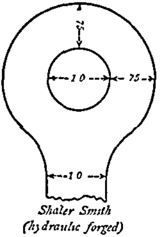


Fig 217

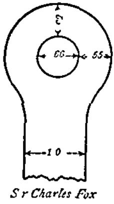


Fig 218

the assumption that the stress is *not* uniformly distributed are far too difficult for this book, and would certainly not be carried out in practice We must therefore resort to experiments Such experiments were made by Sir C Fox, C E, and more recently by Mr David Kirkaldy, C E, and by Mr C Shaler Smith

made 1″ in diameter; this necessitates re-calculating for the previous eye.

Eye of FG.—It will be found that for this eye

$$a = \frac{3{\cdot}88}{5} = 0{\cdot}78,$$

or say $\frac{7}{8}$″ to allow for a loose pin. We now have

$$\frac{\text{diam. of pin}}{\text{width of bar}} = \frac{d}{a} = 1 \div 1\tfrac{3}{16} = 0{\cdot}84.$$

This number does not appear in Table IX., but by interpolation we find that 0·7 is the corresponding number of column 2. Hence

$$\frac{ca \times 5}{3{\cdot}88} = {\cdot}7,$$

$$\therefore\ c = \frac{0{\cdot}7 \times 3{\cdot}88 \times 8}{5 \times 7} = 0{\cdot}62,$$

and

$$ba = \frac{3{\cdot}88}{5}.$$

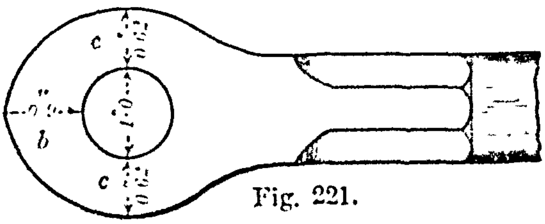


Fig. 221.

Elevation of Eye of FG.

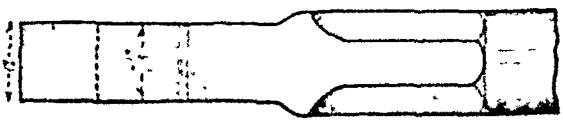


Fig. 222.

Plan of Eye of FG.

thick. But the ratio $\frac{\text{diam}}{\text{width}}$ for each half of the double eye is $1 \div 1\frac{3}{8} = 0.73$,

so that
$$\frac{ca \times 5}{1 \times 7.83} = 0.67,$$
$$c = 0.6 \text{ inch}$$

Further
$$la = \frac{(1 \times 7.83)}{5},$$
$$l = 0.9 \text{ inch},$$

or very nearly the same dimensions as those found for the single eye. Practically the double eye would be of the form shown in Fig. 221, and the single eye also of the same shape.

Rod FC.—There remains to be designed the double eye for the small tension rod FC having a stress of 4.13 tons. In this case we have
$$a \times 1 \times 5 = \frac{4.13}{2},$$

or
$$a = .413 \text{ inches}$$

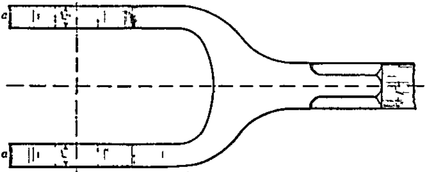


Fig. 221

Plan of Eye of FC

But as this thickness would be barely sufficient to make the arms of the jaw of equal strength to the body of the tie-rod, we will make $a = \frac{1}{2}''$. An increase of strength in this part will be judicious.

If the diameter of the rod FC be taken as 1″ as shown in Figs. 373, 374, and the width of the arms of the jaw also 1″, then $\frac{\text{diam. of pin}}{\text{width of tie bar}} = 1 \div 1 = 1$, and the corresponding number in column 2, Table IX., is 0.75. Hence $ca = \frac{4.13}{5 \times 2} \times 0.75 = 0.31$ and $c = 0.62$. The total width of the eye will therefore be the same as that of the jaw shown in Fig. 221, which will do very well.

The cross section at the back of the eye is as before equal to that of the tie bar, or = .78 square inch—b therefore equals 0.78″ or say $\frac{7}{8}''$, but as it is not desirable to make minute alterations in the patterns of these eyes, we will take $b = .9''$, and the shape of the eye will then be similar to that in Fig. 221. The complete jaw is shown in Fig. 224.

Fig. 220 shows the complete joint. It is wide in the direction of the bolt, and therefore somewhat clumsy in appearance. It would be improved by taking $a = \frac{3}{4}''$ instead of $\frac{7}{8}$. The student is recommended to try this.

Joints at Apex and Feet of Truss.

Example 26a.—The connections at the apex and feet of the roof-truss are sometimes made by castings.

These, however, are clumsy and unreliable, and apt to be broken in transit.

As the principals are usually sent to their place of erection in parts (so as to avoid injury, and for convenience of carriage), it is better in the case of small roofs where there are not many principals to use bolts rather than rivets, because there would be so few rivets that it would not be economical to send a gang of workers with their forge and tools, and because with bolts under ordinary circumstances all the necessary work in erection can be done by a carpenter.

For the roof, Fig. 373, p. 215. The rafter is a T iron $4\frac{1}{2} \times 4\frac{1}{2} \times \frac{1}{2}$, and the stress it sustains $= 7 \cdot 83$ tons (Table H, p. 213).

Taking $r_b = 5$ tons, bearing area required for bolts $= \dfrac{7 \cdot 83}{5} = 1 \cdot 56$ square inch.

For such roofs it is better therefore to use bolts, and connections with

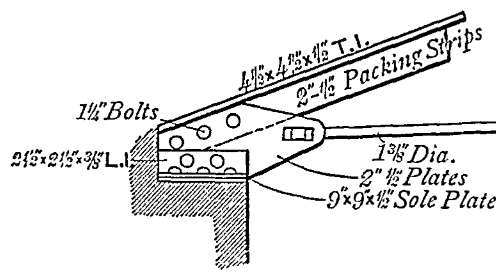


Fig. 225.

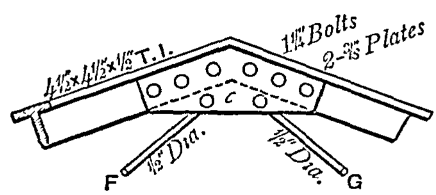


Fig. 225a.

bolts are shown in Figs. 225, 225a. That at the apex will now be calculated.

Joint at apex.—Assuming $1\frac{1}{4}''$ bolts,

Bearing area of each bolt $= \frac{1}{2}'' \times 1\frac{1}{4}'' = \cdot 62$ inch.

Number of bolts required on each side of the connection $= \dfrac{1 \cdot 56}{\cdot 62} = 2 \cdot 5$, say 3.

As there will be a plate on each side of the T iron the bolts will be in double shear, and the shearing strength will not require investigation, as it will be very much in excess of what is required.

The united thickness of the plates should be at least equal to the thickness of the stem of the T iron, *i.e.* $\frac{1}{2}''$; but as it is better to make them a little in excess, in this case two plates each $\frac{3}{8}''$ thick will do.

Having calculated the joint for the T irons, the bars FC and CG (see Fig. 373, p. 215) must be considered. The diameter of the bolts should first be calculated in proportion to the bars, as explained at p. 140, then the bearing area of the bolt on the connection plates investigated. If the thickness of plates calculated for the T irons is not sufficient, then the plates or the diameter of the bolts connecting FC and CG must be increased; in this case the bars FC and CG are only $\frac{1}{2}''$ diameter, and it can be seen by inspection that the plates are amply thick. Fig. 225a shows the connection.

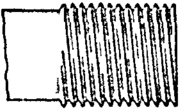
Joint at foot.—The shoe of the principal can also be made of wrought iron, as shown in Fig. 225. The stress and the bolts will be the same as at the apex.

SCREWS

Screws are of constant use in building construction in the shape of nuts and bolts, screw connectors, etc.

The thread usually employed is that shown in Fig. 226 *a*; it will be observed that it is cut *into* the bolt or rod and is therefore called a *minus* thread. The strength of the bolt or rod is thereby

Minus Thread
Fig. 226 *a*



Plus Thread
Fig. 226 *b*

weakened, and to obviate this loss what is called a *plus* thread is sometimes used, as shown in Fig. 226 *b*. For such a thread the end of the bolt or rod has to be thickened by upsetting, and it is a matter for consideration in each case whether the saving in metal is worth the additional cost of cutting a plus thread. As a matter of fact their use is somewhat limited.

The thread on the bolt is called a "male" thread and the thread in the nut into which it fits is called a "female" thread.

The resistance of the screw to being torn out of its nut depends on the resistance of the threads to being sheared off. With the V thread generally used in building construction the area to be sheared is the surface area of a cylinder of height l and of diameter D (Fig. 227). D is the diameter of the bolt taken at the bottom of the threads and is called the

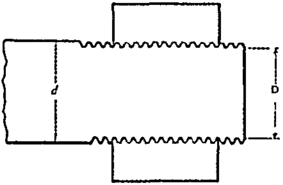


Fig. 227

effective diameter and the ratio between D and d depends on the

pitch and the kind of thread employed. In the case of Whitworth's screws, which are in general use in this country, the ratio is

$$D = 0 \cdot 9d - 0 \cdot 06 \quad . \quad . \quad . \quad (79),$$

which can also be written

$$d = \frac{D + 0 \cdot 06}{0 \cdot 9} = 1 \cdot 1 \times D + 0 \cdot 07 \text{ nearly} \quad . \quad (80),$$

D and d being expressed in inches.

Now the area of the cylinder, that is the shearing area required, is

$$\pi D \times l.$$

Hence
$$R_s = \pi Dl \, . \, r_s.$$

As regards the value to be given to r_s, it is to be observed that the metal is weakened in cutting the thread, and for this reason Humber recommends a value of 2 tons to the square inch (instead of 4 tons, the usual value). Substituting for r_s and D we get

$$R_s = \pi(0 \cdot 9d - 0 \cdot 06)l \times 2.$$

Practical formula.—This equation will be found to reduce approximately to the following, which is a form suitable for practical use—

$$R_s = (5 \cdot 6d - 0 \cdot 4)l \quad . \quad . \quad . \quad (81).$$

Theoretically the resistance to stripping of the thread should be equal to the tensile resistance of the bolt. The latter is

$$R_T = \frac{\pi D^2}{4} \cdot r_t.$$

So that
$$\frac{\pi D^2 r_t}{4} = \pi D l r_s,$$

$$l = \frac{r_t}{r_s} \cdot \frac{D}{4},$$

$$= \frac{r_t}{4r_s}(0 \cdot 9d - 0 \cdot 06).$$

Substituting 5 tons for r_t and 2 tons for r_s and reducing, we find

$$l = 0 \cdot 56d - 0 \cdot 04 \text{ nearly} \quad . \quad . \quad (82).$$

It thus appears that l requires to be rather more than $\frac{d}{2}$. The usual practice is to make $l = d$, which is a good, safe rule.

COTTER JOINTS.

A usual form of these joints is shown in Fig. 229 (see also Part I.) Such a joint tends to fail as follows—

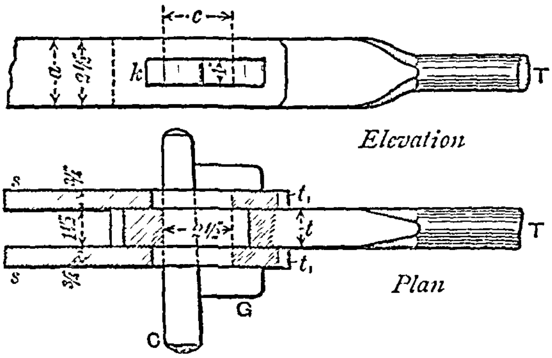


Fig. 229.

1. By shearing of the gib G and cotter C (double shear).

2. By the cotter cutting into the tie-rod T.

3. By the gib G cutting into the sleeve *ss*.

Let k be the thickness of the cotter or of the gib bearing against the rod or the sleeve, and c the width[1] of the gib and cotter; t the thickness of the flattened part of the rod, and t_1 the thickness of each side of the sleeve, then

$$R_S = 2kc \times r_s.$$

For the rod $\qquad R_B = kt \times r_b.$

For the sleeve $\qquad R'_B = 2kt_1 \times r_b.$

Moreover, the effective section of the flat part of the tie and of the sleeve must be proportioned to the resistance to tension of the tie-rod, namely R_T.

For a theoretical joint these resistances should each be equal to R_T, hence

$$2kcr_s = R_T \qquad . \quad . \quad . \quad (83),$$

$$ktr_b = R_T \qquad . \quad . \quad . \quad (84),$$

and clearly

$$t = 2t_1 \qquad . \quad . \quad . \quad (85).$$

If the value of k is assumed, the remaining dimensions for a given value of R_T can be found.

Example 28.—The tie-rod of a roof is connected to the principal, and to the shoe at the abutments, by means of a cotter joint, as shown in Fig. 229. The tension in the tie-rod is 7·83 tons (see Example 42, p. 209, and Table H, p. 213). Find suitable dimensions for the cotter joint.

Let $k = \frac{1}{2}''$, then from Equation 84, taking $r_b = 5$ tons per square inch

$$\tfrac{1}{2} \times t \times 5 = 7{\cdot}83,$$

whence $t = 3{\cdot}1$ inches. This is too large a value for t. It will therefore be necessary to take a larger value for k, for instance 1 inch, then

$$t = \frac{7{\cdot}83 \times 1}{5} = 1{\cdot}56.$$

Practically $1\frac{1}{2}''$ will do. Hence (85)

$$t_1 = \frac{t}{2} = \tfrac{3}{4}''.$$

And $\qquad 2 \times 1 \times c \times 4 = 7{\cdot}83,$

$$c = 1 \text{ inch nearly.}$$

[1] A taper of 1 in 24 to 1 in 48 is usually given to the cotter.

This is the minimum value of c. Practically, more room is required to fit in the gib and cotter, not less than 2 or $2\frac{1}{2}$ inches.

Lastly, to find the depth a of the sleeve. The effective section of the sleeve is

$$2 \times (a - l)t_1 = \frac{7.83}{5},$$

or

$$2(a - 1)\tfrac{3}{4} = \frac{7.83}{5},$$

$$a - 1 = 1.04,$$

or

$$a = 2.04 \text{ inches.}$$

Some allowance ought to be made for possible errors in manufacture, so that practically it would be well to make $a = 2\frac{1}{4}$.

The depth of the flat part of the tie rod should also be made $2\frac{1}{2}$ inches.

The length of the sleeve should be made sufficient to prevent the cotter shearing a piece out of each side of it, and the same is true of the flat part of the tie rod; the calculations are, however, left to the student. The dimensions given in the figure are slightly more than those theoretically necessary.

JOINTS IN WOODEN STRUCTURES

Numerous kinds of joints are employed by the carpenter in framing wooden structures, and a large number of these were described in Part I. The proper proportions for such joints have been determined by long experience, and the necessity to make any calculations in connection with them is never likely to arise. It is, however, thought that the consideration of the principles on which the strength of such joints could be calculated will be a useful exercise for the student, and a few of the joints given will therefore be examined with this object in view.

"Fished" Joint for Wooden Beam

Example 20.—For instance, let us inquire into the joint shown in Fig. 230, which is shown in most books on Carpentry, and in Part I.

If the bolts fit tightly into the wood it can be considered that part of the stress is transmitted by them, and the remainder will therefore have to be transmitted by the tabling. Clearly only the resistance to bearing of the bolts need be considered. If the fish plate is made of the same wood as that to be joined, the thinner part t should be made equal to $\frac{b - 2i}{2}$, i being the depth of the indent. Hence

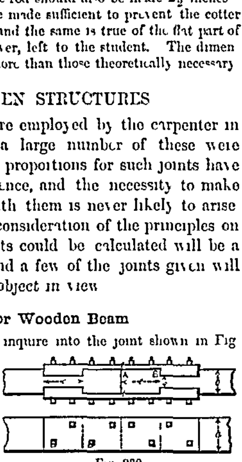


Fig. 230

$$2t = b - 2i \qquad (86)$$

The resistance to tearing of the fish plates will then be equal to the resistance to tearing of the beams to be joined. It will also be seen that then the resistance to bearing of each bolt of diameter d is

$$2t \times d \times r_b$$

So that if there are n bolts on each side of the joint the total resistance to bearing is

$$2ntdr_b.$$

Therefore the resistance to be demanded of the tabling is

$$R_T - 2ntdr_b.$$

But (remembering that we must deduct one bolt hole),

$$R_T = 2t(a-d)r_t.$$

Therefore the resistance of the tabling must be equal to

$$2t(a-d)r_t - 2ntdr_b.$$

Now the indent must be of sufficient depth to prevent the fibres being crushed; this condition is fulfilled by

$$2t(a-d)r_t - 2ntdr_b = 2iar_c \quad . \quad . \quad . \quad (87).$$

Again, the length of the indent and the distance AB must *each* be sufficient to prevent shearing off of the wood. The resistance to shearing of either is

$$ear_s,$$

therefore $$t(a-d)r_t - ntdr_b = ear_s \quad . \quad . \quad . \quad (88).$$

From these equations the various dimensions can be found.

As an example take the following data, which assume that the wood is of fair quality Baltic fir.

$a = 12''$,
$b = 8''$,
$n = 4$, as in Fig. 230.
$d = \frac{3}{4}''$,
$r_t = 12$ cwts. per square inch, Table IA, p. 328.
$r_c = 10$ cwts. " "
$r_s = 1{\cdot}3$ cwt. " "
$r_b = 12$ cwts. " "

Substituting these values in Equations 86 and 87, we get

$$2t = 8 - 2i,$$

or $$t = 4 - i.$$

And $$t(12 - \tfrac{3}{4})12 - 4t \times \tfrac{3}{4} \times 12 = i \times 12 \times 10,$$

or $$99t = 120i.$$

Therefore $$99(4 - i) = 120i,$$

$$219i = 396,$$

$$i = 1{\cdot}8 \text{ inch},$$

and $$t = 2{\cdot}2 \text{ inches}.$$

Again, from Equation 88 we have

$$2{\cdot}2 \times (12 - \tfrac{3}{4}) \times 12 - 4 \times 2{\cdot}2 \times \tfrac{3}{4} \times 12 = e \times 12 \times 1{\cdot}3,$$

whence $$e = 14 \text{ inches}.$$

It would probably be better in practice to neglect the resistance offered by the bolts, and take them as only holding the joint together. To adapt the above equations to this case it is only necessary to omit the term $2ntdr_b$. The equations then become

$$2t = b - 2i,$$

$$t(a-d)r_t = iar_c,$$

$$t(a-d)r_t = ear_s.$$

Taking the same data as before,

$$t = 4 - i,$$

$$t(12 - \tfrac{3}{4})12 = i \times 12 \times 10,$$

whence

$$(4 - i)135 = 120i,$$

or $$i = \frac{135 \times 4}{255} = 2.1 \text{ inches.}$$

Hence $$t = 4 - 2.1 = 1.9,$$

and from Equation 88

$$1.9(12 - \tfrac{3}{4})12 = e \times 12 \times 1.3,$$

or $$e = \frac{1.9 \times 11.25 \times 12}{12 \times 1.3},$$

$$= 16\tfrac{1}{4} \text{ inches nearly}$$

It appears strange that the value obtained for e in this case should be so little greater than in the first case, but the reason is that the beam is much more weakened than in the first case owing to the greater depth of the indent, as will be seen by the following

In the first case the resistance to tension of the joint is

$$2t_1(a - d)r_t = 2 \times 2.2 \times 11.25 \times 12,$$
$$= 594 \text{ cwts}$$

In the second case it is

$$2t(a - d)r_t = 2 \times 1.9 \times 11.25 \times 12,$$
$$= 513 \text{ cwts}$$

Further, the safe resistance to tension of the full section of the timber, namely $8'' \times 12''$, is

$$8 \times 12 \times 12 = 1152 \text{ cwts},$$

from which it is evident that this is a very uneconomical form of joint, more than half the strength of the timber being lost

Scarfed Joint for Wooden Beam.

Example 30 —Calculate the safe strength of the joint shown in Fig 231, which is a bad form, as pointed out in Part I

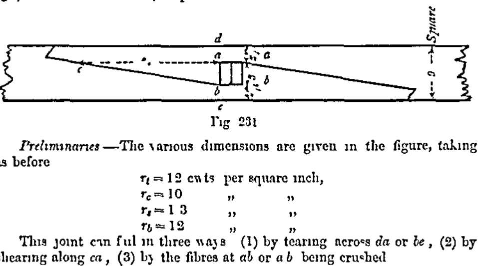


Fig 231

Preliminaries—The various dimensions are given in the figure, taking as before

$r_t = 12$ cwts per square inch,
$r_c = 10$ " "
$r_s = 1.3$ " "
$r_b = 12$ " "

This joint can fail in three ways (1) by tearing across da or be, (2) by shearing along ca, (3) by the fibres at ab or $a\,b$ being crushed

For the first we have

$$R_t = 9 \times 2\tfrac{3}{4} \times 12 = 297 \text{ cwts}$$

For the second

$$R_s = 24 \times 9 \times 1.3 = 280 \text{ cwts}$$

For the third

$$R_c = 3\tfrac{1}{2} \times 9 \times 10 = 315.$$

It thus appears that this joint is most liable to fail by shearing the fibres along *ca*, and that 280 cwts. is the safe stress that it could be subjected to.

The strength of the full beam $= 9 \times 9 \times 12 = 972$ cwts., so that this joint is still less economical than the last.

Joint at Foot of Principal Rafter (Wood).

Example 31.—To ascertain whether the joint shown in Fig. 232 is of sufficient strength to resist a compression of 61 cwts. in the principal.

Preliminaries.—We will assume that this joint is made with inferior timber, and therefore take

$r_t = 10$ cwts. per square inch.
$r_c = 7$ " "
$r_s = 1{\cdot}3$ " "
$r_b = 7$ " "

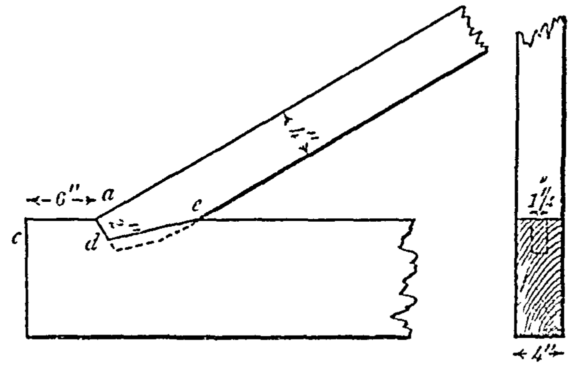


Fig. 232.

The joint may fail in two ways: (1) By crushing the fibres at *ad*; (2) by shearing the tie-beam along *cd*.

As regards (1), the bearing surface is *ade*, allowing nothing for the tenon, which would not be fitted to bear; *de* is inclined to the pressure, and only its equivalent at right angles to the pressure must be taken, therefore the bearing surface is the whole cross section of the principal, less the tenon, that is

$$4 \times 2 = 8 \text{ square inches.}$$

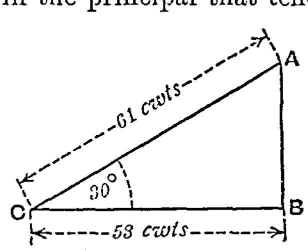
So that

$$R_B = 8 \times 7 = 56{\cdot}5 \text{ cwts.,}$$

which is not quite sufficient.

As regards (2), $R_s = 6 \times 4 \times 1{\cdot}3 = 31{\cdot}2$ cwts.

But it will be observed that it is only the horizontal component of the stress in the principal that tends to shear the tie-beam along *cd*. This horizontal component can be very simply found by a graphic method, which will be more fully explained in the sequel (see Chap. IX.) Draw a triangle ABC (Fig. 233); AB being vertical, BC horizontal, and AC parallel to the principal, then CB will represent the horizontal component required, to the same scale that AC represents the compression in the principal. In Fig. 233 the scale chosen is 50 cwts. to one inch, AC is plotted as 61 cwts., and the horizontal component CB is found to scale 53 cwts.



Scale 50 ȼ to 1 inch.
Fig. 233.

It thus appears that the resistance to shearing of the tie-beam along *cd* (31·2 cwts.) is quite insufficient.

The additional strength required can be obtained by means of a strap as shown in Fig. 234. To find the stress in this strap we must resolve the portion of the horizontal component it has to bear parallel to the strap. The

portion of the horizontal component which the strap has to bear is 53 – 31·2 = 21·8 cwts., and the resolved part of the stress can be found by means of a triangle as before, and as shown in Fig 235 Thus it is found that the stress in the strap is 43·6 cwts = 2·18 tons

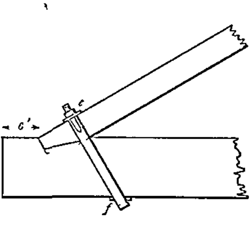


Fig 234

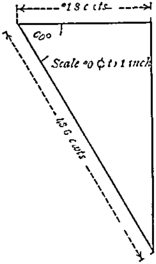


Fig 235

Supposing the strap is made of $\frac{1}{4}''$ iron, then the necessary breadth b can be found from

$$2b \times \tfrac{1}{4} \times 5 = 2{\cdot}18,$$

whence $b = 0{\cdot}87$ inch

A strap 1″ wide would be used

By a calculation similar to that given in Example 27, it will be found that a $\frac{5}{8}''$ screw and nut are required

A bearing plate is required to prevent the strap cutting into the upper side of the principal at *e*, its length will be 4, the breadth of the principal rafter, and to find the least breadth we have

$$4 \times b \times 7 = 43{\cdot}6 \text{ cwts},$$

whence $b = 1\frac{5}{8}$ inch nearly

The bearing plate[1] at *f* would be made of the same width

The above calculation supposes that the horizontal component is partly borne by the strap, and partly by the wood Any shrinkage of the wood would, however, throw the whole stress on the strap, and it is therefore safer to consider that the whole of the horizontal component is resisted by the strap The stress in the strap would then be (referring to Fig 235)

$43{\cdot}6 \times \dfrac{53}{21{\cdot}8} = 106$ cwts $= 5{\cdot}3$ tons, and making the strap of $\frac{3}{8}$ iron,

$$2b \times \tfrac{3}{8} \times 5 - 5{\cdot}3 \text{ tons},$$
$$b = 1{\cdot}4$$

So that a strap $1\frac{1}{2} \times \frac{3}{8}$ would be used

The bearing plates at *e* and *f* would be made about 4″ wide, and the screw bolt of $\frac{3}{4}''$ diameter

---

[1] Unless there is any objection to weakening the tie beam at this point *f* would be better notched in to prevent slipping (see Part I) The strap at *e* must be made as much longer than 4″ as will provide bearing for the nuts and must be thick enough to prevent bending

Chapter VIII.

PLATE GIRDERS.

THE subject of plate girders was discussed to a certain extent in Part I., and it is now proposed to enter more into detail and to show how the various calculations can be effected.

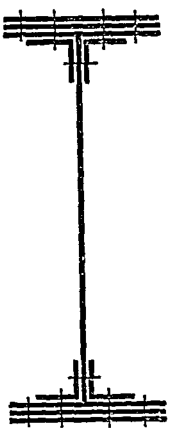


Fig. 236.

General form.—Fig. 236 shows in outline the general form of the cross section of a plate girder with a single web. The flanges consist of one or more plates (the thickness and number depending on the stress to be borne) riveted to a couple of angle irons; the web consists of a thin plate, stiffened, however, against crumpling up, by means of vertical T or L irons. On referring to the distribution of the stresses in a rolled iron girder, p. 83, it will be seen that the web of a plate girder being very thin can bear but a very small part of the direct stresses. Practically the whole of the direct stresses may be taken as borne by the flanges, and the whole of the shearing stresses by the web, when the flanges are parallel.

Practical points to be attended to.—The following practical points should be borne in mind in designing a girder.

The depth of plate girders varies from $\frac{1}{8}$ to $\frac{1}{15}$ the span; $\frac{1}{12}$ is said to be the most economical proportion. In floors of buildings much shallower girders are often used to save height.

For calculations the "*effective span*," that is, the span between the centres of the bearings of the girder upon the abutments, should be taken, and the "*effective depth*," that is, the distance between the centres of gravity of the flanges (see Plates A and B).

The width of the flange under compression should not be less than $\frac{1}{30}$ to $\frac{1}{40}$ of the span, or it will be liable to buckle sideways.

No plates of less than $\frac{1}{4}$ inch thickness should be used, or they will soon be destroyed by corrosion.

There should be as few joints as possible, especially in the tension flange and web.

Plate girders should be constructed with a camber or rise in the centre of about $\frac{1}{180}$ to $\frac{1}{240}$ of their span, or at any rate in excess of their calculated

in like manner the point a''' and c''', where the third plate ought to commence, can be found, and so on for as many plates as may be required to afford the necessary resistance.

The same process is clearly applicable to the compression flange of the girder.

Flanges for Plate Girder of 20 feet span.

Example 32.—The matter will be probably made clearer by an example. Let the effective span be 20 feet and the load 50 tons, equally distributed so that the moment of flexure at the centre is 1500 inch-tons[1] (see Fig. 240). Then we can draw the parabola akc as explained in Appendix III.[2] Assume further that the depth of the girder to the backs of the L irons is 1′ 7″, and that 3″ × 3″ × ½″ L irons and ½″ plates 10″ wide are used.

Tension, or lower flange.—It will be seen from Plate A that the cross section contains one rivet (⅞″) hole in each angle iron, and two rivet holes in each plate.[3] It will be found that the "effective depth," *i.e.* distance between centres of gravity for the L irons, is 17·1 inches.[4] Therefore the moment of resistance of both angle irons in the *tension* flange (taking $r_t = 5$ tons) is

$$2(3 + 2\tfrac{1}{2} - \tfrac{7}{8}) \times \tfrac{1}{2} \times 17{\cdot}1 \times 5,$$
$$= 395 \text{ inch-tons.}$$

Hence we can draw bd by making $ab = 395$ inch-tons to the scale chosen for the figure, namely 3000 inch-tons to an inch, and we then find by measurement that aa' and $c'c = 1{\cdot}4$ foot.

Again, the effective depth of the first plate is 19·5″, hence its moment of resistance is

$$[10 - (2 \times \tfrac{7}{8})] \times \tfrac{1}{2} \times 19{\cdot}5 \times 5,$$
$$= 402 \text{ inch-tons.}$$

Of the second plate

$$[10 - (2 \times \tfrac{7}{8})] \times \tfrac{1}{2} \times 20{\cdot}5 \times 5,$$
$$= 423.$$

Of the third

$$[10 - (2 \times \tfrac{7}{8})] \times \tfrac{1}{2} \times 21{\cdot}5 \times 5,$$
$$= 443.$$

We can therefore draw a series of parallel straight lines at distances representing these several values in inch-tons, until we just pass beyond the apex of the parabola. In this manner we obtain the points f, g, etc., and by dropping perpendiculars the points a', a'', a''', etc., so that the number of plates and length of each are shown in Fig. 239, and can be found by measurement.[5]

Compression, or upper flange.—On the *compression* side the moment of resistance of the two angle irons (taking $r_c = 4$ tons) is

---

[1] $M_c = \dfrac{wl^2}{8}$ (see p. 33) $= \dfrac{\frac{50}{240} \times 240^2}{8} = 1500$ inch-tons.

[2] A circular arc can be substituted for the parabola if the rise of the arc does not exceed ⅛th of the span (see p. 30).

[3] In many cases, when the plates are wider, they are riveted together also at their edges, so that there would be four rivets to deduct.

[4] It is as easy and more accurate to take "effective depths" separately for the pair of L irons and for each plate.

[5] As the sum of the above moments is 1663 inch-tons, and by the data only 1500 are required, it is evident that the third plate need not be more than $\frac{5}{16}$″ thick. It will be found that the moment of resistance of such a plate is 275 inch-tons.

$$2(3+2\tfrac{1}{2}) \times \tfrac{1}{2} \times 17 \cdot 14 \times 4,$$
$$= 377 \text{ inch tons}^{1}$$

And of the first plate $10 \times \frac{1}{2} \times 19\frac{1}{2} \times 4$,
$= 390$ inch tons

Of the second $= 410$

Of the third $= 430$

Repeating the above process as shown in Fig 239 we obtain the number and lengths of plates given in Fig 240

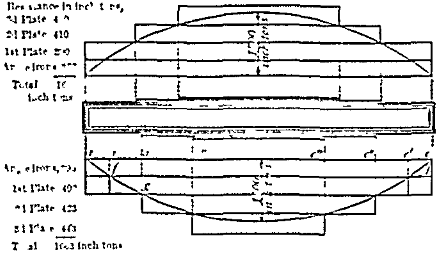


Fig 240

Fig 241

Fig 239

As already observed, the first plate is continued in both the tension and compression flanges for the whole length of the girder, and is so shown in Fig 239 241

It is to be observed that in practice the plates (after the first) will be somewhat longer than shown on the diagram, for they must include the next row of rivets beyond the point indicated in theory

WEB

CALCULATIONS FOR THE WEB—*Shearing stress*—As already mentioned, the web can be considered as taking the whole of the shearing stress, and no great error will be involved by further assuming that the stress is uniformly distributed over the cross section The methods of finding the amount of the shearing stress have already been explained in Chap III, p 55 *et seq*

The thin webs of plate girders tend to fail, much in the same way as long compression bars do, by bending or buckling, as shown in Fig 242, and, to understand this, we must consider for a moment the stresses acting on a small portion of the web Let Fig 243 represent a small square marked on the web We know from p 61 that vertical shearing stresses V act along

¹ The rivet holes are not deducted as the rivets are supposed to fill them so that the pressure is transmitted over the whole section Some engineers would take $r_c = 5$ tons over the section deducting the rivet holes $\bar{M}$ of the angle irons would then be the same as for the tens on flange 395 inch tons

AB and CD, and that horizontal shearing stresses H act along CB and AD. Now it is clear that these forces will distort the square and make it take the shape of a lozenge, as shown in an exaggerated manner in Fig. 244. This

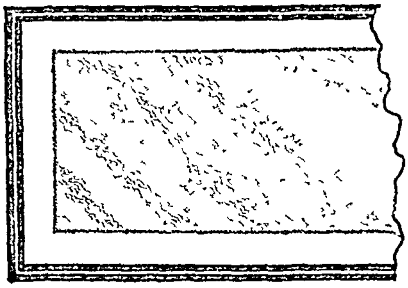


Fig. 242.

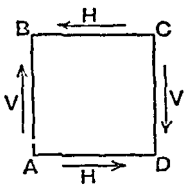


Fig. 243.

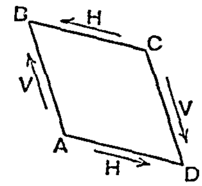


Fig. 244.

distortion clearly extends the material in the direction BD, and compresses it in the direction AC; and the tensions and compressions thus excited in the material will just balance the shearing stresses. This may perhaps be more clearly understood by imagining the square AC formed of four rods jointed

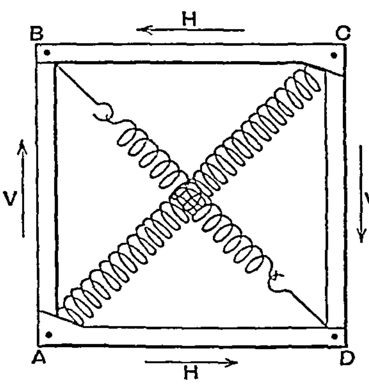


Fig. 245.

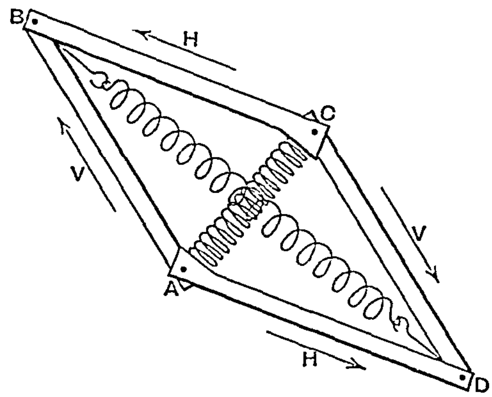


Fig. 246.

at A, B, C, and D, and two springs introduced as shown in Fig. 245. When the forces H, H, V, V commence to act, the square will be deformed as shown in Fig. 246; the spring BD will be stretched and the spring AC compressed; and (supposing the spring AC is not allowed to bend) the extension and compression of the springs will continue until they balance the applied forces.

Now Professor Rankine has shown that the *intensity* of tension and compression excited as described above is equal to the intensity of the shearing stresses which produce them. We may therefore replace the vertical and horizontal shearing stresses, acting on the web of a plate girder, by a tensile and compressive stress of equal intensity, and acting in directions inclined at an angle of 45° (see Fig. 247).

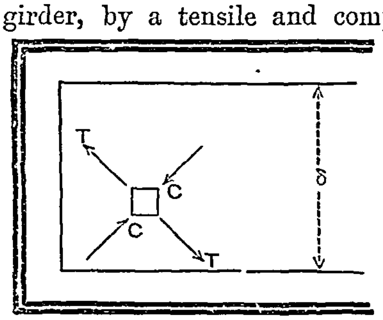


Fig. 247.

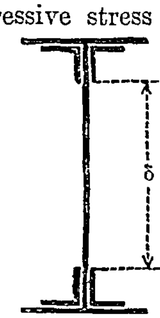


Fig. 248.

Now it is the compressive stress CC which tends to cause the buckling of the web, and for purposes of calculation we can consider the web as made up of a series of strips, inclined downwards towards the supports at an angle of 45°,

and each acting as a long column. The length of these long columns is clearly [1]

$$\sqrt{2} \times \delta,$$

where δ is the unsupported depth of the web between the angle irons (see Fig 248), and they can be considered as "fixed" at the ends The thickness (t_w) of the web is the least diameter of the long columns in question We can therefore apply Gordon's formula (see p 113, or Tables V and XI) to calculate the necessary thickness to be given to the web It, however, frequently happens that an undue thickness would have to be given to the web to make it stiff enough In such a case the additional resistance to buckling can be obtained by riveting vertical T or L iron stiffeners to the web as shown in Fig 249,

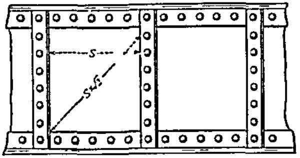


Fig 249

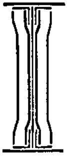


Fig 250

and if S is the clear space between the stiffeners, the length of the long columns into which the web is supposed to be divided is

$$S \times \sqrt{2},$$

and Gordon's formula or Table V can be applied as before

As stated below,[2] the base and perpendicular of the triangle, of which the column is the hypothenuse, are equal Therefore, when the distance S between the stiffeners is less than δ, S will represent the base and perpendicular of the triangle, and the length of the column will be shortened From this it will be seen that stiffeners are theoretically useless unless they shorten the length of the assumed columns, and to be effective must be closer together than the depth

Example 33—The depth between centres of rivets of a web is 21 and the unsupported depth (δ) is 18", the thickness $\frac{5}{16}$, and the shearing stress to be resisted is 9 3 tons Find whether any stiffeners are necessary, and if so, at what distance apart they ought to be placed. Find also what thickness ought to be given to the web to obviate the necessity of stiffeners [3]

---

[1] Length of the long column $= \dfrac{\delta}{\sin 45°} = \sqrt{2} \times \delta$

[2] The length of the columns is represented by the length of a line drawn across the web *in the direction that the web will fail, and as this direction is at an angle of* 45°, its length can be represented by the hypothenuse of a right angled triangle whose perpendicular is equal to the depth of the web The formula $\sqrt{2} \times \delta$ which expresses the length of the hypothenuse in terms of the depth of the web is arrived at from *Euclid* Book I, Proposition 47

[3] The web being very thin the shearing stress may practically be considered as uniformly distributed not as in Fig 97, the depth of the web to resist shearing being taken as equal to the distance between the centres of the rivets connecting it to the flanges 21" in the present example

Substituting in Gordon's formula (see p. 113) as follows—

$$r_c = 4 \text{ tons},$$
$$a = \frac{1}{2500},$$
$$l = 18 \times \sqrt{2},$$
$$d = \tfrac{5}{16}'',$$

we obtain—

Safe intensity of compression in the web $= \dfrac{4}{1 + \left(\dfrac{1}{2500} \times \dfrac{(\sqrt{2} \times 18)^2}{(\frac{5}{16})^2}\right)}$

$= 1 \cdot 1$ tons per square inch.

Using Table V., n being 3·5, the ratio is $3 \cdot 5 \dfrac{\sqrt{2} \times 18 \times 16}{5}$; or 285, whence the safe intensity of compression is 0·88 ton.

Now the intensity of shearing stress per square inch is

$$\frac{9 \cdot 3}{21 \times \frac{5}{16}} = 1 \cdot 42 \text{ ton.}$$

Evidently, therefore, the web is not quite stiff enough.

To find the distance apart of the stiffeners we have

$$\frac{4}{1 + \dfrac{1}{2500} \times \dfrac{(\sqrt{2} \,.\, \mathrm{S})^2}{(\frac{5}{16})^2}} = 1 \cdot 42.$$

Whence we find by reduction

$$\mathrm{S}^2 = 221,$$
$$\mathrm{S} = 14 \cdot 9 \text{ inches.}$$

The distance apart of the stiffeners can also be found by means of Table V. The ratio corresponding to a stress of 1·42 ton is 215. Hence

$$3 \cdot 5 \frac{\sqrt{2} \,.\, \mathrm{S}}{\frac{5}{16}} = 215, \text{ where } 3 \cdot 5 = n \text{ in Table V.}$$

or $$\mathrm{S} = 13 \cdot 5 \text{ inches.}$$

To find the thickness of web required to obviate use of stiffeners we have

from Table V. as above $$3 \cdot 5 \frac{\sqrt{2} \times 18}{t_w} = 215,$$

or $$t_w = 0 \cdot 413 = \tfrac{7}{16} \text{ inch nearly.}$$

We can also use Table XI. for this purpose. The stress per foot of depth of the girder is $\dfrac{9 \cdot 3 \times 12}{21} = 5 \cdot 3$ tons, which for a net unsupported depth of 18″ corresponds to a thickness of web of $\frac{3}{8}$″.

The shearing stress we have been dealing with is the maximum stress at the ends. The intermediate stresses diminish towards the centre (see Fig. 90), and the thickness of web may accordingly be reduced; but it is never advisable, for practical reasons, to use less than $\frac{1}{4}$″ plates even at the centre of the girder.

When the load on the girder is concentrated at one or more points it is usual to add stiffeners to the web at such points, to transmit the load equally to both flanges.

Joints in the Web—These joints are arranged as shown in Fig 251. One or two covers can be used as shown in the sections, but it is better work to use two covers The thickness of one cover should be the same as that of the web When two covers are used their combined thickness should be at least equal to that of the web, so long as neither of them is made less than $\frac{1}{4}$ inch thick

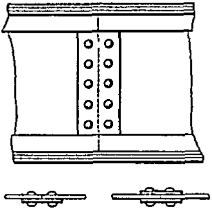


Fig 251

As regards the number of rivets required to make the joint, it is to be observed that they must be sufficient on each side of the joint to take up the shearing stress at the section of the girder where the joint is made

Example 34—For instance, taking the same data as in the two previous examples, namely—

Shearing stress = 9 3 tons,
Thickness of web = $\frac{3}{8}$ inch,
Diameter of rivets = $\frac{7}{8}$ „

the bearing resistance of one rivet is 2 62 tons, so that the number of rivets required on each side of the joint is

$$\frac{9\ 3}{2\ 62} = 3\ 5 \text{ nearly}$$

These rivets are arranged as shown in Fig 251, and the pitch would be

$$\frac{18}{3\ 5} = 5 \text{ inches nearly,}$$

which is too great, being 13 times the thickness of the plates riveted The pitch may be taken at 12 times that thickness,[1] or practically at 4

Width of cover plates—As regards the width of the cover plates, following the rule, p. 136, that the centres of rivets are not to be nearer the edge of a plate than $1\frac{1}{2}$ times their diameter, it will be seen that the width must not be less than $4 \times 1\frac{1}{2}\ d = 6\ d$

In the above example, therefore, the least width of cover plate is

$$6 \times \tfrac{7}{8} = 5\tfrac{1}{4} \text{ inches, say 6 inches,}$$

and this would be a suitable width to take

Connection of the Web with the Flanges.

The web is connected to the flanges by being riveted to the angle irons, and the number of rivets should be sufficient to withstand the horizontal shearing stress The easiest way is to calculate the amount of the shearing stress per foot, and then to find the number of rivets required for bearing and for shearing

Example 35—In the above example, for instance, the shearing stress at each end per foot

[1] See Stoney s Rule, p. 136

$$= \frac{9 \cdot 3 \times 12}{21} = 5 \cdot 3 \text{ tons.}$$

Now if $\frac{7}{8}''$ rivets are used, the resistance of one rivet to bearing in web (taken as $\frac{6}{16}''$ thick) is $= 2 \cdot 62$ tons.
(The resistance to shearing need not be considered, as it is much greater.)
Hence the number of rivets required per foot

$$= \frac{5 \cdot 3}{2 \cdot 62} = 2,$$

so that a 6-inch pitch would be sufficient as regards bearing, but this would be too great in proportion to the thickness of the plates, and a pitch of not more than $12 \times \frac{3}{8} = 4\frac{1}{2}''$ should be adopted. In practice $\frac{3}{4}''$ rivets with 4″ pitch would probably be used in both this and Example 34.

We have now considered the main points in the calculations required for plate girders. In the two following examples some further matters of detail will be considered.

Example of Plate Girder with Single Web.[1]

Example 36.—Design the girder shown in outline in Fig. 26, p. 21.

Load.—The load to be borne by the girder is 23 tons placed 10 feet from the left support, and from Fig. 54 it appears that the greatest bending moment occurs at this point. The weight of the girder itself must, however, be taken into consideration, and this weight can be found by using either of the formulæ given in Appendix XV. These formulæ can only be expected to give approximate results, but the error is, as a rule, too small to be of any practical importance; they are based on the supposition that the load is uniformly distributed. To apply them in the present case we must therefore first find the "equivalent" uniformly distributed load, that is the uniformly distributed load that would produce the same bending moment at the centre of the girder that the actual loading does. Now if W is this load, we know by Case 7, p. 33, that the bending moment in the centre is

$$= \frac{Wl}{8} = \frac{W \, . \, 30}{8} \text{ foot-tons.}$$

And from Equation 2, p. 25, the bending moment due to the actual loading is

$$R_A \times 10' = 15 \cdot 33 \times 10 = 153 \cdot 3 \text{ foot-tons.}$$

So that

$$W = \frac{8}{30} \times 153 \cdot 3 = 40 \cdot 9 \text{ tons.}$$

This does not include what would be required to carry the weight of the girder itself, but the additional material would be small (about ·16 ton), so that we can take $W = 41$ tons.

Approximate weight of girder.—Using Unwin's formula (see Appendix XV.) and taking the data given there, and taking $D = 30''$ as below, we obtain

$$\text{Weight of girder} = \frac{41 \times 30^2}{1500 \times 4 \times \frac{30}{12} - 30^2},$$
$$= 2 \cdot 62 \text{ tons.}$$

As a check we may use Anderson's formula, and thus we obtain

$$\text{Weight of girder} = \frac{1}{560} \times 41 \times 30,$$
$$= 2 \cdot 2 \text{ tons.}$$

---

[1] See Plate A, p. 352.

The former result agrees with the weight of the girder given at p 21 (2 6 tons), and we will assume it to be the weight of the girder

Depth of girder.—The depth of girders is usually taken at from $\frac{1}{8}$ to $\frac{1}{15}$ of the span It should be observed that the deeper the beam the more liable is the web to buckle, and therefore a thicker web or a greater number of stiffeners will be required On the other hand, a shallow girder requires more metal in the flanges, and is less stiff We will assume a depth of 30″, that is $\frac{1}{12}$ of the span The distance between the centres of gravity of the flanges may be taken as 30″

Deflection.—It remains to be seen whether the depth chosen (30″) is sufficient to ensure that the deflection does not exceed a certain amount, say $\frac{1}{40}$″ per foot of span The total admissible deflection would be

$$=\tfrac{1}{40}\times 30=0\ 75 \text{ inch}$$

Now, as the beam is to be subjected to certain limiting stress, we can use Equation 46, p. 67, namely—

$$\Delta=\frac{n'(r_c+r_t)l^2}{ED}$$

As regards the value to be given to n', it is to be observed that the beam will be stiffer than a beam of uniform strength, but less stiff than a beam of uniform section, for the former $n'=\frac{1}{8}$, and for the latter (with central load) $n'=\frac{1}{12}$ It will therefore be a fair approximation to take $n'=\frac{1}{10}$

Further, let

$$r_c=4 \text{ tons},$$
$$r_t=5 \text{ tons},$$
$$l=360 \text{ inches}.$$

As regards the value of E it should be observed that the full value (29,000,000 lbs.) for wrought iron cannot be reckoned on in built up girders, owing to the unavoidable imperfections of riveting A value between 16,000,000 and 18,700,000 is generally assumed We will take 17,920,000 lbs., or 8000 tons Hence

$$\Delta=\frac{\frac{1}{10}\times 9\times 360^2}{8000\times 30},$$
$$\Delta=0\ 49 \text{ inch}$$

So that the girder is amply stiff enough

Width of flanges.—The width of the compression flanges should be sufficient to prevent buckling sideways. It is found by experience that for this purpose the compression flange should be made about $\frac{1}{30}$ to $\frac{1}{40}$ of the span, that is in this case 12 to 9 inches We will take 9 inches. The width of the tension flange is not subject to the same condition, but for uniformity we will assume it to be also 9 inches wide

Thickness of flange plates.—The thinner the plates used the more nearly can the girder be made to approximate to a beam of uniform strength On the other hand, corrosion affects thin plates to a serious extent, and the number of plates to be joined together is increased It should also be observed that very thin plates are liable to buckle along the edges when placed in the compression flange Professor Unwin says that the thickness

should not be less than $\frac{1}{700}$ of the span. In practice the thickness is limited between $\frac{1}{4}''$ and $\frac{3}{4}''$; usually $\frac{3}{8}''$ or $\frac{1}{2}''$ is found to be suitable. We will in this example take $\frac{3}{8}''$ plates.

Angle irons for connecting the flanges to the web.—The thickness will be taken the same as that of the flange plates, namely $\frac{3}{8}''$. A table 3 inches wide gives sufficient room for riveting, but in the present case we will use angle irons $3\frac{1}{2}'' \times 3\frac{1}{2}'' \times \frac{3}{8}''$, as they will afford more room for riveting.

Calculation of flanges.—We are now in a position to proceed with the calculation of the flanges in the manner explained at p. 156.

From Fig. 54, p. 35, it will be seen that the diagram for the moment of flexure due to the single load consists of a triangle, the height of which is the maximum moment of flexure. This maximum moment of flexure due to the load is in the present case equal to

$$R_A \times 10 \times 12 = 15{\cdot}33 \times 10 \times 12,$$
$$= 1840 \text{ inch-tons.}$$

We thus get the triangle A*d*B shown in Fig. 252.

The weight of the girder itself produces a different diagram of bending moments, namely the arc of a circle (see p. 34); and the height of the arc is the bending moment at the centre, that is

$$\frac{2{\cdot}6 \times 30 \times 12}{8} = 117 \text{ inch-tons.}$$

We thus obtain the arc A*c*B shown in Fig 252.

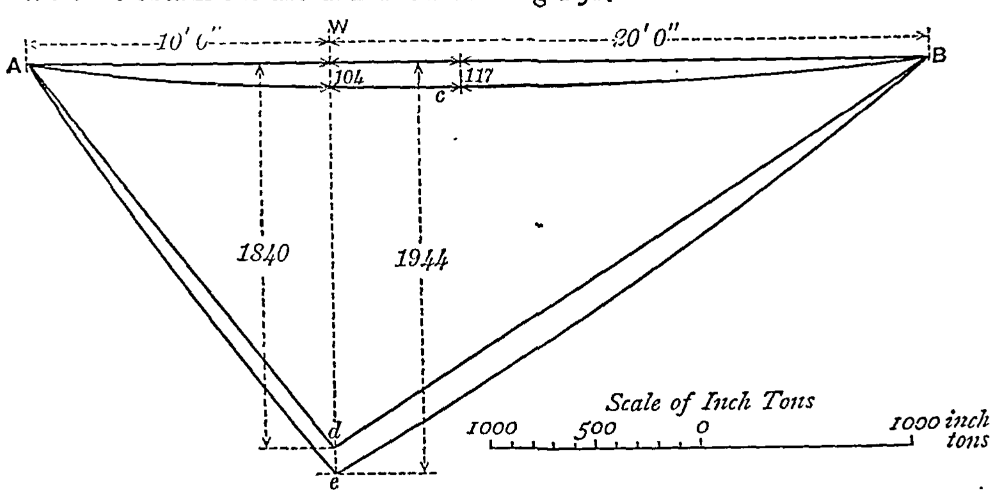


Fig. 252.

The diagram for the total bending moments can of course be found by adding the ordinates of the arc to those of the triangle, and thus, finally, we get A*e*B as the diagram of bending moments.

Tension flange (see Plate A).—The rivet holes must be deducted from the cross section to obtain the effective section. Deducting one $\frac{3}{4}''$ rivet hole from each angle iron, and taking the depth to the backs of the angle irons at 29″, *i.e.* 1″ less than the distance between the centres of gravity of the flanges, whence the "effective"

depth[1] of the angle irons is 27" we obtain the moment of resistance of both angle irons

$$=2(3\tfrac{1}{2}+3\tfrac{1}{2}-\tfrac{3}{8}-\tfrac{3}{4})\tfrac{3}{8}\times 27\times 5,$$
$$=595 \text{ inch tons.}$$

Two rivet holes must be deducted from each plate, so that moment of resistance of the first plate is

$$=(9-2\times\tfrac{3}{4})\tfrac{3}{8}\times 29\tfrac{3}{8}\times 5,$$
$$=413 \text{ inch tons nearly}$$

In the same way it will be found that the moment of resistance of the second plate is $=423$ inch tons, and of the third plate 434 inch tons. The total moment of resistance of the L irons and the three plates is therefore 1865 inch tons, which is less than the maximum bending moment of 1914 inch tons, but practically sufficient.

Following out the process explained at p 155, we obtain the completed diagram given in Fig 253, which shows the number of plates required and the length of each On referring to the Tables in Part III, giving the market sizes of iron it will be seen that plates of the full length can readily be obtained, and that therefore no joints are needed, but as an exercise for the student the joints

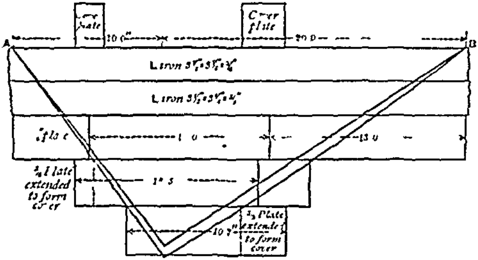


Fig 253

have been so arranged in Fig 253 that no plate will exceed 13 feet in length, and, moreover, the joints have been so placed that 'grouped' joints can be formed Each of these joints requires to be designed and calculated on the principles explained in Chap VII, and in fact one of the joints has been worked out in Example 24, p 132

Compression flange—The diagram of bending moments just

[1] *i e* distance between centres of gravity of angle irons.

obtained for the tension flange holds good for the compression flange, and is re-drawn in Fig. 254, but upside down, so as to show the plates in their proper relative positions.

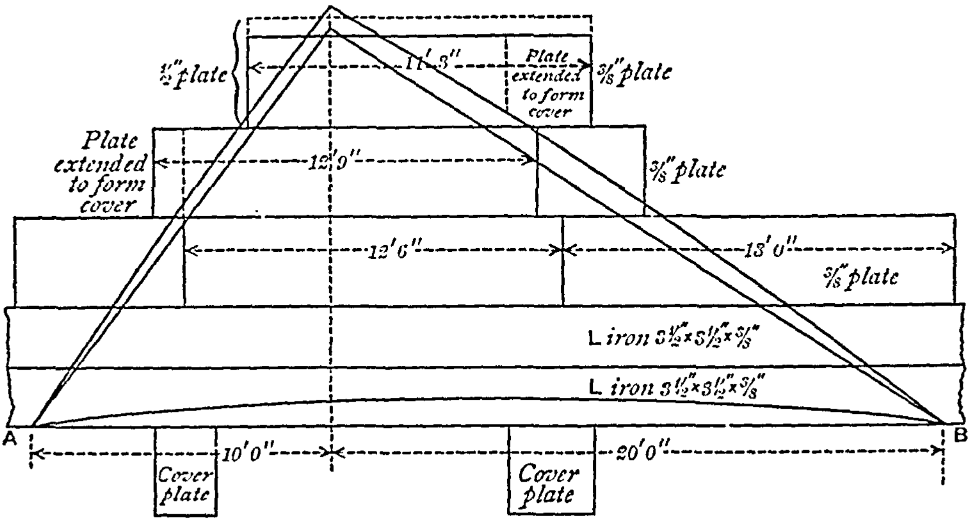


Fig. 254.

As already remarked at p. 136, it is not necessary in this flange to deduct the rivet holes to find the effective section, but the safe stress is reduced to 4 tons per square inch.

The moment of resistance of both angle irons will therefore be

$$2(3\tfrac{1}{2} + 3\tfrac{1}{2} - \tfrac{3}{8})\tfrac{3}{8} \times 27 \times 4,$$
$$= 537 \text{ inch-tons.}$$

The moment of resistance of the first plate will be

$$9 \times \tfrac{3}{8} \times 29\tfrac{3}{8} \times 4 = 396,$$

and so on.

The total moment of resistance of the L irons and of the three plates will be found to be 1754 inch-tons, but the maximum bending moment is 1944 inch-tons. It will be therefore seen that three $\frac{3}{8}''$ plates do not give sufficient resistance at the point of maximum moments of flexure, but practically they are quite safe. By making the outside plate $\frac{1}{2}''$ thick the moment of resistance will be increased to 1897 inch-tons, which is near enough.

Repeating the operation we can obtain the number and length of plates required, and also ascertain in what position the joints ought to be placed if full-length plates cannot be obtained.

The flanges can now be considered as designed, with the exception of the joints, for which the student is referred to Example 24, p. 132.

Web.—To design the web the first step is to find the shearing stress in each portion of it.

The simplest way is to make use of the diagrams given in Chap. III. As the girder is subjected to two loads, we must combine the diagram for a uniform load (Fig. 90, p. 59) with that for concentrated load placed at any point (Fig. 92).

Thus, in Fig 255, $AefB$ represents the shearing stress due to the weight of the girder, and $AghklB$ the shearing stress due to the load of 23 tons, so that $AlmnoB$ represents the shearing stress to which the web is subjected.

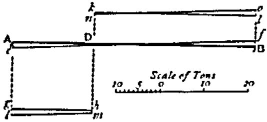


Fig 255 [1]

It will be observed that owing to the weight of the girder being small in comparison with the load to be carried, the shearing stress is almost constant from A to D and again from D to B It will therefore be theoretically best to make the web of a certain uniform thickness from A to D, and of another less thickness from D to B, these thicknesses must now be found

For the part AD we have a maximum shearing stress of 16 6 tons to deal with (see p 22), and the depth of the web (δ) between the angle irons may be taken as 22" and the distance between the centres of rivets as 25 1 (see Plate A)

Assuming the web to be $\frac{1}{2}$" thick, the intensity of shearing stress is

$$\frac{16\ 6}{25\ 1 \times \frac{1}{2}} = 1\ 32 \text{ ton per square inch}$$

Now referring to Table V, we find the value of $\frac{l}{\kappa}$ for the equivalent long column to be, taking $n = 3\ 5$,

$$3\ 5 \times \frac{22 \times \sqrt{2}}{1} = 218,$$

and hence the safe intensity of stress (for a column fixed at both ends) to be nearly 1 10 ton, so that a $\frac{1}{2}$" web will do very well.

We can also find the thickness of the web by means of Table XI The net unsupported width of the plate is 22 and the shearing stress per foot is $\frac{16\ 6 \times 12}{25\ 1} = 7\ 94$ tons, and it will be seen on referring to the Table that $\frac{1}{2}$ is the thickness required

It should be noticed that the $\frac{3}{4}$" rivets, which have been selected, are a little small for a $\frac{1}{2}$" plate, but this difficulty can be overcome by drilling the holes, or by using a $\frac{3}{8}$" plate for this part of the web, supported by stiffeners

Pitch of rivets to connect the web to the flanges —The stress per foot is

$$\frac{16\ 6}{25\ 1} \times 12 = 7\ 94 \text{ tons nearly}$$

Taking $r_s = 4$ tons and $r_b = 8$ tons the resistance to bearing of a $\frac{3}{4}$" rivet in a $\frac{1}{2}$" plate is 3 0 tons (Table VIII), and the resistance to double shear is $1\ 8 \times 2 = 3\ 6$ tons, so that the number of rivets per foot

$$= \frac{7\ 9}{3\ 0} = 2\ 6$$

Practically a pitch of 4" would do

For the part DB of the web we have a maximum shearing stress of 9 0 tons (see p 22) Assuming the web to be $\frac{3}{8}$" thick, the intensity of shearing stress is

$$\frac{9\ 0}{25\ 1 \times \frac{3}{8}} = 0\ 95 \text{ ton}$$

But the value of $\frac{l}{\kappa}$ is

[1] See Appendix V, p 298

$$3{\cdot}5 \times \frac{22 \times \sqrt{2}}{\frac{3}{8}} = 290.$$

And from Table V. the safe intensity of stress is 0·86 ton, so that a $\frac{3}{8}''$ plate is somewhat thin. It should be observed, however, that the maximum shearing stress occurs only at the abutment B, and it will also be found that $\frac{\cdot 7}{16}''$ is thicker than necessary according to the Table. There can, therefore, be no doubt that $\frac{3}{8}''$ is thick enough.

To use Table XI. we have: net unsupported width of plate $= 22''$, and shearing stress per foot $= 4{\cdot}4$ tons, which correspond to a thickness of $\frac{3}{8}''$.

Pitch of rivets to connect the web to the flanges.—The stress per foot is

$$\frac{9{\cdot}0}{25{\cdot}1} \times 12 = 4{\cdot}3 \text{ tons},$$

and the resistance to bearing of a $\frac{3}{4}''$ rivet in a $\frac{3}{8}''$ plate is 2·2 tons. Hence the number of rivets per foot

$$\frac{4{\cdot}3}{2{\cdot}2} = 1{\cdot}9,$$

or a 6″ pitch will do as regards bearing; but in order to keep the plates together the pitch should not exceed $12 \times \frac{3}{8}'' = 4\frac{1}{2}''$, and may practically, for the sake of uniformity, be made 4″ as in Plate A.

Joints in the web.—Two joints will be required in the web, so as to keep the plates within the ordinary market sizes. One of these joints will be best placed immediately under the load, and the joint can be made by means of two ⊥ irons, which will also act as a stiffener to transmit the load to both flanges of the girder. Referring to p. 136 we see that the least width of the table of the ⊥ iron should be $6 \times \frac{3}{4} = 4\frac{1}{2}$ inches. A 6″ ⊥ iron will however afford more room for the riveting as shown in Fig. 256.

It will be noticed that this joint is placed at the point where there is no shearing stress in the web, so that theoretically there is no need for a cover; but practically it would not do to dispense with it, and, moreover, in this case, as before said, it fulfils the important duty of assisting to transmit the load of 23 tons.

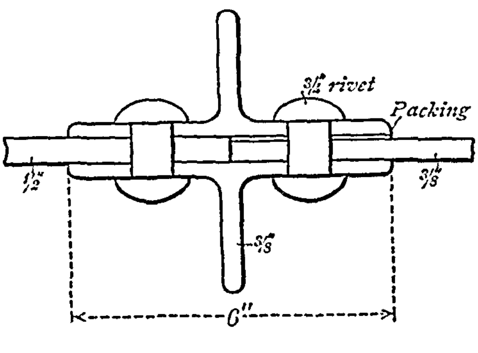


Fig. 256.

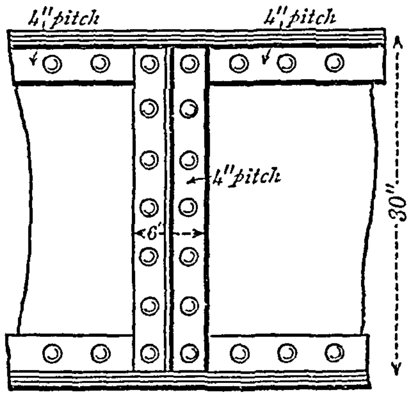


Fig. 257.

Theoretically, no rivets are required to connect the ⊥ iron to the web, but practically they would be placed at about 4″ pitch to put the girder together. Fig. 257 represents the joint.

The second joint will be best placed midway between D and B. The shearing stress at this point found from Fig. 255 is 8·4 tons.[1] The cover plates

[1] The student is recommended to draw Fig. 255 to a large scale to obtain this value.

can be made 6" wide, and $\frac{5}{16}$" or $\frac{6}{16}$" is a suitable thickness To find the number of rivets, the bearing resistance of $\frac{3}{4}$" rivets in a $\frac{3}{8}$" plate is 2 25 tons, therefore number of rivets

$$=\frac{8\ 4}{2\ 25}=4 \text{ nearly},$$

or a 4" pitch will do

End pillars.—The ends of the girder should be stiffened to resist the shearing stress at those points The most *economical* way is to stop the longitudinal angle irons at the ends of the girders, and to rivet two *vertical angle irons* to the web, using packing pieces as shown in Fig 258

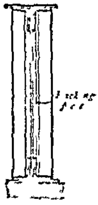


Fig 258

The ends of the girder are generally bedded on sheet lead The rivet heads should be countersunk to allow of this and four of the rivets would be left out at one end to enable the girder to be bolted down to the top of the column

Fig 259 and Plate[1] A show the girder as arranged to comply with the calculations made above The total length of the girder is 31′ 0″, 1′ 0′ more than the distance between centres of bearings, or 2′ 0″ more than the clear span, thus allowing 12″ at each end to rest on the cap of the C I column and on the wall The bearing area is therefore $\frac{9}{12}\times 1 = 0\ 75$ square foot, so that the pressure at the wall end is $\frac{9\ 0}{0\ 75}=12\ 0$ tons per square foot, and this is not more than a hard stone can bear If, however, the stone is soft a casting ought to be placed under the end of the girder to distribute the pressure

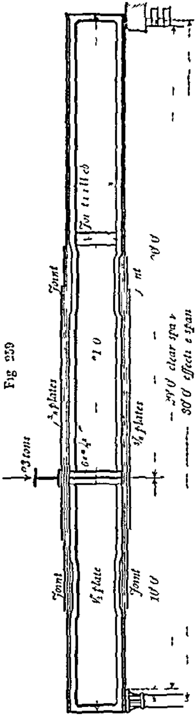


Fig 259

[1] In Plate A the joggles in the angle iron are omitted for the sake of simplicity and economy in construction

Example of Box Girder.

Example 37.—Calculate and design a box plate girder to act as a bressummer, as shown in Fig. 260 (see Plate B, p. 353).

Load.—The first step is to find the load to be borne by the girder. The portion of the wall supported by the girder is shown in dotted lines[1] in Fig. 260, and the cubic content of this portion is

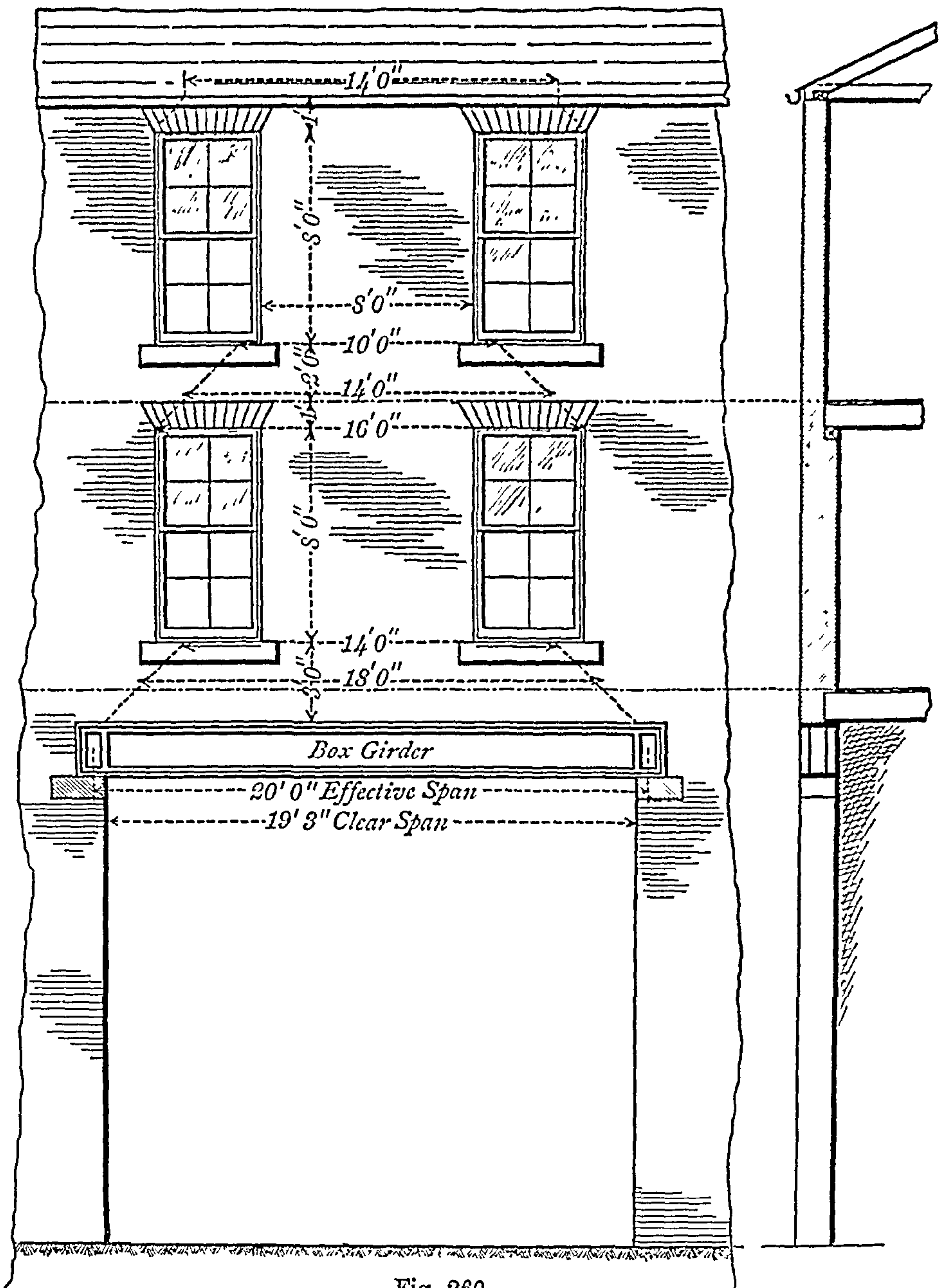


Fig. 260.

[1] It might be objected that the load due to the wall ought to be taken as the weight of the wall contained between vertical lines drawn from the ends of the

$$\left\{\frac{20+11}{2}\times 3+8\times 8+\frac{16+11}{2}\times 1\right\}\frac{14}{12}$$
$$+\left\{\frac{14+10}{2}\times 2+8\times 8+\frac{16+14}{2}\times 1\right\}\frac{9}{12}$$
$$=229 \text{ cubic feet}$$

A cubic foot of brickwork weighs 1 cwt. (see Table XVII.),[1] so that weight of brickwork to be supported = 229 cwts.

The first floor is supported over a length of 18 feet, and since the floor is 20 feet wide, area supported = 18 × 10 = 180 square feet.

The second floor is supported for a length of 14 feet by the girder, hence floor area supported = 14 × 10 = 140 feet.

The total for both floors is therefore 320 square feet.

Assuming that these are warehouse floors, and that the load is $3\frac{1}{2}$ cwts. per square foot, the total load produced by the two floors is

$$320\times 3.5=1120 \text{ cwts.}$$

Lastly, the length of the roof supported is 14 feet, that is, a horizontal area of 14 × 10 = 140 square feet is supported. Allowing 40 lbs. per square foot (Tredgold), the load due to the roof is

$$\frac{140\times 40}{112}=50 \text{ cwts.}$$

The total load on the girder (except its own weight) is therefore

$$229+1120+50=1399 \text{ cwts.},$$
$$=70 \text{ tons very nearly.}$$

Approximate weight of girder.—Clearly this load can be taken as uniformly distributed.

Using Anderson's formula (Appendix XV.) to obtain the weight of the girder we get the weight per foot run of girder

$$\frac{2240\times 70}{560}=4\times 70 \text{ lbs.}=280 \text{ lbs.}$$

Hence total weight of girder

$$=280 \text{ lbs.}\times 20=5600 \text{ lbs.}$$
$$=2.5 \text{ tons.}$$

Therefore the total distributed load the girder has to sustain is

$$70+2.5=72.5 \text{ tons.}$$

Deflection.—As the load is considerable we will take the depth at $\frac{1}{10}$ of the span, that is 24". It remains to be seen whether this depth will give sufficient stiffness. In the formula (Equation 16)

$$\Delta=\frac{n'(r_c+r_t)l}{\mathrm{ED}}$$

We have in this case

$$r_c=4 \text{ tons},$$
$$r_t=5 \text{ tons},$$
$$l=240 \text{ inches},$$
$$\mathrm{E}=8000 \text{ tons (see p. 163)},$$
$$\mathrm{D}=24 \text{ inches},$$

girder, it is considered, however, that the bond of the brickwork causes it to take part of the weight.

[1] Under some circumstances it would be better to take 130 lbs. per cubic foot as the weight of the brickwork, to allow for the bricks being saturated with water.

So that

$$1{\cdot}90 = \frac{4}{1 + \frac{1}{2500}\cdot\left(\frac{16c\sqrt{2}}{7}\right)^2}$$

From this equation the value of c can be found, an exercise which is left to the student.

The result can be obtained more rapidly by using Table V. It will be found that a safe intensity of stress of 1·90 ton corresponds to a value of $\frac{l}{\kappa}$ of 170. Hence taking $n = 3{\cdot}5$ we have

$$\frac{16c\sqrt{2}}{7} = \frac{170}{3{\cdot}5},$$

whence $c = 15$ inches nearly.

The stiffeners can therefore be arranged as shown in Fig. 264, one stiffener being placed immediately at the edge of the abutment, the other just where the $\frac{7}{16}''$ plate commences to be strong enough without assistance.

Joints in the web.—There will be two joints; the calculations for these are precisely similar to those already made, and are left to the student.

Joints in the flanges will not be necessary, as the longest plate is only 20′ long.

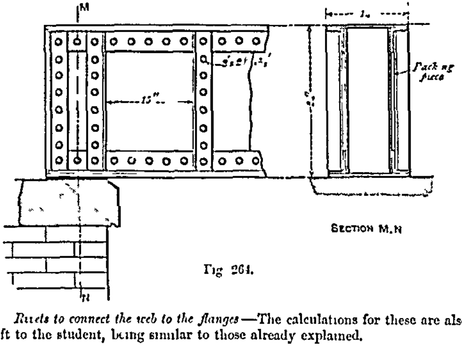


Fig. 264.

Rivets to connect the web to the flanges.—The calculations for these are also left to the student, being similar to those already explained.

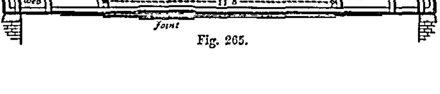


Fig. 265.

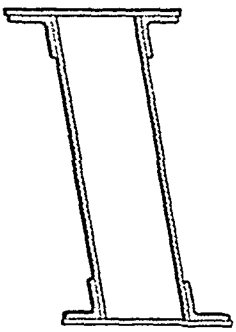


Fig. 266.

End pillars.—The clear span is 20 feet, but the girder must be made longer at each end so as to bear on the stone sills, say 9 inches; and as the shearing stress is considerable, it would be advisable to adopt the arrangement shown in Fig. 264.

The completed girder is shown in Fig. 265, the thickness of the plates being considerably exaggerated, also in Plate B.

Intermediate diaphragms.—In a large girder, two or three of these should be inserted. They are pieces of plate iron riveted to both webs to prevent racking, that is, to prevent the girder bending over as shown in Fig. 266.

Saddle-backed or Hog-backed Girders have a curved upper flange such as that of the Cast-iron Girder, Fig. 144. Both flanges can be made of nearly uniform section throughout instead of as shown in Fig. 238.

Such girders are however very seldom used in connection with buildings, and need not be further described.

Alternative Rules for the thickness and spacing of stiffeners of Plate Webs.

Sir Benjamin Baker in his remarks on the practical strength of beams says:—[1]

"So far as plate webs of medium size are concerned, the author is of opinion that the general conditions laid down by Mr. Chanute in his specifications for the Erie Railroad bridges, meet all the requirements indicated by experiment, and he cites these in preference to his own practice as being independently deduced.

"These are, that the 'shearing strain' shall not exceed half that allowed in *tension on the bottom flange of a riveted girder, and that when the least thickness* of the web is less than $\frac{1}{80}$ of the depth of the girder, the web should be stiffened at intervals not over twice the depth of the girder."

Mr. Theodore Cooper,[2] M.Am.Soc.C.E., gives the following rule for wrought iron girders:—

"The webs of plate girders must be stiffened at intervals of about the depth of the girder, wherever the shearing strain per square inch exceeds the strain allowed by the following formula:—

$$\text{Allowed shearing strain in lbs.} = \frac{12{,}000}{1 + \frac{H^2}{3000}},$$

where H = ratio of depth of web to its thickness; but no web plates shall be less than three-eighths of an inch in thickness."

[1] *Min. Proc. Inst. C.E.*, vol. 62.

[2] *General Specifications for Iron and Steel Railroad Bridges and Viaducts.*

CHAPTER IX

BRACED OR FRAMED STRUCTURES

General Remarks —Before proceeding to the consideration of lattice girders and roofs, which are braced structures, we must investigate a few general propositions in connection with such structures and we will also give two methods of finding the stresses in them

Let us consider in the first place the simplest case of a weight W supported by two rods, as shown in Fig 267, A and B being

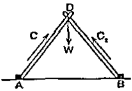


Fig 267

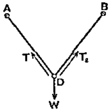


Fig 268

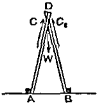


Fig 269

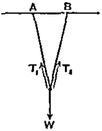


Fig 270

fixed points The two rods are evidently in compression and as they are equally inclined to the direction of the weight they will be subject to equal stresses The same remarks apply to the rods shown in Fig 268, with the exception that they are in tension instead of in compression

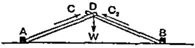


Fig 271

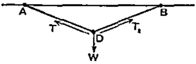


Fig 272

If the inclination of the rods to the *vertical* is diminished, as in Figs 269 and 270, the stress in them will also be diminished, and on the other hand, if the inclination is increased as in Figs 271 and 272 the stress will be increased.

In Figs. 273 and 274 the rod AD is shown at a greater inclination to the vertical than the other rod. Clearly the stress is less

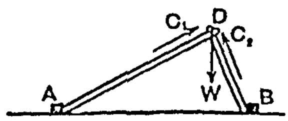


Fig. 273.

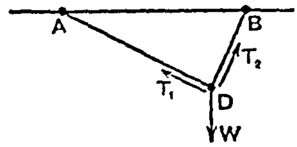


Fig. 274.

in AD than in BD. If, as in Figs. 275 and 276, BD is vertical,

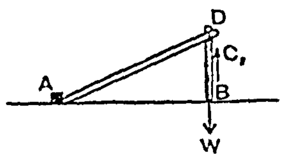


Fig. 275.

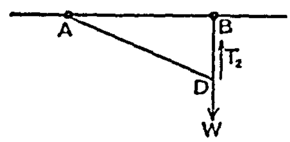


Fig. 276.

that is, the inclination to the vertical is nothing, the stress in BD will be equal to W, and there will be no stress in AD.

Having thus obtained an idea of the manner in which the position of the rods relatively to the weight affects the stresses in them, we proceed to show how the actual value of the stresses (in terms of W) can be found.

Returning to Fig. 267, draw *ba* (Fig. 277) parallel to W, *bd* parallel to BD, and *ad* parallel to AD, then by a theorem called the "triangle of forces,"[1] if *ba* is drawn to scale to represent W, *db* will represent C_2 and *ad* will represent C_1 to the same scale. Supposing, for instance, that W = 1·5 ton, and that *ba* is made 1·5 inch, then it will be found that

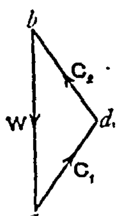


Fig. 277.

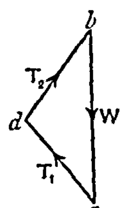


Fig. 278.

$$db = 0{\cdot}93 \text{ inch}, \ ad = 0{\cdot}93 \text{ inch},$$

or

$$C_2 = 0{\cdot}93 \text{ ton}, \ C_1 = 0{\cdot}93 \text{ ton}.$$

Further, *ba* represents the direction of W, and *ad* will give the direction of C_1 and *db* that of C_2, as indicated by the arrowheads in Fig. 277. Transferring these directions to Fig. 267 we see that the rods AD and BD are in compression.

It is very important to observe that in Fig. 277 the arrows follow each other, as it were, in the same direction of rotation.

Again, applying the theorem to Fig. 268, we obtain Fig. 278. As before, if

$$W = 1{\cdot}5 \text{ ton, and } ba = 1{\cdot}5 \text{ inch},$$

we obtain $$db = 0{\cdot}93 \text{ inch}, \ ad = 0{\cdot}93 \text{ inch},$$

$$T_2 = 0{\cdot}93 \text{ ton}, \ T_1 = 0{\cdot}93 \text{ ton}.$$

[1] A proof of this theorem will be found in any work on Statics.

In this case the arrows follow each other in the reverse direction of rotation as compared with the previous case, and transferring to Fig. 268 we find that the rods are in tension.

Applying the theorem in succession to Figs. 267 to 270 and Figs. 273 to 276, we obtain Figs. 279–286,[1] and assuming in each case that W = 1·5 ton as before we find the stresses marked on the figures. The arrows showing the directions of the forces are also marked. The student is recommended to compare these results with the preliminary remarks.

The theorem of the triangle of forces can be thus stated:—

If three forces acting on a point are in equilibrium then, if a triangle be drawn whose sides are parallel to the directions of the three forces, the length of the sides will represent the magnitude of the forces, and the directions of the forces will follow each other round the triangle.

Thus, taking a general case: if F, G, H are three forces acting on the point P as in Fig. 287, then Fig. 288 will be the triangle of

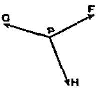


Fig. 287

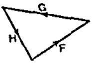


Fig. 288

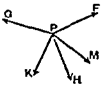


Fig. 289

forces, and if the magnitude of *one* of the forces be known, that of the other two can be found.

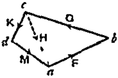


Fig. 290

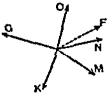


Fig. 291

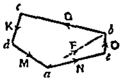


Fig. 292

Now supposing we replace the force H (Fig. 289) by two forces K and M, which together have the same effect as H (in other words, H is the resultant of K and M), then the triangle *cda* (Fig. 290) would be the triangle of forces for K, M, and H (strictly H reversed) so that we obtain the polygon *abcd* to represent the forces G, F, M, and K. In the same way, if F were replaced by two forces N and O (Fig. 291), we should obtain the polygon *aebcd* (Fig. 292), and so on.

[1] Figs. 279–286. See Plate I. at end of volume.

This result is called the "polygon of forces," and it can be stated as follows:—

If any number of forces acting at a point are in equilibrium, then, if a polygon be drawn having its sides parallel to the forces, the sides of this polygon will represent or be proportional to the magnitude of the forces; and the directions of the forces will follow each other in rotation round the polygon.

In the case of the *polygon* of forces it is only possible to find the magnitude of two of the forces when the magnitude of all the others is known. Thus in Fig. 290, if only the magnitude of G were known, all that could be done would be to draw *cb* and the direction of *cd*, but the point *d* could not be fixed, as the magnitude of K is not known; but if the magnitude of K is known, then *d* can be fixed, and drawing *da* and *ba* we see that the magnitudes of M and F are found.

The student is recommended to study the above very carefully, as these results will be found of the utmost importance in the sequel.

Some different forms of framed structures.—A framed structure is composed of a number of bars jointed together. It is usual to assume that these joints are hinged, and offer therefore no resistance to the movement of the bars. Such an assumption, unless pin joints are used, is not true, but it errs on the side of safety.

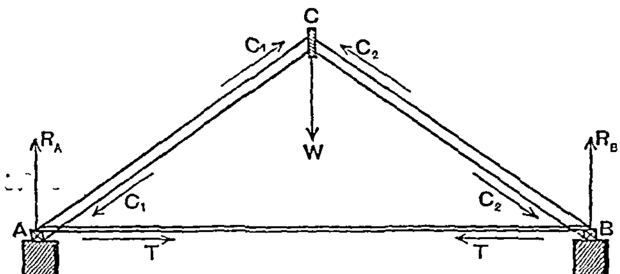


Fig. 293.

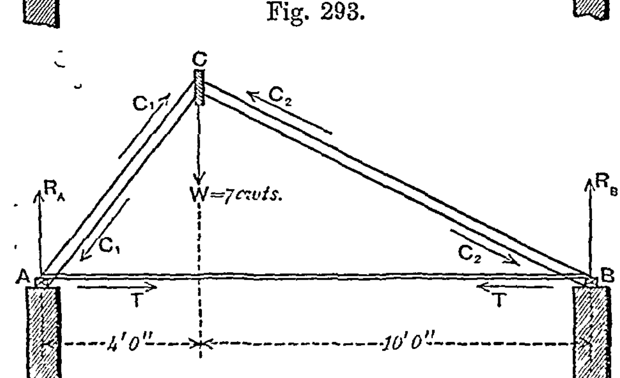


Fig. 294.

The simplest frame is a triangle, as in Figs. 293 and 294, and such a frame is perfectly braced; that is, it is rigid or stiff, and cannot change its form.[1] This being the case, the reactions R_A and R_B can be found by the

[1] A very minute change takes place in consequence of the elongation and shortening of the members produced by the stresses, but this may practically be disregarded.

rule given at p 18 for beams, and thus, taking the numerical data given in Fig 294, we find

$$R_A = \tfrac{10}{14} \times 7 = 5 \text{ cwts}$$
$$R_B = \tfrac{4}{14} \times 7 = 2 \text{ cwts}$$

A frame composed of four bars, as in Fig 295, is, however, not perfectly braced, and would collapse under the action of the load W But if an additional bar DB be added (Fig 296), it will become perfectly braced A moment's consideration will show that this additional bar, called a "brace," is in tension The frame could also have been braced by adding a bar AC (Fig 297), which would, however, have been in compression C_1 and C_3 would then be unnecessary

Fig 295

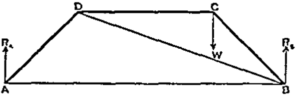


Fig 296

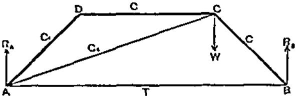


Fig 297

If the bars AD and CB are equally inclined, and another weight W be added at D (Fig 298), the frame would retain its shape, but the least deviation would cause it to collapse, it is, in fact, in unstable equilibrium, and practically a brace would have to be added If only one brace is added, it must be capable of resisting both compression and tension, that is, a simple rod would not do, or two rods capable of resisting tension only may be added, as in Fig 299 In this case the frame is *over braced,* and it is possible, by drawing up the rods by means of coupling screws, to introduce stresses in

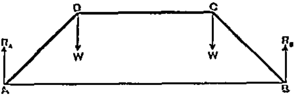


Fig 298

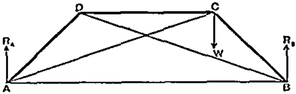


Fig 299

the frame in addition to those produced by the weight. In such a case the frame is said to be self-strained—a state of things which is of course to be avoided as much as possible.

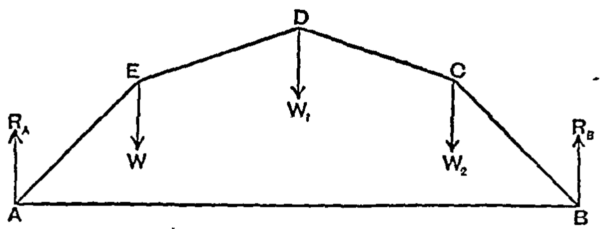


Fig. 300.

In Fig. 300 is shown a frame composed of five bars. If a brace EC be added (Fig. 301), the frame is still incompletely braced, and another brace, either AC or EB, must be introduced. Figs. 302 and 303 show different ways in which a five-sided frame could be perfectly braced, and Fig. 304 shows the frame over-braced.

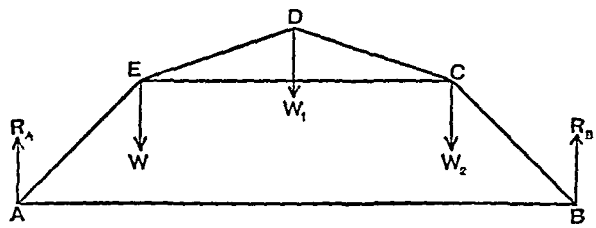


Fig. 301.

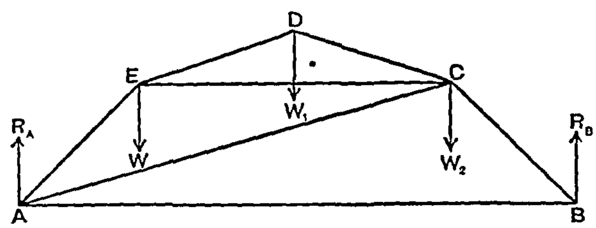


Fig. 302.

From the above it will appear that to *perfectly* brace a frame it must be divided up into *triangles.*

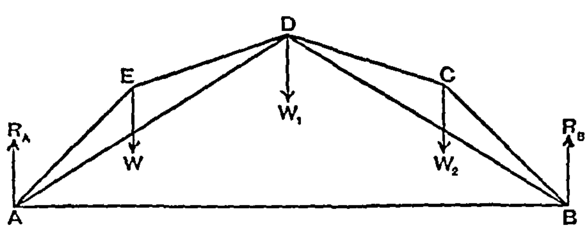


Fig. 303.

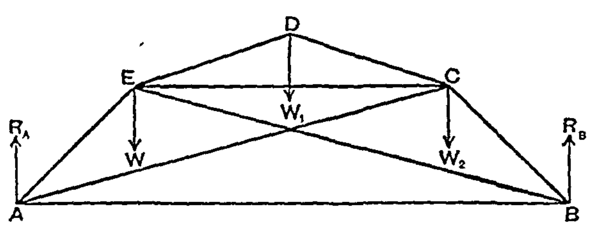


Fig. 304.

Maxwell's Diagrams.

Returning to Fig. 294, p. 180, which is redrawn in outline in Fig. 305,[1] and applying the triangle of forces to the joint A, we obtain the triangle shown in Fig. 306, and therefrom the stresses given. The triangle also shows that the bar CA is in compression, as represented by the arrow pointing *towards* A, and that AB is in tension, as represented by the arrow pointing *away* from A.

[1] Figs. 305-311. See Plate II. at end of volume.

For the joint B we obtain in the same manner the triangle shown in Fig 307

And lastly, at C we obtain the triangle given in Fig 308, and we see that, as should be the case, the values obtained for C_1 and C_2 are the same as before

Method of drawing a stress diagram [1]—The above three triangles can be combined together in one diagram, Fig 309, thus Commence by drawing the triangle for joint A We now have C_1, and on this side of the triangle we can draw the triangle for the joint C, as in Fig 308, taking care to reverse the direction of C_1 Lastly, the triangle for joint B can be drawn on C_2 Fig 309, and if the diagram has been correctly drawn, it will finish off or 'close" where it commenced Such a diagram is called a *stress* or *force diagram*, and is one of the methods of finding the stresses in a framed structure These diagrams were originally proposed by Professor Clerk Maxwell

It will be noticed that in this example we could have commenced by drawing the triangle for the joint C, and then obtain the other two triangles, thus finding the values of R_A and R_B without previous calculation, but it is not always possible to do this, as will be seen by the following example —

In Fig 310, which is an outline copy of Fig 297, but loaded with two weights W and W_1, if we were to commence at the joint C we should have four forces to deal with namely, the weight W_1 and the stresses in the three bars connected together at that joint Therefore, as already seen at p 180, we cannot determine the stresses Finding however, R_A and R_B as usual, we can draw the triangle for joint B, as shown in Fig 311 Now proceeding to joint C we form a polygon and obtain the stresses C_3 and C_4 This polygon is drawn by reversing the direction of C_2 drawing W_1 vertically to scale, then drawing C_3 to meet C_4 drawn parallel to their respective bars Taking D as the next joint, we find C_1 and the polygon for the joint A can then be added

This diagram could, however, have been drawn by commencing at the joint D, without determining R_A or R_B.

The student is recommended to see what effect altering the values of W and W_1 has on the shape of the diagram He will find, for instance, that when $W = W_1$ then $C_3 = T$ and $C_4 = 0$, or, again when $W_1 = 0$, then $C_3 = 0$ and $C_1 = 0$, which is, moreover, evident on inspection

[1] In Appendix XVI will be found more detailed rules for drawing a Maxwell s diagram

The above method will be applied in the sequel to the determination of the stresses in roofs.

Bow's system of notation has not been used for the simple roof trusses here dealt with, but it is admirable for complicated structures. It is explained in Appendix XVII. and used for the braced girders at p. 354.

Method of Sections.

We can now proceed to the consideration of the second method of determining the stresses in a framed structure.

Imagine the frame shown in Fig. 312 cut in two parts along the line PP; then, considering the left portion, it is clear that this portion could be kept in equilibrium by applying to each bar that is cut, a force of the same magnitude and direction as the stress existing in the bar before the section is made.

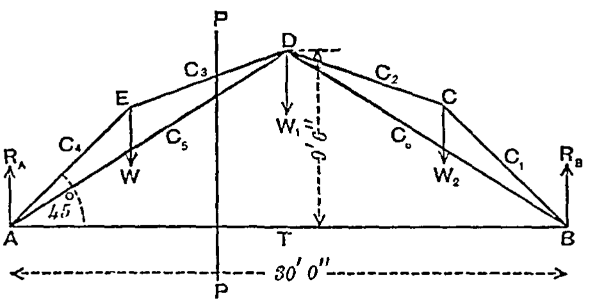


Fig. 312.

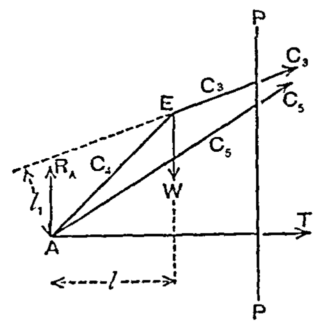


Fig. 313.

Thus the forces R_A, W, C_3, C_5, and T, acting on the portion of the frame shown in Fig. 313, are in equilibrium. Hence the sum of the moments of these forces about any point is zero (see App. II.) Now it is quite immaterial about which point moments are taken. If, therefore, we wish to find C_3, we can select the point A, which is the intersection of C_5 and T (the other two unknown forces), and so we obtain an equation containing C_3 in terms of W, and the perpendicular distances from A on to the directions of C_3 and W, which are called the lever arms (R_A does not appear in the equation, as it also passes through A).

If l (see Fig. 313) is the lever arm for W, then lW is the moment of W; and if l_1 is the lever arm for C_3, l_1C_3 is the moment of C_3—the sign being negative, because C_3 tends to turn the portion of the frame in what we have assumed to be the negative direction (see p. 26).

Hence $$lW - l_1C_3 = 0,$$

or $$C_3 = \frac{l}{l_1} W.$$

l and l_1 could be found by calculation, when the dimensions of the frame are known, but the process is rather complicated, and, moreover, they can be obtained graphically very simply and rapidly, and with sufficient accuracy for all practical purposes, by *drawing* perpendiculars from A (which can be called the "turning point") to the directions of W and C_3, and measuring the lengths of these perpendiculars to the same scale that the drawing of the frame is made to. Carrying this out in the present example we find (Fig. 314)

$$l = 6{\cdot}70 \text{ feet}, \; l_1 = 3{\cdot}93 \text{ feet},$$

and consequently

$$C_3 = \frac{6.70}{3.93} \cdot W = 1.7W$$

To find T we must take moments about the point D, the intersection of C_3 and C_5. Measuring the lever arms we find[1]

$$-9.5T - 8.3W + 15R_A = 0$$

Hence

$$T = 1.58R_A - 0.87W$$

And lastly, to find C_4 we must produce T and C_3 to meet at O (Fig. 315), and dropping perpendiculars from O on to the directions of W and R_A we find

$$-6.7C_5 + 19.3W - 12.6R_A = 0$$

Hence

$$C_5 = -(1.88R_A - 2.9W)$$

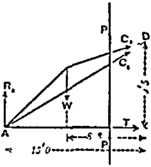


Fig. 314

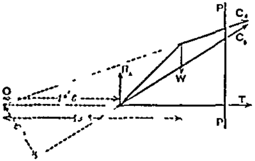


Fig. 315

This method is called the Method of Sections, and is a modification of that originally proposed by Professor Rankine. In the present form it was, it is believed, originally suggested by Professor Ritter.[2]

RULE FOR METHOD OF SECTIONS.—This method can be stated as follows:—

Sever the structure in two parts by means of an imaginary line (which need not be a straight line), cutting *if possible* through three bars of the structure. Select one of the parts, and apply to the severed ends of the bars forces equal in magnitude and direction to the stresses in those bars. This part of the structure will then be in equilibrium. To find the stress in any one of the severed bars, take moments about the point of intersection of the other two bars. An equation will thus be obtained containing the unknown stress it is required to find, any loads that may be acting on the part of structure selected, any reaction that may be acting on it, and the lever arms of the various forces involved.

The following terms are used in connection with this method, and are tabulated as follows:—

Turning point is the point about which moments are taken, and is generally the intersection of two of the unknown stresses.

Lever arm of a force is the perpendicular distance from the turning point

[1] It will be observed that the forces applied to the ends of the several rods in Figs. 313, 314, and 315 are all tensions. Hence clearly tensions are *positive* but compressions are *negative*, if the forces are thus drawn.

[2] See *Iron Bridges and Roofs*, by Professor Ritter, translated by Lieut. H. R. Sankey, R.E.

to the direction of the force. Practically these lever arms are obtained graphically.

It was mentioned above that the section should if possible be taken through three bars. This is not always possible, and the method must then be slightly modified.

The above is only an outline of the method, which is a very powerful one. The student is referred to Professor Ritter's work above mentioned, where he will find the method very fully described.

We have now considered two methods of determining the stresses in a braced structure, and it will be found that in some cases the first, and in other cases the second method, gives the result in the quickest and simplest manner. It is better to use Maxwell's diagrams when the stresses in all the bars of a frame have to be found, and Ritter's method when the stress on one only is required.

It should be observed that both methods require a drawing to scale of the structure, and to obtain accurate results this drawing should be made to as large a scale as possible.

We now proceed to apply these methods to some simple cases of framed structures.

CHAPTER X.

BRACED OR OPEN WEBBED GIRDERS.

BRACED or open webbed girders are much used in engineering works, but have only a limited application in building construction. It will therefore only be necessary to consider one or two simple cases. Moreover, the general theory of these girders is difficult, and beyond the scope of this book.

Warren Girders.—A very usual form of braced girder is shown in Fig

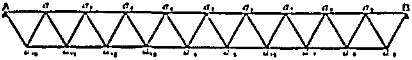


Fig. 316.

316, and names are given to the different parts as follows:—

AB is the *compression* boom or flange.

$a_{10}a_{19}$ is the *tension* boom or flange.

a_1a_{11}, a_2a_{12}, a_3a_{13}, etc., are called *braces*, and are tension braces or compression braces according to the stress in them

a_1, a_2, etc., are the *apices*.

A modification of this girder would be obtained by turning it upside

Fig. 317.

down, as in Fig. 317, when the nature of the stresses in the diagonals is of course reversed. These forms of braced girders are known as Warren girders.

Fig. 318.

Lattice Girder.—Fig. 318 shows another form of girder, which is called a lattice girder. Vertical braces are added at the ends, and are called end pillars.

It is most usual to incline the braces of a Warren girder at 60° and those of a lattice girder at 45°.

N Girder.[1]—Fig. 319 shows another form of braced girder, called the N girder. In this the upright braces are called *verticals*.

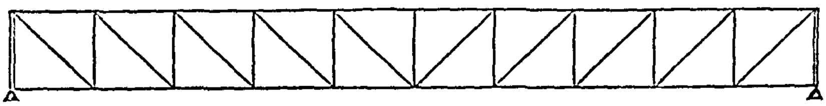


Fig. 319.

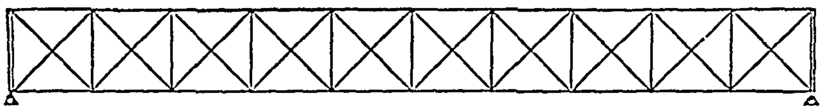


Fig. 320.

N (braced) girder.—Sometimes additional braces are added, as in Fig. 320; but, as explained at p. 181, the girder is then over-braced and may be self-strained, so that these braces should not be added unless the girder is subject to a moving load, and then only to a certain number of bays next the centre, depending on the proportion of the moving load to the weight of the girder.

Application of the Load.—The load is supposed to be concentrated at the apices, either at the top or at the bottom; occasionally, but unusually, at both top and bottom. To do this, arrangements must be made to transmit the load to the apices by means of secondary bearers. The weight of the braces and booms can also be considered as applied at the apices, and *practically* the weight of the girder can be taken as uniformly distributed.

Proportion between span and depth of girder.—As in former cases, the depth of the girder must be sufficient for stiffness, and it is found that a ratio of about $\frac{1}{12}$ gives the best results. The exact depth must be arranged so as to get an exact number of triangles, and will therefore depend on the angle of inclination chosen for the braces.

Stresses in Warren Girder.

Example 38.—We will take as an example the girder shown in Fig. 321 supporting loads of W_1, W_2, W_3, etc., tons on the upper apices.

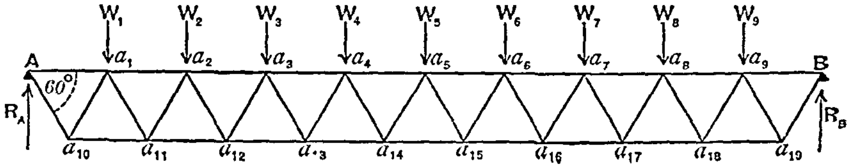


Fig. 321.

To find the number of triangles we have, d being the depth,

$$\frac{d}{\frac{1}{2}\,Aa_1} = \tan 60° = \sqrt{3},$$

or

$$Aa_1 = \frac{2d}{\sqrt{3}}.$$

---

[1] Also called *Whipple-Murphy girder.*

If n is number of triangles,

$$n(Aa_1) = \text{span} = 12d = n\frac{2d}{\sqrt{3}},$$

whence $\qquad n = 10.4$

Therefore ten triangles will do, and will give a depth rather greater than $\frac{1}{12}$ span, as shown in the figure.

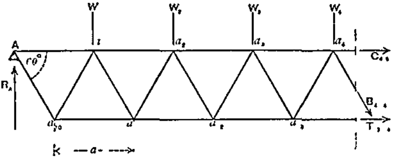


Fig 322

To find the stresses, take a section and select the left portion of the girder for calculation (Fig 322). We will adopt the following notation —

$C_{4,5}$ the compression in the bar a_4a_5 of the top boom

$B_{4,14}$ the stress in the brace a_4a_{14}

$T_{13,14}$ the tension in the bar $a_{13}a_{14}$ of the lower boom

a, the distance between the apices

And similarly for the other bars

To find the stress $C_{4,5}$ we must take moments about a_{14} and we thus obtain the equation

$$+d \times C_{4,5} - W_4 \times \frac{a}{2} - W_3 \times \frac{3a}{2} - W_2 \times \frac{5a}{2} - W_1 \times \frac{7a}{2} + R_A \times \frac{9a}{2} = 0$$

Supposing that

$$W_1 = W_2 = W_3 = \qquad = W_9 = W,$$

as is most usually the case, then

$$R_A = 4.5W,$$

and

$$+2d \times C_{4,5} - a(W + 3W + 5W + 7W - 9 \times 4.5W) = 0$$

Or $\qquad +2d \times C_{4,5} + 24.5W \times a = 0,$

and $\qquad C_{4,5} = -24.5W\,\frac{a}{2d}$

But $\qquad \frac{\frac{1}{2}a}{d} = \text{cotan } 60° = \frac{1}{\sqrt{3}}$

Therefore $\qquad C_{4,5} = -\frac{24.5}{\sqrt{3}}W$

The $-$ sign shows that $C_{4,5}$ is in compression

Again, to find $T_{13,14}$ we must take moments about a_4, and we obtain

$$-d \times T_{13,14} - W_3 \times a - W_2 \times 2a - W_1 \times 3a + R_A \times 4a = 0$$

As before, when the apices are equally loaded, we get

$$-d \times T_{13,14} - a(W + 2W + 3W - 4 \times 4.5 \times W) = 0$$

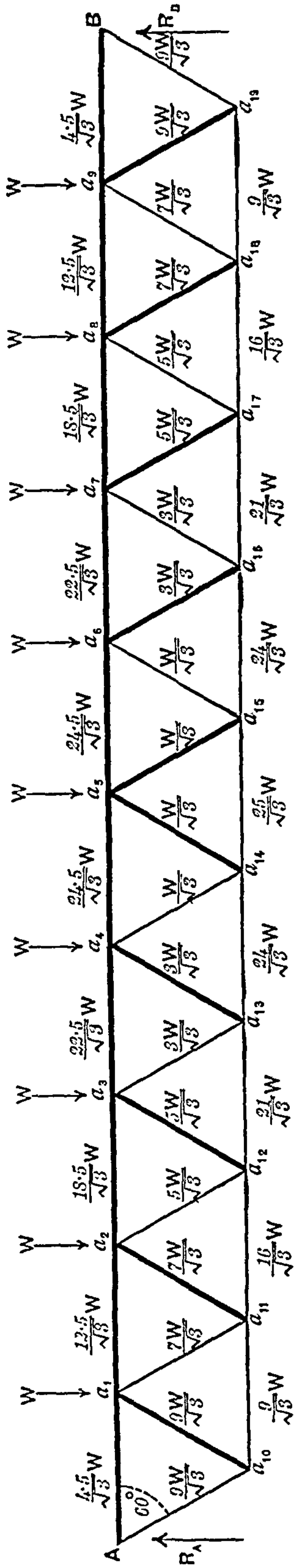


Fig. 323.

That is,

$$T_{13,14} = \frac{12a}{d}W = \frac{\frac{1}{2}a}{d} \times 24W,$$

$$= \frac{24}{\sqrt{3}}W.$$

According to the rule, to find $B_{4,14}$ we ought to take moments about the intersection of a_4a_5 and $a_{14}a_{15}$. But these lines do not meet, or, in other words, their intersection is at infinity. All the lever arms would therefore be infinite. Although the result can be obtained by following the rule, it is simpler in this case to resolve vertically (see Appendix II.) the forces acting on the portion of the girder shown in Fig. 322. It will be observed that both $C_{4,5}$ and $T_{13,14}$ have no resolved part vertically, since they are both horizontal, and that the resolved part of $B_{4,14}$ is

$$B_{4,14} \times \sin 60°,$$

$$= \frac{\sqrt{3}}{2}B_{4,14}.$$

Hence

$$-\frac{\sqrt{3}}{2}B_{4,14} - W_4 - W_3 - W_2 - W_1 + R_A = 0.$$

Or, if the apices are equally loaded,

$$B_{4,14} = 0{\cdot}5W\frac{2}{\sqrt{3}},$$

$$= \frac{W}{\sqrt{3}}.$$

In the same manner the stresses in all the other bars can be found. These have been worked out, and are marked on Fig. 323.

In the above example the loads are supposed to be applied to the upper apices. If they were applied to the lower apices, the stresses would be slightly modified. The student is recommended to re-work the example with the loads applied to the lower apices.

Stresses in Whipple-Murphy Girder.

Example 39.—Find the stresses in the girder shown in Fig. 324, the lower apices being loaded with 3·0 tons each.

To find the stresses in the bars a_3a_4, $a_{11}a_3$, and $a_{11}a_{12}$, take an oblique section along the line OO, as shown in Fig. 325.

The stress in a_3a_4 can be found by taking moments about a_{12}, thus (the depth being 4 feet)—

$$+4C_{3,4} - 4 \times 3{\cdot}0 - 8 \times 3{\cdot}0 + 12 \times 13{\cdot}5 = 0.$$

Whence $C_{3\,4} = -31{\cdot}5$ tons.

a_3a_4 is therefore in compression

To find $T_{11,12}$, take moments about a_3,

$$-4T_{11,12} - 4 \times 3\,0 - 8 \times 3\,0 + 12 \times 13\,5 = 0,$$

$$T_{11,12} = +31{\cdot}5 \text{ tons},$$

so that $a_{11}a_{12}$ is in tension.

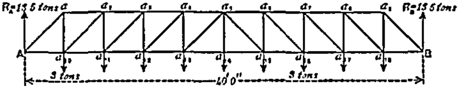


Fig. 321.

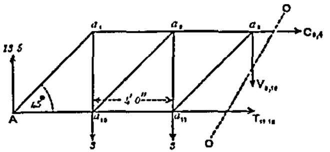


Fig 325

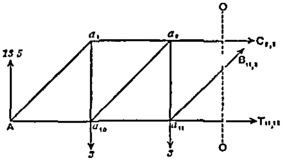


Fig 326.

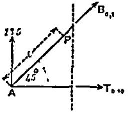


Fig 327

The stress in the vertical a_3a_{12} can be found by resolving vertically, thus—

$$-V_{3\,12} - 3\,0 - 3\,0 + 13\,5 = 0,$$

$$V_{3\,12} = +7\,5 \text{ tons}.$$

Another section must be taken to find the stress in the brace a_3a_{11}, as shown in Fig 326.

Resolving vertically,

$$B_{3\,11} \times \cos 45° + 13\,5 - 3\,0 - 3\,0 = 0,$$

$$B_{3\,11} = -7\,5 \times \sqrt{2},$$

$$B_{3\,11} = -10\,6 \text{ tons}$$

To find the stresses in the bars a_0a_1 and a_0a_{10}, we can take a section

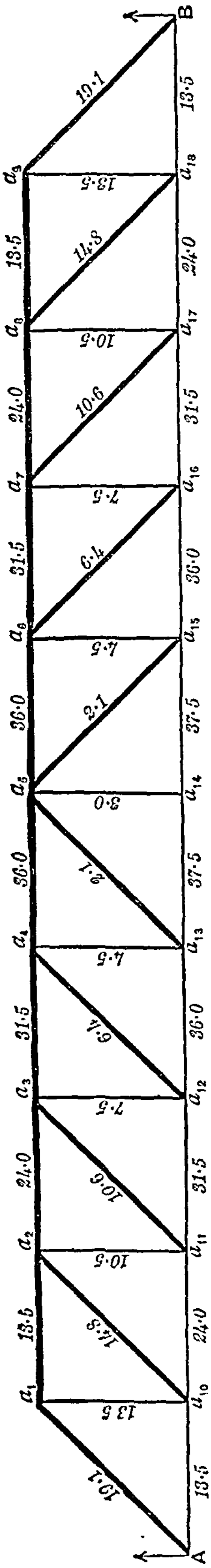


Fig. 330.

cutting through only two bars (Fig. 327). Resolving vertically,

$$B_{0,1} \times \cos 45^\circ + 13{\cdot}5 = 0,$$

or
$$B_{0,1} = -13{\cdot}5 \times \sqrt{2},$$
$$= -19{\cdot}1 \text{ tons.}$$

To find $T_{0,10}$ we can take moments about any point in a_0a_1, for instance, about P, whose distance from a_0 is c, then

$$-T_{0,10} \times c \sin 45^\circ + 13{\cdot}5 \times c \cos 45^\circ = 0.$$

But
$$\sin 45^\circ = \cos 45^\circ,$$

so that
$$T_{0,10} = +13{\cdot}5 \text{ tons.}$$

The stresses $B_{0,1}$ and $T_{0,10}$ can also be found very simply, by diagram, as shown in Fig. 328.

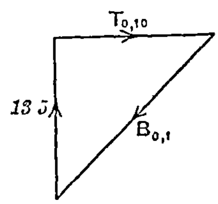


Fig. 328.

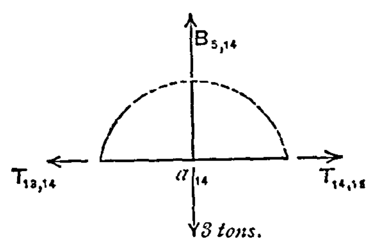


Fig. 329.

It will be observed that a straight section taken to obtain the stress in a_5a_{14} will cut through four bars, but by taking a circular section, as shown in Fig. 329, we need only cut through three bars, and by resolving vertically we obtain at once

$$B_{5,14} = +3 \text{ tons,}$$

a result which is self-evidently correct.

The stresses in the remaining bars of the girder can be obtained in precisely the same manner to the above, and as an exercise the student is recommended to carry out the calculations for each bar and check the results he obtains by those given in Fig. 330. He is also advised to find the stresses when the girder is turned upside down, as shown in Fig. 331.

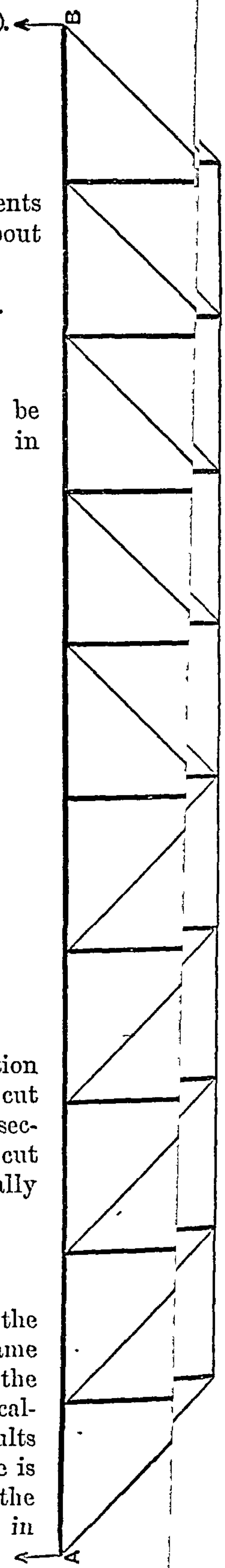


Fig. 331.

General Remarks.—It is to be noticed that in both the above examples all the top horizontal bars are in compression, and all the lower horizontal bars in tension. Further, all the braces sloping downwards away from the centre are in compression, and all the braces sloping downwards towards the centre in the Warren girder and the verticals in the Whipple-Murphy girder, supported as in Fig. 330, are in tension. The stress in both top and lower booms is greatest at the centre, and gradually diminishes towards the ends; the stress in the braces is least at the centre, and increases towards the ends.

The student is recommended to compare carefully these stresses with those in a plate girder.

Dimensions of the various bars.—The stresses being obtained, the dimensions of the various bars can be found as explained in Chap. VI.

For the small braced girders used in building construction, the booms are generally made of horizontal plates riveted together as in plate girders, and connected by **L** irons with a vertical web plate, to which the braces are secured. The ties are generally made of flat bar iron, and the struts of **L**, **T**, or **⊔** iron.

Short practical Formula for all open webbed Girders.

The stresses in the flanges B_s may practically be taken as in a plate girder.

Applying this to Example 38, p. 188, for a plate girder (uniformly loaded) we have from Equations 15 and 89

$$M_c = \frac{wl^2}{8} \quad M = r_t A d = B_s d,$$

$$B_s \times d = \frac{wl^2}{8} \text{ and } B_s = \frac{wl^2}{8d},$$

$$w = \frac{W}{a} \text{ in Example 38}, \ d = \frac{\sqrt{3}}{2},$$

$$\therefore B_s = \frac{W}{a} \times \frac{(10a)^2}{8} \times \frac{2}{\sqrt{3}},$$

$$= \frac{25aW}{\sqrt{3}},$$

$$aW \text{ here} = W \text{ in Example 38},$$

$$\therefore B_s = \frac{25W}{\sqrt{3}},$$

which is practically near enough to the result in Example 38, viz. $\frac{24 \cdot 5\text{W}}{\sqrt{3}}$.

The stress on any bar is equal to the shearing stress at the point multiplied by the cosecant of the angle of inclination of the bar to the horizontal, and divided by the number of triangulations. Thus in Example 38 the shearing stress at $B_{4,14} = -W_4 - W_3 - W_2 - W_1 + R_A = -4W + 4 \cdot 5W = \cdot 5W$.

By above rule the stress in $B_{4,14} = \cdot 5W$ cosecant $60°$

$$= \cdot 5W \frac{2}{\sqrt{3}} = \frac{W}{\sqrt{3}},$$

which agrees with the result by the other method (see p. 190).

In vertical bars (the cosecant of $90°$ being 1) the stress is equal to the shearing stress at the point (see $V_{3,12}$, p. 191).

Graphic Method.

Figs. 1 and 2, Plate C, show Example 38 worked out by a Maxwell's diagram, and Figs. 3 and 4 show the same for Example 39.

In Example 38, 2 tons per upper apex has been assumed as the load. The notation is by Bow's method, which is explained in Appendix XVII.

By measuring the different lines of the diagrams on the scales given it will be seen that the stresses found graphically agree with those found by calculation.

The thick lines denote compressions, the thin lines tensions.

Lattice girders may be calculated by the method given for Warren girders at p. 188, or by the graphic method.

A separate calculation or diagram should be made for each triangulation.

Bowstring Girders have a curved upper flange or *bow*, which for an uniformly distributed load should theoretically be a parabola, but is practically made an arc of a circle (see p. 34); the lower flange or *string* is horizontal.

For an uniformly distributed dead load only vertical bars are required, suspending the string (on which is the load) from the bow.

Where l=span, w=weight of bow and load per unit of length, d=depth of girder at centre, m=distance between the vertical bars.

The *horizontal* thrust (T) throughout bow=tension in string$=\frac{wl^2}{8d}$.

The compression along the bow at any point distant x horizontally from the centre$=\sqrt{w^2x^2+T^2}$.

Thrust on bow at springing$=T \times$ secant of angle of tangent of bow at this point with horizontal.

Tension on each vertical bar$=mw$.

Advantages.—The stress on the string and verticals is uniform, that on the bow varies only from 8 to 12 per cent, so that the bars and plates may be of uniform dimensions. As the girder can be made of a depth equal to $\frac{1}{8}$ its span, the sections of the flanges will be light and economical in comparison with those of shallower girders.

Bowstring girders are very seldom used in connection with buildings and need not be further described.

Chapter XI

TRUSSED BEAMS

Several kinds of trussed beams were described in Part I and it will be noticed that the construction of these beams is simple; the calculations to obtain the stresses accurately are however, difficult, and beyond the scope of these Notes. It is clear that the beam itself bears a certain proportion of the load, and the trussing makes up the difference. The part borne by the beam depends on the comparative stiffness of the beam and of the trussing, and thus varies with the temperature, which affects the length of the tie rods. Moreover, the beam is not in a simple state of bending stress, but is also subject to direct compression arising from the trussing, and liable at any time to alteration by tightening or slackening the nuts of the tie rods. These considerations will show that the accurate calculations are complicated and practically useless as they would have to be based on assumptions which might at any time cease to be correct.

It is therefore usual in practice to assume that the beam does not bear any of the load *as a beam*, and this assumption amounts to the same thing as to suppose that the beam is jointed at the point or points where the struts are connected to it. Thus the trussed beam shown in

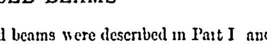


Fig 332

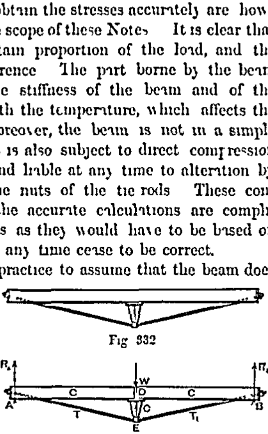


Fig 333

Fig 332 is assumed to be jointed as in Fig 333. This assumption clearly errs on the safe side, and makes a rough allowance for the compression produced in the beam by the tension in the tie-rod.

With this assumption there is no difficulty in finding the

stresses by either of the methods given in the preceding chapter. For instance, employing Maxwell's diagrams it will be seen that Fig. 334 is the diagram of the stresses in the trussed beam shown in Fig. 332.

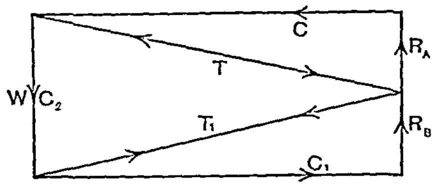


Fig. 334.

Having thus obtained the stresses, the dimensions of the various parts can be found as explained in Chaps. III. and VI.

Trussed beams of the form shown in Fig. 335 are, as explained at p. 181, incompletely braced, and are therefore unsuitable if the load is not uniformly distributed.

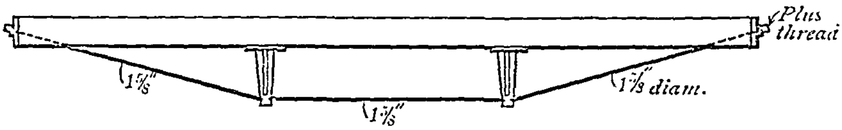


Fig. 335.

Fig. 336 shows the change of form which would take place if the beam were jointed and one point only were loaded;

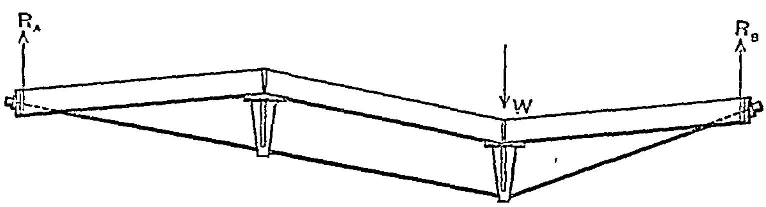


Fig. 336.

this figure exhibits, therefore, the tendency when the beam is continuous, although the stiffness of the beam controls the deformation. For uniform loads, however, such beams are suitable, as the stiffness of the beam would be quite sufficient to counteract any small unevenness in the load that might occur.

Example 40.—Design a trussed beam 18 feet span to carry a load of 8 cwts. per foot run.

The first step is to find how many struts to employ. If we place one strut at the centre, the beam will be divided into two portions of 9 feet each, and there will be a load of $9 \times 8 = 72$ cwts. on each of these portions. Considering *each* portion as a beam loaded uniformly and *supported* at both ends, it will be found on referring to Tables II. and III. that for strength $6'' \times 9''$ deep, and for stiffness $3\frac{3}{4}'' \times 11''$ deep, is required. If, therefore, timber $3\frac{3}{4} \times 11$, or its equivalent in section, is available, one central strut will be sufficient, and the trussed beam would be of the shape shown in Fig. 332. We will, however, suppose that timbers of this section are not obtainable; it will therefore be necessary to use more than one strut. If two struts are used, as shown in Fig. 335, the total span will be divided into three portions of 6 feet each, and if *each* of these portions is looked upon as a beam uniformly

loaded and supported at both ends, it will be found that for strength and for stiffness a beam 1" × 7" deep is needed We will decide on this section

The depth of the trussing must next be fixed If it be made deep, the stresses will be much reduced, but in many cases a deep trussing is inconvenient A rule applicable in general cases is to make the depth of the trussing from centre of beam to tie rod about $\frac{1}{10}$ of the span In the present

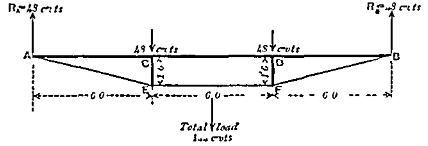


Fig 337

case, therefore, a depth of 1' 6" would do very well, so that the struts will be 1' 2" long Fig 337 shows the trussed beam in outline

Distribution of the Load —We may consider that half the load on AC and half the load on CD is concentrated at C , that is, the load at C is

$$\tfrac{1}{2}(6 \times 8 + 6 \times 8) = 48 \text{ cwts}$$

The remaining half of the load on AC is transmitted direct to the point of support, and it therefore does not affect the stresses in the trussing At D there will clearly be the same load as at C, hence the reactions R_A and R_B are each equal to 48 cwts

We will now find the stresses both by the method of sections and by Clerk Maxwell diagram

STRESSES BY METHOD OF SECTIONS —To find the stresses in AE and CE take the section shown in Fig 338 and for the stress in AE take moments about C

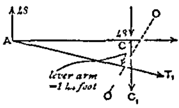


Fig 338

$$-1\ 44T_1 + 6 \times 48 = 0,$$

$$T_1 = -\frac{6 \times 48}{1\ 44} = 200 \text{ cwts},$$

$$= +10 \text{ tons}$$

To find the stress in CE take moments about A.

$$+6 \times 48 + 6 \times C_1 = 0,$$

$$C_1 = -48 \text{ cwts},$$

$$= -2\ 4 \text{ tons}.$$

The stress in EF can be found by taking the section shown in Fig 339, and taking moments about C, then

$$-1\ 5 \times T_2 + 6 \times 48 = 0,$$

$$T_2 = -\frac{6 \times 48}{1\ 5} \text{ cwts},$$

$$= 9\ 6 \text{ tons}.$$

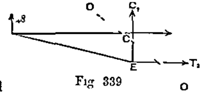


Fig 339

The stress in FB will clearly be equal to that in AE, and the stress in DF to that in CE We do not require the direct compression in AC, CD, or DB for the approximate method we are employing

Stresses by Clerk-Maxwell Diagram.—Commencing at the abutment A we obtain the triangle *fgh* (Fig. 340), from which we find $T_1 = 10$ tons as before. Proceeding to E we obtain the triangle *klm* (Fig. 341), from which we obtain

$$T_2 = 9 \cdot 6 \text{ tons},$$
$$C_1 = 2 \cdot 4 \text{ tons}.$$

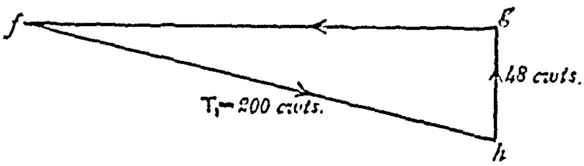


Fig. 340.
Scale 100 cwt. = 1 inch.

Having obtained the stresses, the diameter of the tie-rod can be found as

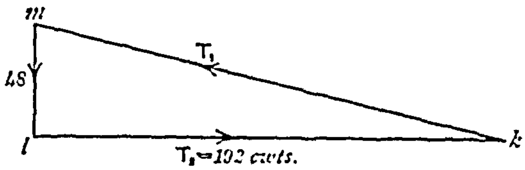


Fig. 341.
Scale 100 cwt. = 1 inch

explained in Example 13, and is given in Fig. 335. The stress in the struts is so small that no calculation of dimensions is needed.

If however we now carry the investigation a step further, and consider the horizontal timber beam as a strut resisting the compression caused by the tie-rods, we shall find the section of 4″ × 7″, found sufficient as a beam, inadequate as a strut, unless the truss were so supported laterally as to be incapable of flexure sideways. For we have compression in CD = tension in EF = 192 cwts. on 28 sq. in. = 6·85 cwt. per sq. in. on a strut in which (see Table VI.) $R = \frac{216''}{4''} = 54$ and $r_c = 0 \cdot 58$ only. If we now increase the section to 9″ × 9″, we shall then have $R = \frac{216}{9} = 24$, $r_c = 2 \cdot 75$ and the compressive stress $= \frac{192}{81} = 2 \cdot 3$ cwt. per sq. in.

Chapter XII

ROOFS

MANY different kinds of roofs were described in Parts I and II and Tables of the dimensions used in practice were given Cases may, however, occur in which the conditions are different from those assumed in these Tables and it then becomes necessary to find the stresses in the various members of the roof and therefrom deduce the dimensions

The general method of finding the stresses in roofs will therefore be described in an elementary manner and will be illustrated by examples

Loads to be borne by Roofs

Roofs are subject to several kinds of loads but these loads can be classed under two heads—

1 PERMANENT LOADS —*These consist of the weight of the* roof truss itself of the purlins rafters and roof covering, frequently also the weight of a ceiling has to be borne by a roof and sometimes the weight of a lantern

The weight of the truss itself must be estimated much in the *same way as the weight of a girder, but there does not appear to* be any formula available corresponding to that of Unwin s for girders Table XII however gives the information required for *several different kinds of roofs and even in a special case the* weight required can be deduced therefrom with sufficient accuracy for practical purposes care being taken in large structures to check the assumed weight by that deduced from the completed design modifying the dimensions if found necessary

The weights of purlins rafters roof covering etc are also given in Tables XII and XIII

The use of these Tables will be better shown when working *out the examples*

2. Occasional Loads.—The occasional loads consist of the weight of snow and of the wind pressure.

Snow.—The weight of snow a roof is likely to have to bear depends on the locality in which it is to be erected, and at best only an approximate estimate can be given. It is usual to assume in England that a roof of very flat pitch may have a depth of 6 inches of snow on it, and the depth will of course diminish as the pitch increases. It can be further assumed that the weight of a cubic foot of snow varies from $5\frac{1}{2}$ to 11 lbs., according to the amount of consolidation, so that an allowance of 5 lbs. per square foot of *horizontal* surface covered by the roof is ample.

Wind pressure.—The wind pressure is more difficult to allow for, and the experimental data on the subject are at present very deficient. It is, in fact, only since the Tay Bridge disaster that English engineers have inquired at all closely into the subject, and the estimated allowance to be made for wind pressure has varied very considerably. In Unwin's *Iron Bridges and Roofs*, however, although published long before the Tay Bridge disaster, the subject is treated at some length, and the following is principally derived from that work.

Assume that the wind blows horizontally. When it meets the inclined surface of a roof it will produce a pressure, normal (*i.e.* perpendicular) to the surface of the roof, and there will also be a force exerted *along* the surface of the roof. Now as air is almost a perfect fluid, this force along the surface of the roof will be extremely minute and can be neglected. Therefore we can state that the wind pressure acts perpendicularly to the surface of the roof.

Let P be the horizontal wind pressure per square foot, that is, P is the force exerted by the wind on a vertical surface one square foot in area, and let P_n be the pressure normal to the surface of the roof per square foot, then according to Hutton's experiments

$$P_n = P(\sin i)^{1 \cdot 84 \cos i - 1} \quad . \quad . \quad . \quad . \quad (90),$$

where i is the angle of pitch of the roof.

Amount of wind pressure to be calculated for.—As regards the value to be given to P, the data at our disposal are not so numerous or so trustworthy as might be wished; the general practice, however, is to give P some value between 40 and 60 lbs. per square foot.[1] For ordinary roofs, unless in positions of exceptional exposure, 50 lbs. per square foot will be sufficient to take.

Table XIV. is derived from Equation 90, which it must be remembered is only empirical, on the assumption that P = 50 lbs. per square foot.

Stresses in roofs of special shape.—It may occur in large roofs of special shape that the wind pressure produces (say) compression in some member of the roof, in which the permanent load and the snow produce tension or *vice versa*. In such a case the weight of the snow actually assists the member in question, but as the maximum wind pressure may occur when there is no snow on the roof this assistance from the snow should not be reckoned on.[1] Hence the stresses due (1) to the permanent load, (2) to the snow, (3) to the wind pressure should be determined separately, and that combination of (1), (2) and (3) which produces the greatest stress in any member must be adopted in designing it. However, in the comparatively small roofs used in ordinary building construction, such as those illustrated in Parts I and II, the wind pressure does not reverse the stresses in any of the members.

Stresses in ordinary roofs.—In these it is only necessary to obtain the stresses produced—

(1) By the permanent load and the snow combined, (2) by the wind pressure, and from a consideration of these stresses the maximum stress to be borne by each member can be found, as will be shown in the sequel.

Tredgold assumed that the wind pressure acted vertically and uniformly over the whole surface of the roof. Such an assumption is clearly erroneous, nevertheless the scantlings given on his authority in the Tables, Parts I and II, are dependent on this assumption. The lighter scantlings for wooden roofs given in Parts I and II, and in Table XV, were obtained on the assumption that the wind pressure acts perpendicularly to the surface of the roof, and only on one side, and are recommended for use.

Distribution of the Loads

The weight of the roof covering, of the snow, and of the wind pressure is borne in the first instance by the common rafters and by them transmitted to the purlins. Taking a simple case, in which there is only one purlin on each side of the roof, each common rafter will be supported in the centre by the purlin, at the top by the ridge piece and at the bottom by the wall plate. If the rafter is continuous, and these three points are *accurately* in a straight line, it can be shown (see Appendix XI) that $\frac{5}{8}$ths of the load on the rafter will be supported by the purlin, $\frac{3}{16}$ths by the ridge piece and $\frac{3}{16}$ths by the wall plate. If, however, the three points of support are not in a straight line, this distribution would be changed. Supposing the rafters were not continuous but (as it were) jointed just over the purlin, then clearly the dis

[1] "It is suggested that, except perhaps in very cold climates, the snow need not be considered at all. Since long before such a wind force as 50 lbs. per foot square could take effect upon a roof all the snow would have been blown off it."—Wray, *Instruction in Construction* (Seddon).

tribution would be as follows:—half the load on the purlin, quarter on the ridge piece, and quarter on the wall plate. This latter distribution causes greater stresses in the roof, and it will therefore be better to accept it. Now let a, b, c (Fig. 342) be three purlins (and we will suppose, as is usually the case, that $ab = bc$ and that w is the load[1] on the portion ab of the rafter and also on the portion bc), then, if the rafter is considered to be jointed over each purlin, half the load on the portion $ab\left(\text{or } \frac{w}{2}\right)$ will be transmitted to the purlin b, and half the load on the portion bc $\left(\text{or } \frac{w}{2}\right)$ will be transmitted to the same purlin. On the whole, therefore, the purlin b will have a load $= w$ transmitted to it from the rafter in question. Applying this to the case where there are two purlins on each side of a roof, as shown in Fig. 343, the two purlins will each support a load w and the ridge piece and wall plate $\frac{w}{2}$.

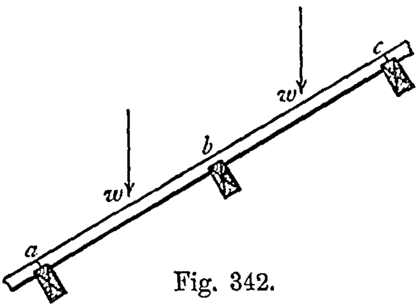


Fig. 342.

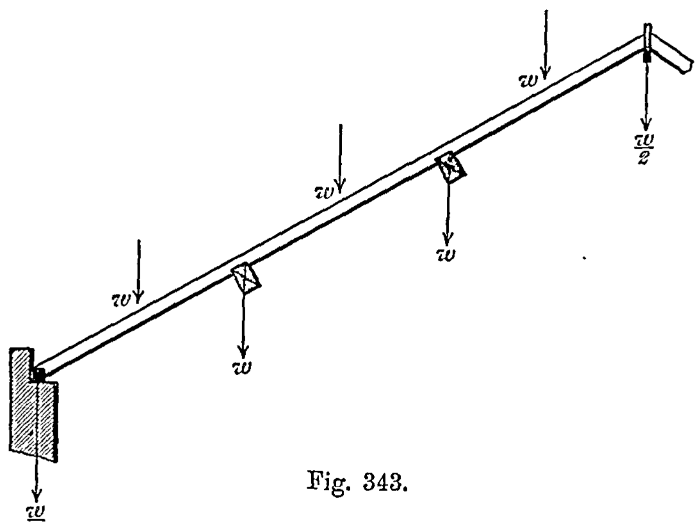


Fig. 343.

The *total* load on a purlin will be that due to the sum of the loads transmitted by all the common rafters supported by that purlin. Referring back to Fig. 342 it will be seen that purlin

[1] The loads are shown acting vertically, but the argument holds equally for the wind pressure.

b supports the load on a strip of the roof $\frac{ab+bc}{2}$ wide, and extending the whole length of the purlin, that is, from one truss supporting the purlin to the next. This is indicated in Fig. 344.

The purlins transmit the load to the principal truss and, looking upon the purlin as a beam supported at each end, half the load on the purlin will be borne by each truss supporting it.

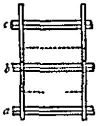


Fig. 344

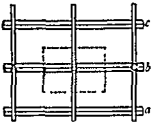


Fig. 345

Thus, referring to Fig. 345, at each point where a purlin is fixed to the principal rafter of the truss there will be a load

$$= \tfrac{1}{2} \text{ load on purlin AB} + \tfrac{1}{2} \text{ load on purlin BC},$$

that is the load due to the hatched portion of the roof Fig. 345.

Usually the trusses are equidistant. In such a case the load at every point of the principal rafter where a purlin is fixed is equal to the load on a strip of the roof equal to

distance apart of purlins × distance apart of trusses.

So far we have tacitly assumed that the purlins are placed immediately over the joints of the truss as shown in Fig. 346. Sometimes, however, smaller purlins are used, and they are distributed along the principal rafter as shown in Fig. 347. In such a case we can consider ab as a beam supported at b, and bc as another beam supported also at b, so that half the load on each of these beams will be transmitted to the joint at b. The load on the joint at b will therefore be the same as if there were only one purlin at b. The above refers strictly only to the loads due to roof covering, rafters and

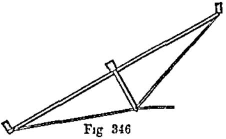


Fig. 346

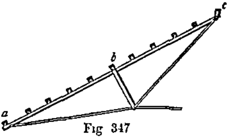


Fig. 347

purlins, wind and snow. The weight of the truss itself should strictly be considered as applied at *all* the joints, but it is sufficiently accurate to consider this part of the load as applied to the same joints (*i.e.* the upper joints) as the other loads.

In every case, therefore, the load acting on a joint of the principal rafter (and *in almost every case* these are the only joints that have to be considered as loaded) is the load on a strip of roof whose width extends half-way from the joint in question to the joints on either side of it, and whose length is the distance between the trusses.

Example 41.—To take a numerical example, let it be required to find the loads on the joints of the iron roof shown in Fig. 348, the trusses being supposed to be 10 feet apart.

Permanent Load and Snow.—First as regards the permanent load and the snow, we want to know the load per square foot of *roof surface.*

Referring to Table XIII. we find

	Per square foot of roof surface.
Roof covering (countess slates)	8 lbs.
Slate boarding (1″ thick)	3·5 „
	11·5 lbs.

From Table XII.

	Per square foot of covered area.
Common rafters and purlins	2·0 lbs.
Principal	3·5 „
Snow	5·0 „
	10·5 lbs.

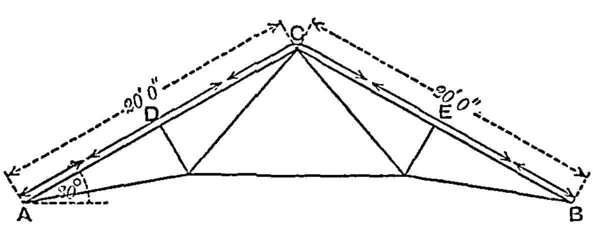


Fig. 348.

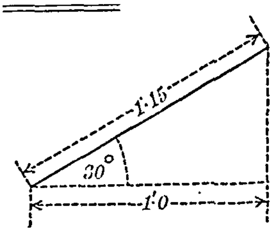


Fig. 349.

This last part of the load must be reduced to the load per square foot of roof surface. Referring to Fig. 349 it will be seen that it is the load on $1 \times 1·15$ square foot of roof surface, so that this part of the load is, per square foot of roof surface,

$$\frac{10·5}{1·15} = 9·1 \text{ lbs.}$$

Therefore the total load per square foot due to permanent load and snow $= 11·5 + 9·1 = 20·6$ lbs.

The load at joint A is due to a strip of roof 5 feet wide as indicated by the arrows in Fig. 348, and 10 feet long, but as this load is supported directly by the abutment, it does not cause any stresses in the roof, and need not therefore be considered.

The load on joint D is due to a strip of roof 5 + 5 = 10 feet wide by 10 feet long Hence load at D

$$= 10 \times 10 \times 20.6 = 2060 \text{ lbs.},$$
$$= 0.92 \text{ ton}$$

The load at C is due to two strips, each 5 feet × 10 feet, so that the load at C is the same as at D

Clearly the load at E is also the same

Load due to Wind —Next, to find the load due to the wind Supposing that the wind is blowing from left to right, then the left side only of the roof will have any wind pressure to bear As already mentioned, the direction of the wind pressure is taken normal to the surface of the roof, and from Table XIV we see that for a pitch of 30° the wind pressure to be allowed for is 33.0 lbs per square foot

The load at joint A is due, as before, to a strip of roof 5 × 10, so that the load equals

$$5 \times 10 \times 33 = 1650 \text{ lbs.},$$
$$= 0.73 \text{ ton}$$

The load at joint D is

$$10 \times 10 \times 33 = 3300 \text{ lbs.},$$
$$= 1.46 \text{ ton}$$

And the load at joint C is

$$5 \times 10 \times 33 = 1650 \text{ lbs.},$$
$$= 0.73 \text{ ton}$$

The load on any roof can be found in a similar manner

Reactions at the Abutments

As regards the permanent load and the snow, the reactions at the abutments can be found as explained at p 23 (Example 3)

The reactions due to the wind pressure cannot, however, be found in so simple a manner

Let Fig 350 represent a roof (the arrangement of the bracing, being immaterial, is omitted), then, regarding the roof as a ... the total wind pressure

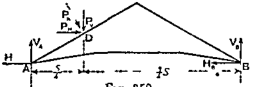


Fig 350

...sidered as concentrated at D, the centre point of ... roof

... can be replaced by two forces—one a vertical ... the other a horizontal force, P_H The reaction at ... can also be replaced by two forces, one vertical ... orizontal as shown in the figure

... can be found at once, as being the reactions due ... ad P_V and a horizontal load P_H thus if d is the ... int D above the abutments

$$V_A = \frac{\frac{3}{4}S}{S} \times P_V - d \times P_H = \tfrac{3}{4}P_V - d \times P_H$$

and similarly

$$V_B = \tfrac{1}{4}P_V + d \times P_H$$

As regards H_A and H_B, however, all we can say is, that

$$H_A + H_B = P_H \quad . \quad . \quad . \quad (91),$$

but we cannot, *from the conditions of equilibrium, determine* the value of H_A or H_B.

This will be easily understood by considering that, if the support B were quite smooth, and therefore unable to offer any horizontal resistance, the support A would have to furnish *all* the horizontal reaction. We must therefore endeavour to find some other condition to determine the relative values of H_A and H_B, or else make some assumption. Now in the case of *wooden roofs* a fair assumption to make would be, that each abutment will just afford sufficient horizontal reaction to make the total reaction parallel to the normal wind pressure, as is shown in Fig. 351. In this case

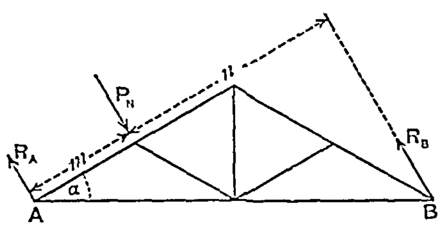


Fig. 351.

$$R_B(m+n) = P_N \times m,$$

$$R_B = \frac{m}{m+n}P_N \quad . \quad . \quad (92),$$

and

$$R_A = \frac{n}{m+n}P_N \quad . \quad . \quad (93).$$

And m and n can be found by measurement.

In *iron roofs*, however, the case is rather different. To providé for the expansion and contraction due to variations of temperature, one end of the roof is fixed and the other end left free to move; and so to enable this end to move freely it is usual in *large* roofs to place steel rollers under the shoe.

In small roofs, where the shoe at the free end simply rests on a stone template, the horizontal resistance of the free end cannot exceed the frictional resistance. Now the coefficient of friction for iron upon stone can be taken at about 0·45, and to find the resistance to friction we have only to multiply the vertical pressure by this number, so that, supposing B to be the free end, we have, if $\overline{V}_B$ is the vertical reaction due to the wind pressure, roof covering, roof truss, and snow,

$$H_B = 0{\cdot}45\overline{V}_B, \quad . \quad . \quad . \quad (94),$$

and this is the greatest value H_B can have, and $H_A = P_H - 0.45\overline{V}_B$, so that, knowing H_B, we can easily find H_A. The assumption made in the case of wooden roofs is therefore only tenable in the case of small iron roofs, so long as it does not require a greater value of H_B than the above.

Now, for ordinary pitches, the assumption that the reactions are parallel to the wind pressure holds good, and equations 92 and 93 are applicable, but for high pitches it does not hold good.

Let us inquire *at what pitch the change takes place*. Fig 352 is the outline of a roof, the reactions at the abutments being parallel to P_w. It will be seen that the inclination of R_B to V_B is α, the angle of pitch. Hence

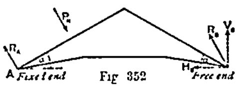


Fig 352

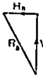


Fig 353

$$R_B \cos\alpha = V_B,$$

also

$$R_B \sin\alpha = H_B$$

Results which are also evident from the triangle of forces, Fig 353.

But by taking moments about A,

$$\tfrac{1}{2}P_w = 2\cos^2\alpha \times R_B$$

Hence

$$H_B = \frac{P_w \sin\alpha}{4\cos^2\alpha}, \text{ and } V_B = \frac{P_w}{4\cos\alpha}$$

Now if L denote the permanent load,

$$\overline{V}_B = \frac{P_w}{4\cos\alpha} + \frac{L}{2}$$

Therefore the maximum value of H_B, from equation 94,

$$= 0.45\left(\frac{P_w}{4\cos\alpha} + \frac{L}{2}\right),$$

and the value of α required can be obtained from the equation

$$\frac{P_w \sin\alpha}{4\cos^2\alpha} = 0.45\left(\frac{P}{4\cos\alpha} + \frac{L}{2}\right)$$

It will be seen that α depends on L, and it will be found on trial that for very light roof coverings (such as zinc) $\alpha = 30°$, and for heavy roof coverings, such as in Example 41, $\alpha = 45°$.

When an iron roof expands by an increase of temperature, the abutments will resist the expansion by exerting horizontal forces h_A and h_B, which will be equal and opposite to each other, but as soon as these forces exceed the frictional resistance of the free end, that end will begin to move. Now supposing that a strong wind is blowing from right to left, whilst the roof is expanding, the wind causes a horizontal reaction H'_B, and the increase of temperature a horizontal reaction

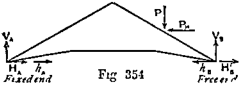


Fig 354

h_B in the opposite direction, as shown in Fig. 354. Supposing, as may well happen, that

$$H'_B = h_B,$$

then clearly there will be no horizontal reaction at the abutment B, or, in other words, abutment A will have to supply the whole of the horizontal reaction, that is

$$H'_A + h_A = P_H.$$

A similar case may occur when the wind is blowing from left to right, and the roof is at the same time *contracting*; then, as in Fig. 355, if

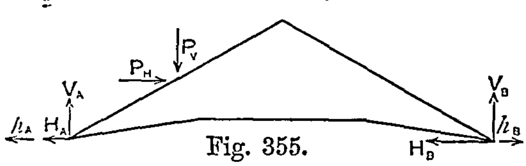


Fig. 355.

$$H_B = h_B,$$

we have

$$H_A + h_A = P_H,$$

and the horizontal reaction at the free end will be zero.

Cases to be Considered.

Reviewing what has been said with regard to *small* iron roofs, supported on substantial abutments and with the free end *not* resting on rollers, it will be seen that there are four cases to be considered, as follows :—

The free end being on the right.

Case 1. Wind on left Case 2. Wind on right	Reactions parallel to normal wind pressure for pitches from 36° to 45°, according to the roof covering, or horizontal reaction at free end equal to frictional resistance for pitches above 45°.
Case 3. Wind on left Case 4. Wind on right	Horizontal reaction at free end zero.

It will be necessary to find the stresses in each case, and then pick out the maximum stresses to which the various bars may be subjected.

On the other hand, in *large* iron roofs, where the free end of the trusses are supported on rollers, or, again, in small iron roofs having one end supported by a column, which can offer little or no horizontal resistance, the horizontal reaction at the free end can be taken as zero. In such roofs, therefore, only two cases need be considered, namely—

Case 1. Wind on left Case 2. Wind on right	Horizontal reaction at free end zero.

We have now shown how to find the loads and reactions acting on roof trusses, and the next step is to show how the stresses can be obtained. This we proceed to do by means of a numerical example.

Stresses in an Iron Roof Truss

Methods of finding the Stresses.—The stresses can be found either by the method of sections or by the graphic method (Clerk Maxwell diagrams). If the stresses in all the bars are required, then the latter method will be the better one to use, but if it is only required to find the stresses in one or two of the bars, then the method of sections will give the result, as a rule, more quickly. As an illustration, however, the stresses will be found by both methods, by the graphic method below, and by the method of sections in Appendix XVIII.

Example 42.—Find the stresses in the iron roof truss shown in Fig. 373, p. 215, and 356,[1] Plate III, assuming the following data:—

Span 40 feet.
Distance apart of trusses 8 feet.
Purlins placed over joints of truss.
Roof covering—Countess slates on $\frac{3}{4}''$ boarding.
Pitch $21\frac{3}{4}^{\circ}$ or $\frac{1}{5}$ span.
Rise of tie-rod $\frac{1}{30}$ of span.

STRESSES DUE TO PERMANENT LOAD AND SNOW

Preliminaries.—To find the permanent load and snow on the roof we refer to Table XIII., whence

	Per square foot of roof surface
Roof covering and boarding	10·5 lbs.
	Per square foot of covered area
Common rafters and purlins	2·0 lbs.
Principal	3·5 ,,
Snow	5·0 ,,
	10·5 lbs.

$$\text{Or } 10{\cdot}5 \times \frac{1}{1{\cdot}08} = 9{\cdot}7 \text{ lbs. nearly per square foot of roof surface.}$$

Hence total load per square foot of *roof surface*

$$= 10{\cdot}5 + 9{\cdot}7 = 20{\cdot}2 \text{ lbs.}$$

Now the length of principal rafter is

$$\sqrt{20^2 + (\tfrac{40}{5})^2},$$
$$= 21{\cdot}6 \text{ feet nearly.}$$

Hence the loads are (see p. 203) at joint D

$$\frac{21{\cdot}6}{2} \times 8 \times 20{\cdot}2 = 1745 \text{ lbs.},$$
$$= 0{\cdot}78 \text{ ton.}$$

And joints C and E will be loaded to the same amount.

The reactions at A and B will be each equal to

$$\tfrac{3}{2} \times 0{\cdot}78 = 1{\cdot}17 \text{ ton.}$$

[1] Figs. 356-359. See Plate III. at end of volume.

Referring to Equations 92 and 93, p 206, and to Figs 360 and 361, v that [1]

$$R_A = \frac{28\ 4}{40} P_x,$$

$$= \frac{28\ 4}{40} \times 1\ 92 = 1\ 36 \text{ ton},$$

and

$$R_B = \frac{11\ 6}{40} \times 1\ 92 = 0\ 56 \text{ ton}$$

The external forces due to the wind from the left are therefore as sho Fig 361 [2]

Maxwell's Diagrams —Commencing at joint A we find two external f R_A and P_A, as, however, these two forces are acting in exactly op directions, we need only plot their difference, namely—

$$1\ 36 - 0\ 48 = 0\ 88 \text{ ton}$$

We thus obtain the triangle *abc*, which gives us S_{AD} and S_{AF} (Fig 362)

Proceeding now to joint D we get the quadrilateral *cade*, in whi represents P_D and is drawn to scale equal to 0 96 ton

For joint F we obtain the quadrilateral *bcef* This quadrilateral is shown separately in Fig 363

Next taking joint C we get the five-sided figure *fedgh* (see also Fig :

Then proceeding to joint E we get the straight line *gh* as the dia; This shows that $S_{CE} = S_{EB}$ and $S_{EG} = 0$

For joint G we obtain the triangle *fhb*, and not a quadrilateral, $S_{EG} = 0$ This triangle will be a check on the accuracy of the drawin the diagram ought to close at *h*

Lastly for joint B we get the triangle *bhg*, and we have thus obtaine stresses in all the bars They can be measured to scale on the diagram are then inserted in Table H, p 213

Case 2 *Wind on right Reactions parallel to normal wind pressure* moment's consideration of Fig 365 will show that this case is exactl

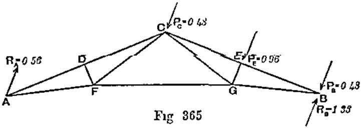


Fig 365

opposite of Case 1, and we can immediately write down the values o various external forces, as has been done in the figure, and it is fu manifestly unnecessary to go through the calculations for the stresses, ε can deduce them from those obtained under Case 1 Thus the stress n in the present case will be equal to the stress in DC in Case 1, and s Column 4 has thus been filled in in Table H

Case 3 (p 208) *Wind on left Reaction at free end vertical* —In this

---

[1] $\frac{n}{m+n} = \frac{28\ 4}{40}$

[2] Figs. 361 364 See Plate IV at end of volume

(Fig. 366) it is a little more difficult to find the values of R_A and R_B, because the external forces acting on the roof are not parallel to each other, and we

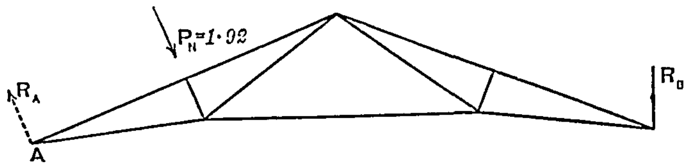


Fig. 366.

know neither the direction nor the magnitude of R_A. Now when three forces acting on a body are in equilibrium, their directions meet in a point,[1] so that, as we know the direction of P_N and R_B, we can find the direction of R_A by joining A with the point of intersection of P_N and R_B. But with flat roofs, as in the roof under consideration, this intersection is a long way below the roof, and the intersection is acute, and therefore cannot be obtained with great accuracy.

In such a case, therefore, a better way is to find the value of R_A, which can be done by taking moments about A. We obtain

$$+10{\cdot}8 \times P_N - 40 \times R_B = 0,$$

$$R_B = \frac{10{\cdot}8 \times 1{\cdot}92}{40} = 0{\cdot}52 \text{ ton.}$$

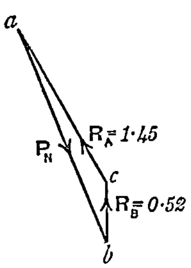


Fig. 367.

Now draw *ab* (Fig. 367) parallel to P_N and measure along it 1·92 ton to scale; next *bc* parallel to R_B, and measure along it 0·52 ton *a* to scale; then *ac* will be the direction of R_A and we also find by measuring *ac* that

$$R_A = 1{\cdot}45 \text{ ton.}$$

We can therefore now proceed to find the stresses.

Maxwell's Diagrams.—Commencing at joint A (Fig. 368)[2] we find two external forces, P_A and R_A. Let *ad* parallel to P_A and *ac* parallel to R_A represent these forces (Fig. 369), then *adec* is the complete diagram for joint A.

It will be found that the remainder of the diagram for the roof can be drawn as in Case 2; the description is not therefore repeated, but the diagram is given in Fig. 369.

Case 4. *Wind on right. Reaction at free end vertical.*—In this case we can obtain the reactions very simply by diagram. Thus produce P_N and R_B to meet at K (Fig. 370) and join AK; then AK is the direction of R_A (see Fig. 367). Now, plotting K*a* to scale to represent

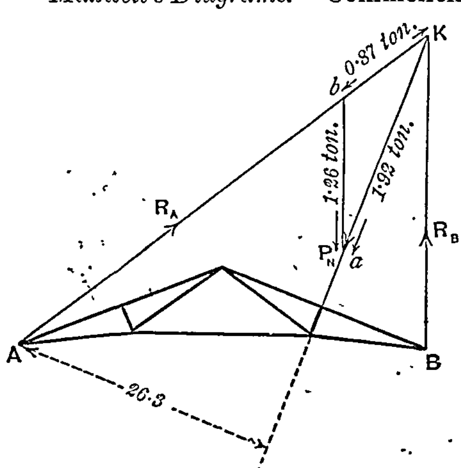


Fig. 370.

[1] This can be easily proved by taking moments about the intersection of two of the forces. The moments of these two forces will be nothing, but for equilibrium the sum of the moments of three forces is nothing, therefore clearly the moments of the third force about this point must be nothing; that is, its direction must pass through the point, or, in other words, the three forces meet in a point.

[2] Figs. 368, 369. See Plate V. at end of volume.

P_x (1 92 ton), and drawing *ab* parallel to R_B, we obtain the triangle of forces for the three forces P_x, R_B, and R_A, and by measurement we find

$$R_A = 0\ 87 \text{ ton},$$
$$R_B = 1\ 26 \text{ ton}$$

As a check, taking moments about A, we have

$$-10 \times R_B + 26\ 3 \times 1\ 92 = 0,$$
$$R_B = 1\ 26 \text{ ton}$$

Maxwell's Diagrams —Fig 371[1] shows the loads for this case, and the corresponding diagram is given in Fig 372, and the stresses obtained therefrom are entered in Table II

TABULATION OF STRESSES

We have thus obtained the stresses in the various members of the roof under various conditions These stresses must now be tabulated, so that we may be able to pick out the maximum stress to which each member of the roof truss may be subjected This has been done in Table II below, and the stresses that are to be added together, to obtain the maximum stress, are printed in italics The seventh column gives the maximum stresses, and it will be seen that the maximum stress to be borne by corresponding members on either side of the roof is not quite the same This is due to the fact that one end of the roof truss is fixed and the other end is free to move But as these differences are very small, and it would be practically unwise, at any rate in so small a roof truss, to make corresponding members on either side of different scantlings, these differences will be ignored and the scantlings will be calculated to meet the stresses given in column 8

It should be noticed also that the stress in the principal rafter is greater in the lower part than in the upper part, but, in a small roof truss like the one under consideration, no difference would practically be made in the scantling of the principal from A to C In column 6, therefore, the stress in the upper part of the principal rafter has been put equal to that in the lower part

TABLE II

Compressions – Tensions + [2]

1 Stresses	2. Permanent load and snow	Wind: Reactions parallel, 3 On left.	Wind: Reactions parallel, 4 On right.	Wind: Reaction on free end vertical, 5 On left.	Wind: Reaction on free end vertical, 6 On right	7 Maximum stresses	8 Stresses to be used in calculating scantlings.
S_{AD}	*−4 40*	−3·27	−1 89	*−3 38*	−1 62	−7 83	−7 83
S_{AF}	*+4 16*	+3 38	+1 57	*+3 67*	+0 83	+7 83	+7 83
S_{DF}	*−0 73*	−0 96	0 00	*−0 96*	0 00	−1 69	−1 69
S_{DG}	*−4 16*	−3 28	−1 89	*−3 33*	−1 62	−7 49	−7 83
S_{FG}	*+1 93*	+2 17	+0 32	*+2 20*	+0 16	+4 13	+4 13
S_{FC}	*+2 34*	+1 30	+1 31	*+1 54*	+0 69	+3 88	+3 88
S_{CG}	*+1 93*	+0 32	*+2 17*	+0 38	+2 02	+4 10	+4 13
S_{CE}	*−4 16*	−1 89	*−3 28*	−1 98	−3 0	−7 44	−7 83
S_{GE}	*−0 73*	0 00	*−0 96*	0·00	−0 96	−1 69	−1 69
S_{GB}	*+4 16*	+1 57	*+3 38*	+1 88	+2 70	+7 54	+7 83
S_{EB}	*−4 45*	−1 90	*−3 27*	−1·98	−3 00	−7 72	−7 83

[1] Figs 371, 372 See Plate V at end of volume

[2] See p 185 Many writers take compressions as + and tensions as −

CALCULATION OF THE SCANTLINGS.

Having obtained the stresses in the various members of the roof truss the next step is to decide on the form, and to calculate the scantlings of the various members, so that they may be able to *safely* resist these stresses.

Suitable forms, such as are used in practice, have been described in Parts I. and II., and the student is therefore referred back for information on that part of the subject.

As regards the calculations of the different members and joints, several examples have already been worked out in Chap. VI. of this Part, and the student is specially referred to Examples 13, 19, 20, 26, 26*a*, 27, and 28.

The roof truss can now be considered as completely designed, and the result is given in Fig. 373. The same truss is shown in Fig. 374, when the purlins are distributed along the length of the principal rafter, in which case the principal rafter is increased in section but no alteration is made in the other members of the truss.

THE CALCULATIONS TO FIND THE SIZE OF THE PRINCIPAL RAFTER IN THE CASE WHERE A NUMBER OF SMALL PURLINS ARE DISTRIBUTED ALONG ITS LENGTH would be as follows:—

We must first find the load on the principal. The permanent load and snow consists of

	Per square foot of roof surface.
Roof covering and boarding . . .	10·5
	Per square foot covered.
Purlins	1·5
Snow	5·0

Or

$$\frac{6 \cdot 5}{1 \cdot 08} = 6 \text{ lbs. per square foot of roof surface.}$$

$$\text{Total} = 10 \cdot 5 + 6 = 16 \cdot 5 \text{ lbs.}$$

This load is vertical and must be resolved perpendicularly to the principal by multiplying by cos $21\frac{3}{4}^\circ$ thus—

$$16 \cdot 5 \times \cos 21\tfrac{3}{4}^\circ = 15 \text{ lbs.}$$

The normal wind pressure is 25 lbs. per square foot (see Table XIV.), so that the total load per square foot is

$$15 + 25 = 40 \text{ lbs.}$$

Hence total load on *one half* of the principal rafter is

$$40 \times \underset{\text{Length.}}{10 \cdot 8} \times \underset{\text{Distance apart of trusses.}}{8} = 3456 \text{ lbs.,}$$

$$= 1 \cdot 54 \text{ ton.}$$

This load produces a bending stress in the principal, the compressive element of which must be added to the direct compression. The moment of flexure is

$$\frac{wl^2}{8} = \frac{Wl}{8} = \frac{1 \cdot 54 \times 10 \cdot 8 \times 12}{8},$$

$$= 25 \text{ inch-tons,}$$

whether we consider the principal as jointed at the strut, in which case the lower half of it is a beam supported at both ends, or whether we suppose it to be continuous over the strut, when the lower half is a beam supported at one end and fixed at the other.

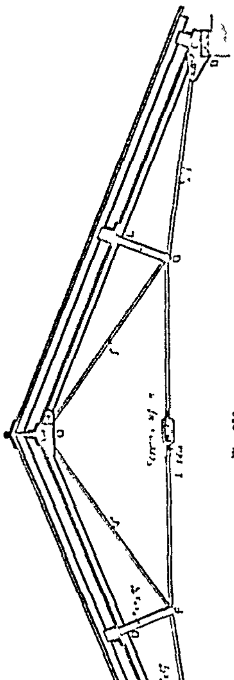


Fig 373.

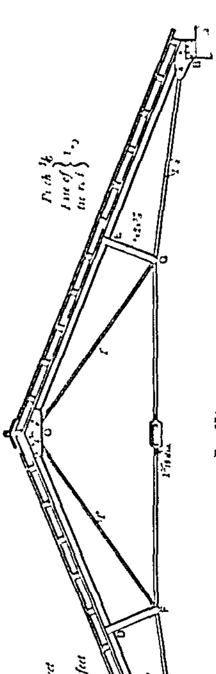


Fig 374

We must now assume some section of ⊤ iron, and see if it is strong enough. Assume, for instance, a section $4'' \times 5\frac{1}{2}'' \times \frac{1}{2}''$, the table being $4''$ wide.

According to the rule given in Appendix IV. the distance of the centre of gravity from the edge of the table is

$$\bar{x} = \frac{4 \times \frac{1}{4} + 5 \times 3}{4 + 5} = 1{\cdot}8 \text{ inch nearly.}$$

And according to Appendix XIV. the moment of inertia about an axis passing through the centre of gravity is

$$\begin{aligned} \mathrm{I} &= \tfrac{1}{3}\left\{4\,(1{\cdot}8^3 - 1{\cdot}3^3) + \tfrac{1}{2}\,(1{\cdot}3^3 + 3{\cdot}7^3)\right\}, \\ &= 13{\cdot}65. \end{aligned}$$

Treating principal as jointed.—Hence the greatest compression due to bending (at the edge of the table), on the supposition that the lower half of the principal is supported at both ends.

From Equation 53 $r_o = \frac{\overline{\mathrm{M}}y_o}{\mathrm{I}} = \frac{25 \times 1{\cdot}8}{13{\cdot}65} = 3{\cdot}3$ tons per square inch.

The direct compression produces a stress per square inch (see note below)

$$= \frac{7{\cdot}83}{\text{area}} = \frac{7{\cdot}83}{(4+5)\,\frac{1}{2}} = 1{\cdot}7 \text{ ton nearly.}$$

Hence total maximum compression

$$= 3{\cdot}3 + 1{\cdot}7 = 5 \text{ tons per square inch.}$$

Practically this section would do.

Treating principal as continued.—If, however, the principal is considered as continuous over the strut, and is supposed to be supported at one end and fixed at the other, the maximum compression due to bending at the edge of the stem becomes

$$\frac{25 \times (5{\cdot}5 - 1{\cdot}8)}{13{\cdot}65} = 6{\cdot}8 \text{ tons.}$$

So that on this supposition the section is too small.

Next, try a section $3'' \times 5\frac{1}{2}''$, the table being $\frac{1}{2}''$ thick and the stem $\frac{3}{4}''$. It will be found that the distance of the CG from the edge of the stem is $3''{\cdot}29$, and that $\mathrm{I} = 15{\cdot}9$. Hence maximum compression (at edge of stem)

$$= \frac{25 \times 3{\cdot}29}{15{\cdot}9} = 5{\cdot}17 \text{ tons.}$$

And the total compression is

$$5{\cdot}17 + 1{\cdot}49 = 6{\cdot}66 \text{ tons.}$$

This section is therefore apparently too small, on the supposition that the principal is continuous over the strut. This supposition, however, requires that the three points A, D, and C shall remain in a straight line, but the maximum compression occurs in the principal AC when the wind is blowing from the left, and this wind pressure bends the roof as shown in Fig. 375 in a very exaggerated manner, and it is evident that this bending of the principal will considerably relieve the compressive stress on

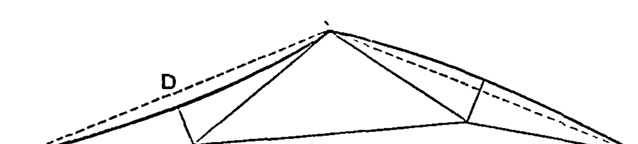


Fig. 375.

the stem of the T iron, so that it can be taken that this section is quite strong enough

Note —It has been assumed that the direct compression is uniformly distributed over the cross section of the T iron. The deflection due to the distributed load tends however to increase the intensity of the stress towards the edge of the table. The increase is however small. If for instance the deflection were ½" the moment of flexure due to the direct compression would be 7·83×0·5=3·9 inch tons, which would cause with a T iron 4"×5½"×½" a maximum compression of $\frac{3·9\times1·5}{13·65}$=0·43 ton per square inch to be added to the 1·7 ton already found

Diagrams for various Roof Trusses

In Figs 376-395[1] diagrams for different kinds of roof trusses are given, with the hope that they may be of assistance to the student. It is strongly recommended that, as an exercise, some span be assumed, and that the stresses be obtained by following these diagrams, the results obtained being occasionally checked by the method of moments.

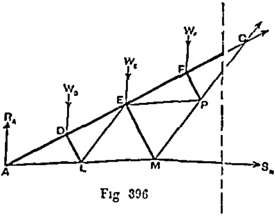


Fig 396

In these diagrams only one case for the wind pressure has been shown, since the other cases produce very similar diagrams

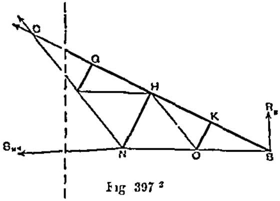


Fig 397[2]

It should be observed that the diagram for the roof truss shown in Fig 392 cannot be drawn by the above method alone, because either at the points E or M the stresses in three of the bars are unknown. But the stress in MN can easily be found by taking a section as shown in Fig 396 for permanent load and snow, and in Fig 397 for wind. The value thus found for S_{MN} can then be inserted, and the diagram completed

WOODEN ROOFS

It is not worth while to give examples of the calculations for timber roofs. These can very seldom be necessary, and, if required, would be similar to those for iron roofs.

Tables of Scantlings for Wooden Roofs are given in Table XV p 340, and in Parts I and II

[1] See Plates VI, VII, VIII, IX and X, at end of volume
[2] For Figs 398-430 see Appendix XVIII

CHAPTER XIII.

STABILITY OF BRICKWORK AND MASONRY STRUCTURES.

IT is proposed in the following chapter to show, in an elementary manner, how the dimensions for various kinds of brickwork and masonry structures can be calculated.

The mortar with which the stones or bricks of a wall are bedded assists the wall by its resistance to tension; but, on the other hand, the resistance to compression of mortar being less than that of stone or brick, it reduces the resistance to crushing of the wall. On the whole, the effect of the mortar is to increase the strength of the wall as *regards overturning.*

In important structures, however, the tenacity of the mortar is not taken into account, because settlements are liable to occur which may dislocate the joints, and such walls are treated as if built up of uncemented blocks.

We will first consider the following problem.

SINGLE BLOCK.—A rectangular block is supported by a flat surface, and is pressed against it by an inclined force F (Fig. 431). Find in what manner the pressure between the block and the flat surface is distributed.

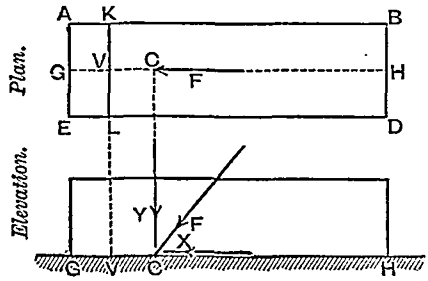


Fig. 431.

It will be observed on looking at the plan that, for simplicity, the force F is assumed to act in the central vertical plane of the block, and is supposed to include the weight of the block.

The point C, where the direction of the force intersects the supporting surface, is called the *centre of pressure.*

Now F may be resolved into two components—one perpendicular to the supporting surface or joint, and the other parallel to it. Y and X are these components. The component X tends to make the block slide along the surface and is resisted by the friction; but at present we are not concerned with this part of the question.

The normal component Y is the one that causes the pressures between the block and the surface

Since C is not in the centre of GH it is clear that these pressures will *not* be uniformly distributed, but will be greater towards AF than towards FD

In the first place, since GH is midway between AB and FD, and C lies on GH, the pressure at all points of any line KL drawn parallel to AE will be the same, and therefore we can take the pressures along GH to represent the pressures over the whole surface of the joint

Secondly, on the supposition that the joint is a perfect one (and in a masonry structure the principal function of the mortar is to make the joint practically perfect), we can say that the pressure along GH diminishes *uniformly* from G towards H The pressures along GH may therefore be represented by a triangle GMO as shown in Fig 432, so that the pressure p at any point V is given by the ordinate VN, and this ordinate will also give the pressure at any point on KVL (Fig 431)

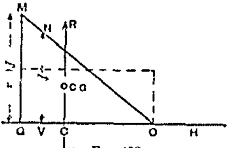


Fig 432

First Let O be situated between G and H this means that there is no pressure on the joint from O to H

Now the system of parallel forces acting over the whole joint and *represented* by the triangle GMO have a resultant R acting through the centre of gravity of the triangle, and for equilibrium R must be equal and opposite to Y, that is

$$R = Y$$

Again, R is *represented* by the area of the triangle GMO, and is equal to the area $\times$ AE. That is

$$R = \frac{MG}{2} \times GO \times AE = Y$$

Therefore
$$MG = \frac{2Y}{GO \times AE}$$

But since Y passes through the centre of gravity of the triangle GMO we have

$$GO = 3GC,$$

and for MG we can write $p_{(max)}$

Hence
$$p_{(max)} = \frac{2Y}{3GC \times AE} \qquad (95),$$

and since Y, GC, and AE are known, $p_{(max)}$ can easily be found

Example 43—As a numerical example let

$$Y = 4 \text{ tons},$$
$$GC = 0\ 5 \text{ foot},$$
$$AE = 1 \quad \text{foot},$$

then
$$p_{(max)} = \frac{2}{3}\ \frac{4}{0\ 5 \times 1}$$
$$= 5\ 3 \text{ tons per } \textit{square foot}$$

Second Let O and H coincide, as in Fig 433 In this case we have

$$GC = \tfrac{1}{3}\ GH,$$

and this is the condition which must exist in order that the pressure may *just* vanish at H, or more strictly along DB

Let GH = 3 feet, and the remaining data be as before, then

$$p_{(\max.)} = \frac{2}{3} \cdot \frac{4}{\frac{3}{3} \times 1} = 2{\cdot}67 \text{ tons per square foot.}$$

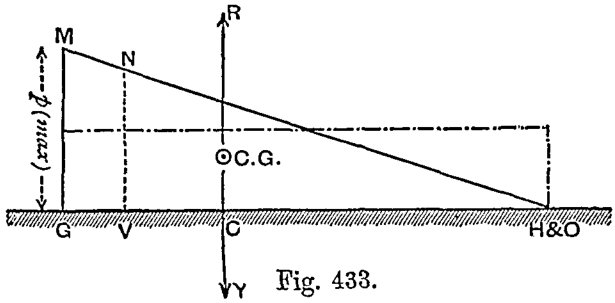


Fig. 433.

Now if C were situated at the middle point of GH, the pressure would clearly be uniformly distributed over the joint, and its intensity would be

$$p = \frac{Y}{GH \times AE},$$

$$= \frac{4}{3 \times 1} = 1{\cdot}\dot{3},$$

from which we see that when the centre of pressure is so situated that the pressure vanishes along one edge of the joint, then the pressure along the opposite edge of the joint will be double what it would have been had the centre of pressure been at the middle point. This is an important result, and should be carefully remembered.[1]

Thus far we have not considered the horizontal component X of the force F. This component is balanced by the resistance to friction of the joint so long as slipping does not occur. Now the resistance to friction is

$$\mu Y,$$

where μ is called the coefficient of friction, and is simply a number (always less than 1) depending on the nature of the surfaces. Thus for stone against stone

$$\mu = 0{\cdot}7.$$

If, however, in a masonry or brickwork joint the strength of the mortar is taken into account, the resistance opposed to X will be the resistance to shearing of the joint. In practice, however, it is not usual to take account of the strength of the mortar. It will be observed that the horizontal component X and its effect on the joint are independent of the position of the centre of pressure.

UNCEMENTED BLOCK STRUCTURES.—If we consider any masonry or brickwork structure, such as a retaining wall for earth or water, a boundary wall or an arch, we see that such a structure can be dissected as it were into a series of blocks separated by joints, such as the one we have just been considering. There will be a resultant external force acting at each joint through the centre of pressure of the joint, and this force will be resisted by the compressions (and tensions in some cases) acting over the surface of the joint.

[1] See Appendix XIX.

Def—The line joining the centres of pressure is called the "line of pressures," or "Moseley's line of resistance"

Thus in Fig 434, representing a portion of an arch the broken line *abcdef* is Moseley's line of resistance, *a*, *b* *c*, *d*, *e*, and *f* being the centres of pressure at the respective joints

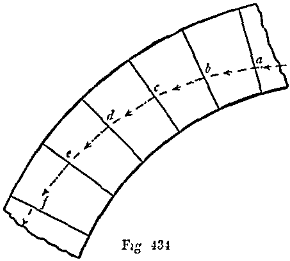


Fig 434

Conditions of Stability

I *For important structures*, or for structures the failure of which might lead to serious consequences, such as loss of life

(*a*) No portion of any joint should be brought into tension Referring to Fig 433 p 220, we see that to conform to this condition, the centre of pressure must not be nearer the edge of any joint than one-third the width of the joint (GH), or in other words, Moseley's line of resistance must lie within the middle third of the structure [1]

(*b*) The intensity of pressure should nowhere exceed what the material is capable of safely resisting

That is, $p_{(max)}$ must not exceed the safe resistance to crushing or cracking of the material and mortar (see Table IA)

(*c*) The resistance to sliding must be greater than the horizontal component of the external force at each joint [2]

II *For unimportant structures* (such as boundary walls)

(*d*) The intensity of pressure should nowhere exceed what the material or mortar is capable of safely resisting

(*e*) The intensity of tension should nowhere exceed what the mortar can resist

(*f*) The safe resistance to shearing of any joint must be greater than the horizontal component of the resultant external force acting at the joint

These conditions will now be applied to some numerical examples

[1] The middle half is sometimes taken for unimportant structures as in Example 51 p 254 The middle third should be taken for all important arches as in the Example Appendix XX

[2] See p 248

Brick Pier.

Example 44.—Find whether a brick pier 14 inches square and 8 feet high can resist a thrust of 3 cwts. inclined at an angle of 55° and applied at the top of the pier, as shown in Fig. 435. If the pier is not strong enough, find what additional thickness of brickwork is required, and how high it ought to be carried up. The tenacity of the mortar not to be taken into account.

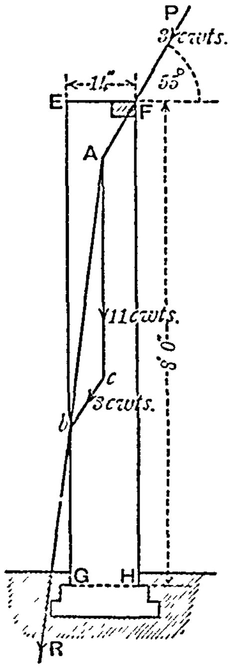


Fig. 435.

Preliminaries.—We will assume that one cubic foot of brickwork weighs 1 cwt. The total weight of the pier will therefore be

$$8 \times \frac{14 \times 14}{12 \times 12} \times 1 = 11 \text{ cwts. nearly,}$$

and acts vertically through the centre of gravity of the pier, which is easily found.

The direction of the weight W intersects that of the force P in the point A, and these are the only two external forces acting on the pier. Their resultant can be found in the usual way by means of the triangle *Abc*.

To find the centre of pressure, produce *Ab* to intersect the plane of the base of the pier. It will be observed that this point falls outside the actual base of the pier, and, since the tenacity of the mortar is not to be taken into account, it follows that the pier is not strong enough to withstand the pressure of 3 cwts.[1]

It will therefore be necessary to increase the thickness of the pier. Suppose the thickness is increased to 18 inches, which would appear to be about what is required, as the centre of pressure will be brought nearer to the wall owing to the additional weight,

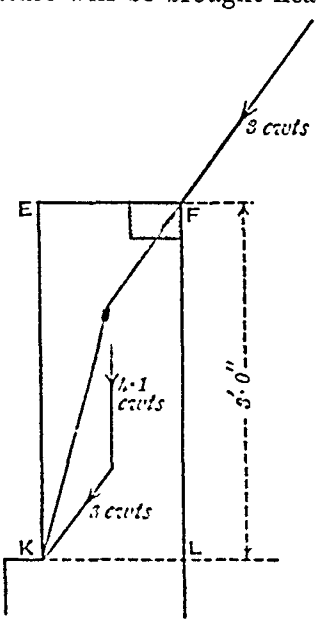


Fig. 436.

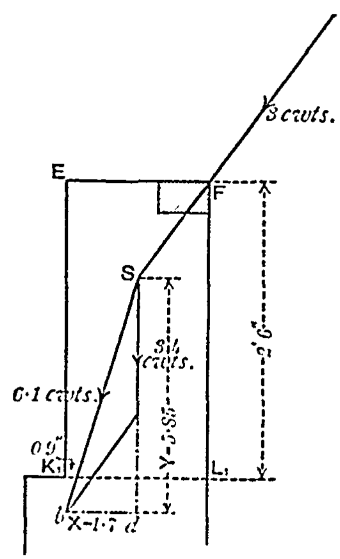


Fig. 437.

[1] The student will also observe what a very small thrust is sufficient to overturn a pier of the given dimensions.

then the first question is how far up is it necessary to make the pier this extra thickness.

Joint KL.—As a trial, suppose the increased thickness commences 3 feet from the top of the pier, then it must be ascertained whether the joint KL at the point of change is safe (Fig. 436). The external forces acting on the upper three feet are the thrust of 3 cwts., and the weight of the portion of the pier EL acting through its centre of gravity. Determining the resultant of these two forces in the usual way, it will be seen that its direction passes very near to K, so that the portion EL of the pier would be on the point of overturning.

Next try if increasing the thickness of the pier 2' 6" from the top will do. Carrying out the same construction as before, we find that the centre of pressure now lies 0.9 inch within the edge K_1.

Crushing.—We must now find the maximum intensity of compression, that is, $p_{(\max.)}$. The vertical component (Y) of the total pressure on the joint K_1L_1 can be found by dropping a perpendicular bd on to Sd the direction of W (Fig. 437), and it is thus found by measurement that

$$Y = 5.85 \text{ cwts.}$$

Hence applying Equation 95 we have

$$p_{(\max.)} = 2 \times \frac{5.85}{3 \times 0.9 \times 14} = 0.31 \text{ cwt. per square inch}$$

And since $p_{(\max.)}$ can safely be as much as 0.5 cwt. per square inch (Table IX.) there is no fear of crushing, and condition (b) is therefore fulfilled.

Sliding.—As regards condition (c), the force tending to cause sliding is the horizontal component of the thrust of 3 cwts. By measurement as above, it is found that

$$X = 1.7 \text{ cwt.}$$

The resistance to sliding is found by multiplying the vertical component (Y) of the pressure on the joint by the coefficient of resistance[1] to sliding, which can be taken at 0.47. Hence the resistance to sliding is

$$0.47 \times 5.85 = 2.7 \text{ cwts.},$$

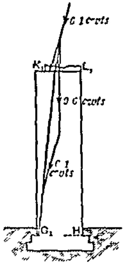


Fig. 438

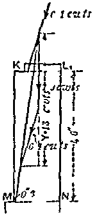


Fig. 439

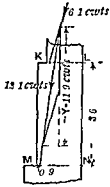


Fig. 440

[1] 0.47 is the co-efficient of friction for masonry and brickwork with wet mortar (see Table XXA.)

so that the joint is safe against sliding.[1] Finally, therefore, it is safe to commence thickening the pier at 2′ 6″ from the top.

Joint GH.—It remains to be seen whether the conditions (*b*) and (*c*) are fulfilled at joint G_1H, Fig. 438. We have now to consider the portion L_1G_1 of the pier. The external forces acting on this portion of the pier are the resultant pressure acting on joint K_1L_1, the value of which is found by measurement from Fig. 437 to be 6·1 cwts., and the weight of the portion L_1G_1 of the pier, namely 9·6 cwts., acting through the centre of gravity of that portion, as shown in Fig. 438.

The centre of pressure of joint G_1H is found to be quite close to G_1, so that the pier is still not thick enough at the bottom. We have therefore to find how far below the joint K_1L_1 the thickness of 18 inches can be carried. Assume that it is carried down 4′ 6″ below K_1L_1 to joint MN (Fig. 439), then, proceeding as before, the centre of pressure of joint MN is found to be 0·3 inches within the edge M, and from Equation 95, p. 219,

$$p_{(\text{max.})} = 2 \times \frac{13{\cdot}7}{3 \times 0{\cdot}3 \times 14},$$
$$= 2{\cdot}18 \text{ cwt. per square inch.}$$

Crushing.—The maximum intensity of compression is therefore more than the safe intensity, and consequently MN must be taken nearer to K_1L_1, say at M_1N_1, 3′ 6″ below K_1L_1 (Fig. 440). We then find

$$p_{(\text{max.})} = 2 \times \frac{11{\cdot}9}{3 \times 0{\cdot}9 \times 14},$$
$$= 0{\cdot}63 \text{ cwts. per square inch.}$$

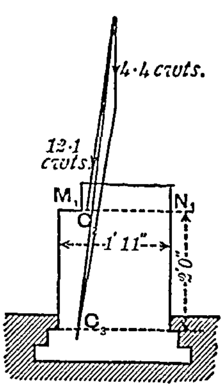


Fig. 441.

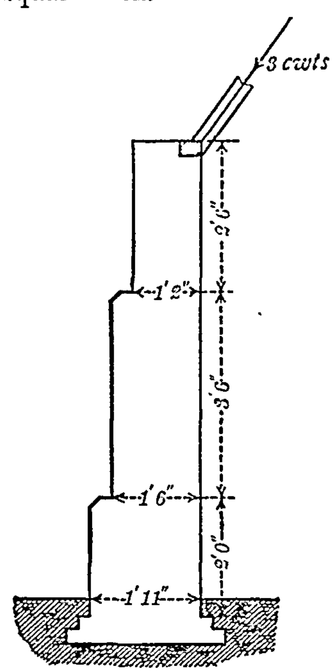


Fig. 442.

[1] According to Rankine, there is no danger of sliding if the angle S (Fig. 437) between the normal to the bed joint and the direction of the resultant pressure is less than from 25° to 36° the angle of repose of fresh masonry. In this case, the angle at S is only 16°.

Below K_1L_1, therefore, the pier must again be thickened to the next brick dimensions, that is to 23 inches.

Lowest portion of pier.—We now have to consider the portion of the pier shown in Fig. 441. The external forces acting on this portion are its weight (1.4 cwts.) and the resultant pressure of 12.1 cwts. acting at C_2 as found by measurement from Fig. 440.

Crushing and sliding.—It is then found that the centre of pressure is C_3, which is 3.5 inches from the edge, and the vertical component of the resultant pressure on the joint is also found to be 16.4 cwts., so that

$$p_{(\max.)} = 2 \times \frac{16.4}{3 \times 3.5 \times 14} = 0.22 \text{ cwt.}$$

Condition (*b*) is therefore fulfilled. As regards condition (*c*), the force tending to cause sliding is found by measurement to be 1.7 cwt., and since the vertical component of the pressure is 16.4 cwts. the resistance to sliding is $0.47 \times 16.4 = 7.71$ cwts.

The pier has now been designed, and is shown in Fig. 442.

Chimney.

Example 45.—Find what wind pressure the chimney shown in Fig. 443 can *safely* resist.

It will be seen at once that AB is the joint most likely to fail.

Now the fall of a chimney may lead to serious consequences, and it is therefore well to lay down the rule that the centre of pressure shall be kept so much inside of one edge that no tension will be excited at the other edge. Referring to Appendix XIX. we see that for a hollow square chimney, AC must not be less than $\frac{1}{4}$ AB in order that the above condition may be fulfilled, and this value can be applied with sufficient accuracy to the case under consideration. But AB is 22.5", so that least value of AC = 5.62" approximately.[1]

It will be found that there are 87 cubic feet of brickwork above the joint AB. Therefore (allowing 1 cwt. per cubic foot for weight of brickwork) the weight of the chimney above the joint AB is 87 cwts.

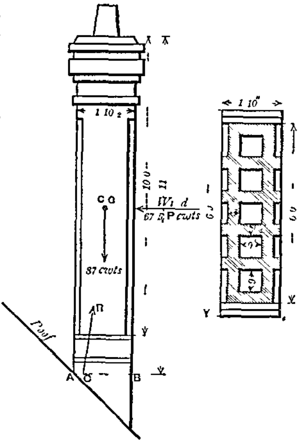


Fig. 443

[1] The student is recommended to ascertain the true value of AC by means of the formula $\delta = \frac{I}{Aj}$ p. 391.

Let P be the normal wind pressure; the total wind pressure on the side of the chimney will therefore be

$$11'3'' \times 6 \times P = 67{\cdot}5P \text{ cwts.},$$

and this force is applied at the centre of gravity of the side of the chimney.

The part of the chimney above the joint AB is therefore under the action of three external forces—namely, 87 cwts. acting vertically through C.G., 67·5P cwts. acting horizontally at E, and R the reaction to the resultant pressure acting upwards at C.

Taking moments about C we have

$$\left(\frac{22{\cdot}5}{2} - 5{\cdot}62\right)87 - \frac{11{\cdot}25}{2} \times 12 \times 67{\cdot}5P = 0,$$

whence

$$P = 0{\cdot}107 \text{ cwt.} = 12 \text{ lbs.}$$

It therefore appears that this chimney under the condition laid down, although the tenacity of the mortar would materially increase the powers of resistance, cannot be considered as safe.

Crushing.—The maximum intensity of compression with 12 lbs. wind pressure should also be found. In this case the vertical component of the resultant pressure on the joint AB is equal to the weight of the chimney, namely, 87 cwts., and this pressure is distributed, but not uniformly, over an area of 9·8 square feet, or 1417 square inches. Following the rule given at p. 220 we have

$$p_{(\text{max.})} = 2 \times \frac{87}{1417} = 0{\cdot}123 \text{ cwt. per square inch nearly},$$

so that there is no fear of crushing with a wind pressure of 12 lbs. per square foot.

Sliding.—As regards sliding, the force tending to cause sliding is the wind pressure equal to

$$\frac{12 \times 67{\cdot}5}{112} = 7{\cdot}2 \text{ cwts.},$$

and the maximum resistance of the joint AB to sliding is $0{\cdot}7 \times 87 = 61$ cwts., so that there is no fear of sliding.

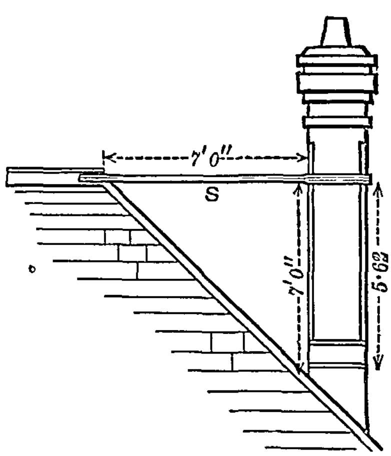


Fig. 444.

Stay.—This chimney can, however, be made perfectly safe by fixing an iron stay as shown in Fig. 444. Let it be ascertained what the cross section of this bar ought to be.

We have seen that the chimney itself can safely resist 12 lbs. wind pressure. Taking 50 lbs. as the maximum wind pressure, it follows that the iron will have to resist $50 - 12 = 38$ lbs. per square foot. So that the wind pressure the stay has to resist is

$$\frac{67{\cdot}5 \times 38}{112} = 23 \text{ cwts. nearly.}$$

To find S, the stress in the stay, take moments about A (Fig. 443)

$$7 \times S - 5{\cdot}62 \times 23 = 0,$$

$$S = 18{\cdot}4 \text{ cwts.} = 0{\cdot}92 \text{ ton.}$$

When the wind is blowing in the direction shown in Fig. 443, the stay has to act as a strut, and it must therefore be calculated as a long column

with ends fixed By the help of Table V it will be found that an L iron $2'' \times 2'' \times \frac{3}{8}''$ can safely bear 0 9 ton (ends rounded), and will therefore be strong enough either as strut or tie

Enclosure Wall.

Example 46—Find the wind pressure per square foot a *long* enclosure wall of the section and plan shown in Fig 445 can safely resist

Ignoring tenacity of the mortar—If the tenacity of the mortar is neglected, it can be shown that, with a wind pressure of 16 2 lbs per square foot, the centre of pressure would be at A, so that, to say nothing of crushing the bricks, the wall would be on the point of overturning This investigation is, however, left as an exercise for the student

Long enclosure walls should therefore be built in good tenacious mortar, and it is well to build the lower courses in cement, especially if the situation is exposed

By a *long* enclosure wall is meant one whose unsupported length is more than about ten times its height Shorter walls than these receive considerable support from each end

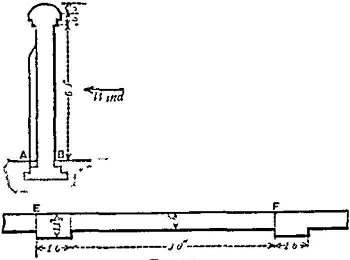


Fig 445

Considering tenacity of the mortar—We will now suppose that this wall is built in a good hydraulic lime mortar, and that the adhesion of this mortar to the bricks can be safely taken at 0 06 cwt per square inch (see Table IA) The tenacity of the mortar itself is greater, but of course we can only reckon on the adhesion, as it is less

Now, looking at the plan, we see that we can take a portion of the wall from E to F as a unit. The weight of this portion of the wall (allowing, as before, 1 cwt. per cubic foot of brickwork) is found to be 59 cwts. This weight is distributed *uniformly* over the area of the joint AB in section and portion EF on plan, which is found to be 1215 square inches The pressure due to the weight of the wall is therefore

$$\frac{59}{1215} = 0\ 05 \text{ cwt. per square inch nearly}$$

Let P be the wind pressure per square foot, then the total wind pressure acting at the centre of gravity of the unit of the wall is

$$10\ 6 \times 6\ 7\frac{1}{2} \times P = 69\ 5P$$

The full height of the wall is not taken on account of the inclination of the coping

The wind pressure will clearly produce tension on the windward side, that is at B, and compression at A, and the intensities of these stresses can be

found by considering the portion of the wall as a beam fixed at one end, namely, at the joint AB. Referring to p. 50 (footnote) we see that we must first find the moment of inertia of the section, and this we can do by the rule given in Appendix XIV. p. 310. We thus find

$$I = 11235.$$

The distance of the extreme fibre in tension from the "mean" fibre is sensibly

$$y_o = 4.95 \text{ inches.}$$

So that moment of resistance

$$= r_o \times \frac{11235}{4.95} \text{ inch-cwts.}$$

But moment of flexure

$$= 69.5\text{P} \times \frac{(6'7\tfrac{1}{2}'') \times 12}{2}.$$

Hence

$$\text{P} = \frac{11235 \times 2}{4.95 \times 69.5 \times 79.5} r_o.$$

To find P we must therefore decide what value to give to r_o. Now we have seen that the *safe* adhesion is 0·06 cwt. per square inch; but r_o can be given a greater value than this, because the compression produced by the weight of wall (viz. 0·05 cwt. per square inch) relieves the tension.[1] Hence we can take approximately

$$r_o = 0.06 + 0.05,$$
$$= 0.11 \text{ cwt. per square inch.}$$

Hence

$$\text{P} = 0.09 \text{ cwt.} = 10 \text{ lbs.}$$

Therefore even if the wall is built in good hydraulic lime it cannot *safely* resist more than 10 lbs. per square foot of wind pressure. It would of course take a greater wind pressure than this to cause the wall to fail. Thus, taking the ultimate adhesion of best hydraulic lime to hard stock bricks as 36 lbs. per square inch or 0·32 cwt., we can put

$$r_o = 0.32 + 0.05,$$
$$= 0.37 \text{ cwts.}$$

And

$$\text{P} = \frac{11235 \times 2 \times 0.37}{4.95 \times 69.5 \times 79.5}$$
$$= 0.30 \text{ cwt.,}$$
$$= 33.6 \text{ lbs. per square foot.}$$

In an exposed situation, therefore, this wall might possibly in course of time be blown down. In an unexposed situation, however, the wall might be safe enough, especially as the friction of the wind against the ground is known to lessen considerably its force near the surface.

The wall can be materially strengthened by building the first ten or twelve courses in cement. In this case the ultimate adhesion can be taken at about 0·5 cwt. per square inch, and $r_o = 0.55$, hence

$$\text{P} = \frac{11235 \times 2 \times 0.55}{4.95 \times 69.5 \times 79.5}$$
$$= 0.45 \text{ cwt.,}$$
$$= 50.4 \text{ lbs. per square foot,}$$

so that the wall is now strong enough against moderate wind pressures.

[1] [illegible] accurately the resistance offered by tension is supplemented by the [illegible] weight of the wall.

It is clearly unnecessary in this case to find the maximum intensity of compression, neither need any calculations be made as regards the resistance to sliding, as the resistance we can now reckon on is not merely the friction of brick against brick, or stone against stone, but the shearing strength of the mortar, and this is much more than enough.

CHAPTER XIV.

RETAINING WALLS.[1]

A RETAINING wall is a wall built for the purpose of "retaining" or holding up earth or water. In engineering practice such walls attain frequently enormous proportions, being used in the construction of railways, docks, water-works, etc. In building construction, however, their dimensions are never very large, and the difficulties attending their design and construction, which in engineering works are often very great, are therefore much reduced.

USUAL CROSS SECTIONS FOR RETAINING WALLS.—The form of cross section given to such walls varies considerably according to circumstances, and often according to the fancy of the designer. A few of these forms, such as might be useful in building construction, are given in Figs. 446-456.

Fig. 446 shows a simple rectangular wall. Such a section is suitable only for very low walls (say not exceeding 5 feet), as it is wasteful of material.

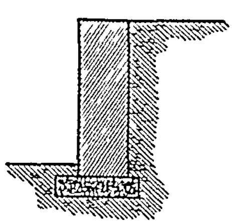


Fig. 446.

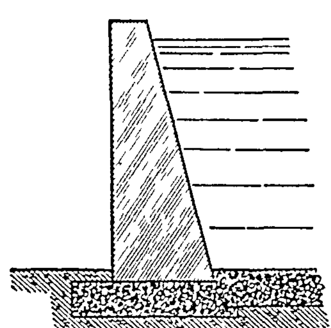


Fig. 447.

Figs. 447 and 448 show a better section, in which the back of the wall is sloped. Except in the case of a wall for retaining water, the alterations in the thickness of the wall would be carried out by means of offsets, as in Fig. 448.

[1] For *Practical Formula* see pp. 235, 239, 244, and App. XXI.

Fig 449 shows a wall with vertical back and with a battered face In point of stability a wall with a battered face may or may not be according to circumstances more stable than a wall with a straight face and sloped back It has the appearance of being more stable and there is the advantage that a small alteration in the batter, caused by rotation or inclining forward is not

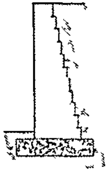


Fig 448

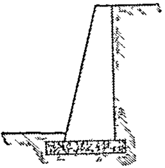


Fig 449

noticeable, whereas the eye is offended if a wall with a vertical face rotates ever so little To prevent lodgment of water however the batter should not exceed $\frac{1}{5}$

In Figs 450 and 451 walls with battered face and sloping

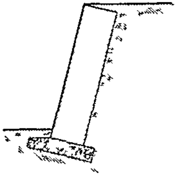


Fig 450

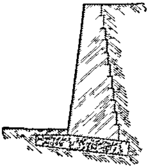


Fig 451

backs are shown It will also be noticed that the footings are inclined thus giving greater resistance against sliding forward The section shown in Fig 450 is however clearly not suitable for retaining water

In the case of walls for retaining earth counterforts are sometimes added as in Figs 452 and 453 and Professor Rankine has shown that a slight saving in masonry is thereby effected

In the preceding cases it has been supposed that the top surface of the earth to be retained is horizontal and also level with the top of the wall Frequently however a bank has to be sup

ported, as shown in Fig. 454 or 455. Such walls are called "surcharged" walls and clearly require greater strength.

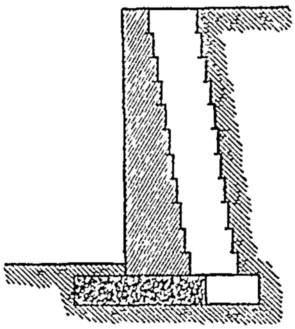


Fig. 452.

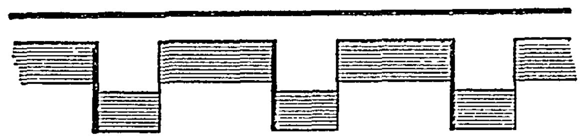


Fig. 453.

Another case in which greater strength is required is when a building is placed close to the wall as shown in Fig. 456, because

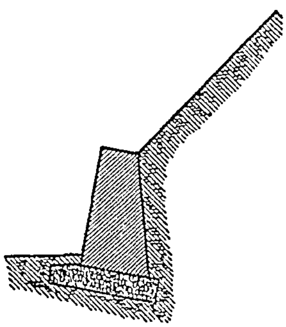


Fig. 454.

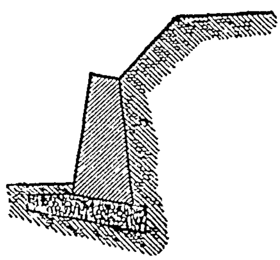


Fig. 455.

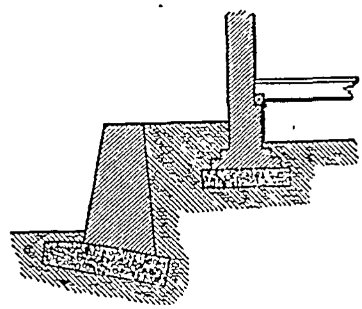


Fig. 456.

the weight of the building tends to overturn the wall by increasing the natural thrust of the earth.

RETAINING WALLS FOR WATER.

The calculations for finding the dimensions of walls for retaining water, so long as the depth of water to be retained is not great, are comparatively simple.

Such a wall is acted on by two external forces, namely, the pressure of the water and the weight of the wall.

As regards the pressure of the water, it is known from hydrostatics that it is:

(1) Directly proportional to the depth below the surface.

(2) Normal to the surface exposed to the pressure.

Applying this to the case of a wall with a vertical back (Fig. 457); at the lowest point B the pressure will be proportional to the depth of water AB, and will be horizontal, since it is normal to the surface pressed. The pressure of the water at B

can therefore be represented by BD, which is drawn horizontally equal to the depth of water at B. At A the pressure is clearly zero, hence ordinates from AB to AD represent the pressures at all points along AB.

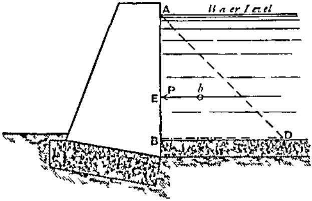


Fig 457

The area of the triangle ABD therefore represents the total water pressure on the wall (see p 219), and the resultant pressure will pass through the centre of gravity of this triangle. The centre of pressure E is therefore two thirds of the way down, that is

$$AE = \tfrac{2}{3}\ AB \qquad (96)$$

And if we consider 1 foot of length of the wall the resultant pressure (P) on *this portion* of the wall will be equal to the weight of a prism of water whose cross section is the triangle ABD and length 1 foot (Fig 458).

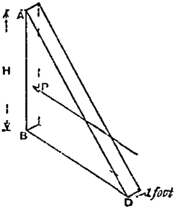


Fig 458

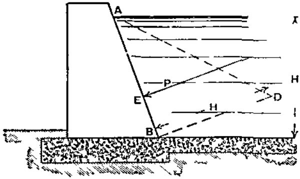


Fig 459

The volume of the prism is

$$= \frac{AB \times BD}{2} \times 1 = \frac{AB^2}{2} = \frac{H^2}{2}$$

where H is the height of water above the point B. And since a cubic foot of water weighs 0 557 cwt we have

$$P = 0{\cdot}557 \times \frac{AB^2}{2} \text{ cwts.} \qquad (97).$$

We therefore know the magnitude, point of application, and direction of P, which is all we require to know.

In the case of a wall with sloping back some modifications must be introduced. Thus in Fig. 459 the pressure at B will be represented by BD drawn normally to AB and equal to H. AD will then represent the pressure along AB, so that the triangle ABD will represent the total pressure, and

$$AE = \tfrac{2}{3} AB.$$

Also

$$P = 0{\cdot}557 \times \frac{AB \times H}{2} \text{ cwts.} \qquad (98).$$

P is therefore completely determined.

As regards the second external force, namely, the weight of the wall, if we assume a trial section the weight can easily be obtained, and we can then ascertain whether this trial section is suitable by applying the rules given at p. 221.

In the case of a wall for retaining water it is of the utmost importance that no cracks should be formed, and for this it is necessary that no tension should be excited. Hence the centre of pressure must not approach the outer edge nearer than one-third the width of the joint.

The various points that have been considered will now be illustrated by means of an example.

Reservoir Wall.

Example 47.—A wall to retain water to a depth of 6 feet is required; the top of the wall is to be 1 foot above the surface of the water and the batter of the face $\frac{10}{1}$. The wall to be built in brickwork and cement.

Preliminaries.—So far as stability is concerned, the top of the wall might be built to a feather edge, but if the wall were thus built the top would soon be in want of repair; moreover, the saving in brickwork would be exceedingly small, and would not compensate for the additional difficulty in building. A width of 9″ at the top, although somewhat narrow, will be assumed.

Since the batter of the face of the wall is given, it remains only to find the batter of the back of the wall, or, what comes to the same thing, to find the width of the wall at the footings.

Now it can be shown that if the wall were of triangular section, *i.e.* with a feather edge at the top, that all joints would be of equal strength, and the effect of giving a certain width to the top of the wall is to make the lowest joint the weakest; we need therefore only consider this joint.

The courses will be built at right angles to the face of the wall, so that the joints will have a slope of $\frac{1}{10}$.

As a first trial, assume the thickness of the wall at the footings to be 4 feet. The weight of one foot of length of the wall will be

$$\frac{0\cdot75+4}{2}\times 6\cdot6\times 1 \text{ cwt} = 15\cdot6 \text{ cwts},$$

and this force will be applied at the centre of gravity marked C G in Fig 460, which can be found graphically by a method explained in Molesworth's *Pocket Book* [1]

The water pressure is equal to

$$0\cdot557\times\frac{6\cdot4\times6}{2},$$
$$=10\cdot7 \text{ cwts}$$

It is perpendicular to AB, and is applied at the point E

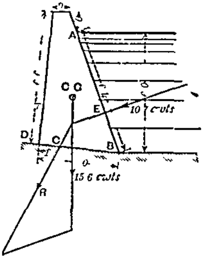


Fig 460

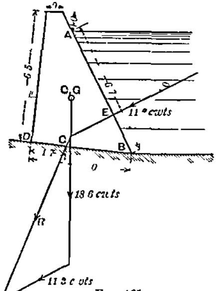


Fig 461

The resultant of these two forces intersects the joint DB in the point C, and as DC is 1·05 foot and is less than $\frac{DB}{3}$ the wall must be made thicker

Next try 5 feet, the weight will be 19 cwts and the water pressure 11·2 cwts It will be found that DC = 1·7 foot, which is within the middle one third of the width of the joint This thickness is therefore sufficient and the section of the wall will be as shown in Fig 461

Strictly it should be ascertained that conditions (*b*) and (*c*), p 221, are fulfilled This is left as an exercise for the student

As a further exercise, the student can find the proper dimensions to give to this wall supposing the back to be vertical

Practical Rule for thickness of walls to retain water—The following is an old rule for retaining walls for water—

Width at bottom = height × 0·7,
Width at middle = height × 0·5,
Width at top = height × 0·3

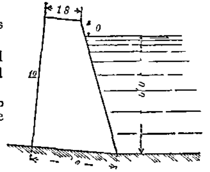


Fig 462

[1] Page 366 23rd edition

and a wall designed accordingly would be as shown in Fig. 462. It will be seen that this section has a smaller base but somewhat greater area than the one just designed. Its weight per foot run is 19·6 cwts. The resultant in this case also falls just at the one-third of the width of the base within the toe. On comparing Figs. 461 and 462 it will be seen that the width at the top has a considerable effect on the cross section of the wall.

RETAINING WALLS FOR EARTH.

The section of a wall for retaining earth could be at once ascertained in the manner just described for walls for retaining water if we knew the direction and magnitude of the pressure exerted by the earth on the back of the wall. We saw in the last example that the water pressure on the back of a wall can be easily found, but it is not so with regard to earth pressure, which varies with every different kind of soil, and also with the conditions in which the soil is; for instance, whether loose or compressed, wet or dry. Various theories on the subject have been propounded by Professor Rankine, Boussinesq, and others, but they do not appear to be very satisfactory, and in any case are far too complicated for these Notes. Under these circumstances it will only be possible to give a mere outline of the subject.

If a steep bank of earth is left to itself it will, under the action of the weather, gradually crumble down until it has taken up a certain slope as shown in Fig. 463. The angle of inclination of the slope at which crumbling ceases is called the "angle of repose," and this angle varies with each different kind of earth. This angle is generally denoted by ϕ. In Table XVI. will be found the angle of repose for several of the usual kinds of earths, but this angle can be found experimentally in any particular instance by shovelling the earth up into a heap and measuring the angle of the slope so formed.

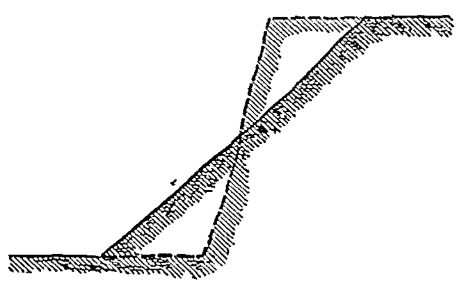


Fig. 463.

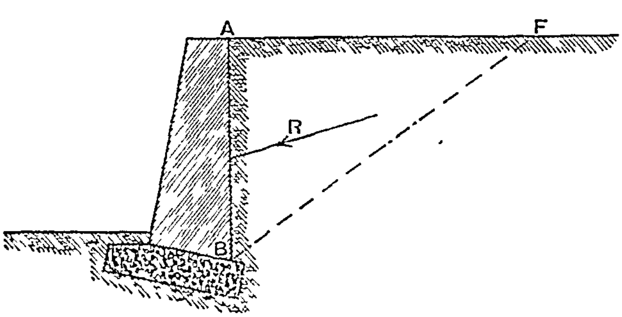


Fig. 464.

Now suppose that the earth behind a retaining wall is loose, and that it would immediately fall down to

the angle of repose were the wall removed, then it will be seen from Fig. 46; that the wall, aided by the friction of the earth supports the wedge of earth represented in the figure by the triangle ABF, which is tending to slide down BF.

The resultant pressure of this wedge of earth will be some force such as R, and it is to find this force R that the various theories already alluded to have been proposed. Once found, the dimensions of the wall can be ascertained precisely as in the case of the reservoir wall (Example 47). If, however, the earth is at all consolidated, so that it would not fall to the natural slope immediately on removal of the wall, the pressure on the back of the wall is much less than that due to the wedge ABF. The earth may even be sufficiently consolidated to stand vertically on removal of the wall, in which case there is practically no pressure on the back of the wall, and if it can be ensured that the earth will always remain in this condition, the wall need only be built thick enough to resist its own weight. It then acts simply as a covering to the earth to prevent its being degraded by the action of the weather. Such walls are called "breast walls." On the other hand, however, infiltration of water will increase the pressure of the earth against the back of the wall, beyond what it is when the earth is quite loose. The student will at once see that to frame a theory to meet all these different conditions must be a difficult matter.

"The presence of moisture in earth to an extent just sufficient to expel the air from its crevices, seems to increase its coefficient of friction slightly, but any additional moisture acts like an unguent in diminishing friction, and tends to reduce the earth to a semi fluid condition, or to the state of mud. In this state, although it has some cohesion, or viscidity, which resists rapid alterations of form, it has no frictional stability, and its coefficient of friction, and angle of repose, are each of them null.

"Hence it is obvious that the frictional stability of earth depends, to a great extent, on the ease with which the water that it occasionally absorbs can be drained away. The safest materials for earthwork are shivers of rock, shingle, gravel, and clean sharp sand, whether consisting wholly of small hard crystals, or containing a mixture of fragments of shells, for those materials allow water to pass through without retaining more than is beneficial. The cleanest sand, however, may be made completely unstable, and reduced to the state of quicksand, if it is contained in a basin of water holding materials, so that the water mixed amongst its particles cannot be drained off.

"The property of retaining water, and forming a paste with it, belongs specially to clay, and to earths of which clay is an ingredient. Such earths, how hard and firm soever they may be, when first excavated, are gradually softened, and have both their frictional stability and their adhesion diminished by exposure to the air. In this respect, mixtures of sand and clay are the worst; for the sand favours the access of water, and the clay prevents its escape.

"The properties of earth with respect to adhesion and friction are so variable that the engineer should never trust to tables or to information obtained from books to guide him in designing earthworks when he has it in his power to obtain the necessary data either by observation of existing earthworks in the same stratum or by experiment."[1]

Mathematical Formulæ.—RANKINE'S THEORY—*Earth, horizontal top.*—It has been shown by Rankine that in the case of a wall retaining loose granular earth (*i.e.* not consolidated), the top surface of the earth being horizontal and the back of the wall vertical, the pressure P of the earth acts horizontally, at a point $\frac{1}{3}$d of the height from the base, and that its magnitude is

$$P = \frac{wH^2}{2} \cdot \frac{1 - \sin\phi}{1 + \sin\phi} \qquad (99),$$

in which ϕ = the angle of repose, w = weight of earth per unit of volume, H = AB.

Graphic Method.—This result can be expressed graphically

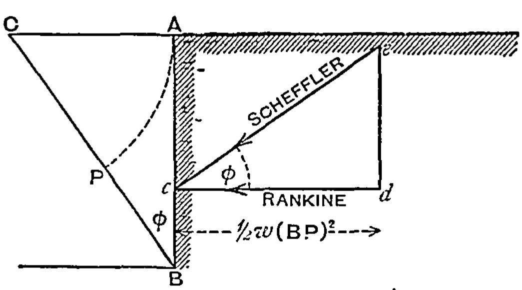


Fig. 465.

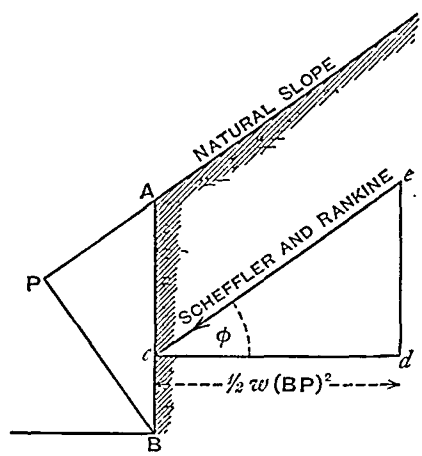


Fig. 466.

as shown in Fig. 465 by laying off the angle ABC = ϕ, and making CP = CA. Then $\frac{1}{2}w(BP)^2$ lbs. is the magnitude of the earth thrust, for it can be shown[2] that

$$BP^2 = H^2 \cdot \frac{1 - \sin\phi}{1 + \sin\phi},$$

so that

$$P = \tfrac{1}{2}w(BP)^2 \text{ lbs.} \qquad (100),$$

and hence the earth thrust is represented by *cd*.

If the back of the wall slopes, AB can be taken as a vertical line just clear of the wall, and the earth resting on the back of the wall adds to its stability.

[1] *A Manual of Civil Engineering*, by Prof. Rankine, F.R.S., etc.

[2] $BP^2 = (BC - AC)^2 = \left(\frac{AB}{\cos\phi} - AB\tan\phi\right)^2 = H^2\frac{(1-\sin\phi)^2}{\cos^2\phi} = H^2\frac{1-\sin\phi}{1+\sin\phi}.$

When the surface of the earth is at a slope instead of being horizontal a similar construction will give the earth thrust, by drawing its direction, according to Rankine, parallel to the slope of the earth. These constructions are shown in Figs. 466 and 467. It will be observed that Fig. 467 shows the general case, and that Figs. 465 and 466 can be deduced from it. According to Dr. Scheffler's theory the direction of the thrust is always inclined at an angle ϕ to the horizontal, but the magnitude is greater than given by Rankine, except in the case shown in Fig. 466.

The overturning power of the thrust as found by Scheffler's hypothesis is less than that of the thrust as found by Rankine's method, but it has been proved by experiments made by G H Darwin, as well as by the results of actual practice, that Scheffler's

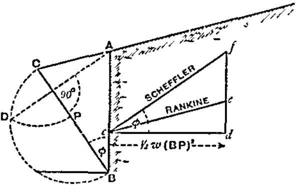


Fig 467.

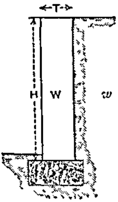


Fig 468 *Case* 1.

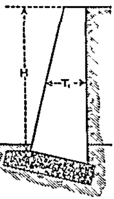


Fig 469. *Case* 2.

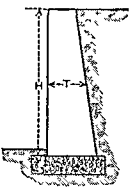


Fig 470. *Case* 3.

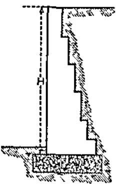


Fig 471. *Case* 3*a*.

theory can be safely depended upon. This hypothesis, in fact, makes some allowance for the cohesion of the soil, whereas, as already stated, Rankine's method is based on the supposition that the earth is granular and loose.

Practical Formulæ.—A formula is given in Hurst's *Pocket-Book*,[1] based on Rankine's investigations, which is shown below, with certain modifications.[2]

[1] Page 123, 14th edition.

[2] The following process is recommended for designing retaining walls, first find the section by means of the formula, and then apply the graphic method as a check.

Let

T = thickness of wall with vertical sides.

T_1 = *mean* thickness of wall with either face or back sloping (*i.e.* thickness half-way up the wall).

H = height of the wall.

w = weight of a cubic foot of the earth at the back of the wall.

W = weight of a cubic foot of the wall.

$K = 0{\cdot}7 \tan\left(\frac{90-\phi}{2}\right)$, where ϕ is the angle of repose (see Table XVI.)

Then

Case (1) *Wall with vertical sides and backing horizontal at the top* (Fig. 468).

$$T = K \cdot H\sqrt{\frac{w}{W}} \qquad \text{(101).}$$

(2) *Sloping walls with vertical back* (Fig. 469).

Batter of face

$\frac{12}{1}$	$T_1 = 0{\cdot}86T.$
$\frac{8}{1}$	$T_1 = 0{\cdot}80T.$
$\frac{6}{1}$	$T_1 = 0{\cdot}74T.$

Case (3) *Wall with vertical face and sloping back* (Fig. 470), or (*Case* 3*a*) *with offsets* (Fig. 471).

$$T_1 = 0{\cdot}85T.$$

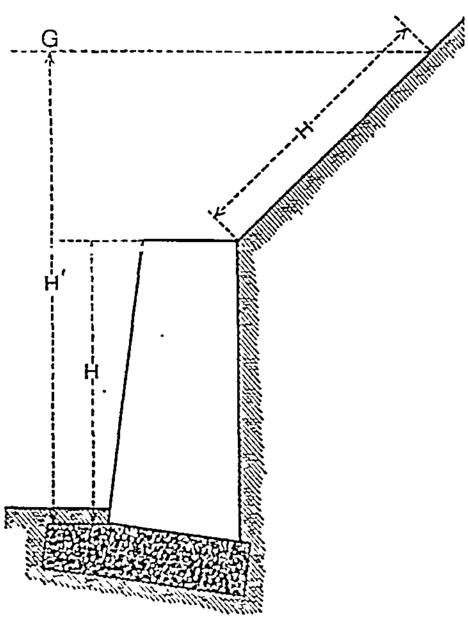


Fig. 472. *Case* 4.

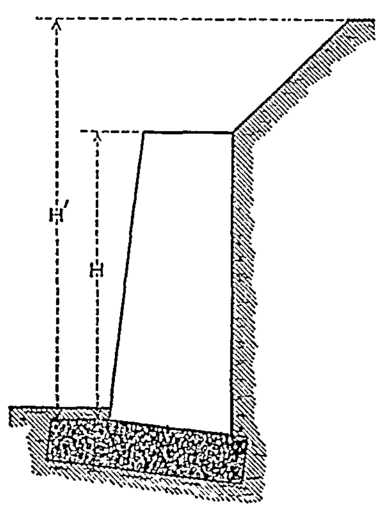


Fig. 473. *Case* 5.

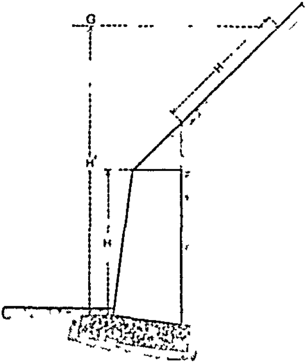


Fig 474 Case 6

Case (4) Surcharged Revetments.—Substitute for H in the formula the vertical height H' measured to the point G, found by setting off the distance H along the earth slope, as shown in Fig 472

Case (5) When the bank is of less height above the wall than the distance H would give, take the actual height of the bank, as shown in Fig 473

Case (6) When the earth slopes from the front edge of the wall the point G should be found as shown in Fig 474

Values for w and W will be found in Table XVII

Retaining Wall for Light Vegetable Earth.

Example 48.—Design a wall to retain light vegetable earth, consolidated and dry, to a depth of 8 feet, the backing being horizontal at the top and flush with the top of the wall. The back of the wall to be vertical and the face to batter $\frac{1}{8}$. The wall to be built in ordinary brickwork.

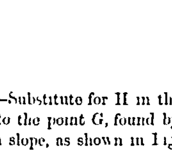


Fig 475

Preliminaries.—From Table XVII we find
$w = 90$ lbs per cubic foot for vegetable earth
W = 112 ,, ,, for brickwork in mortar

Also from Table XVI

K = 0 26 for moist vegetable earth

Calculation by practical formula.—Hence (Equation 101, p. 240),

$$T = 0{\cdot}26 \times 8\sqrt{\frac{90}{112}},$$
$$= 1{\cdot}86 \text{ foot};$$

and
$$T_1 = 0{\cdot}80 \times 1{\cdot}86,$$
$$= 1{\cdot}49,$$

or say 1′ 6″; and as the batter is $\frac{8}{1}$ the thickness at the top will be 1′ 0″, and at the bottom 2′ 0″.

The foundations can, with advantage, be inclined so as to resist the pressure which tends to make the wall slip forward. We thus get a wall as shown in Fig. 475.

Graphic method.—To compare the formula given above with Rankine's and Scheffler's methods, draw a cross section of the wall, and apply the graphic construction as shown in Fig. 476, taking the angle of repose as 49°. It will be found that BP is 2·96 feet, so that

$$\tfrac{1}{2}w(BP)^2 = \frac{90}{2}(2{\cdot}96)^2 = 394 \text{ lbs.}$$

Also the weight of one foot length of the wall = 1345 lbs.

Taking Rankine's method first, we find R as the resultant of the weight

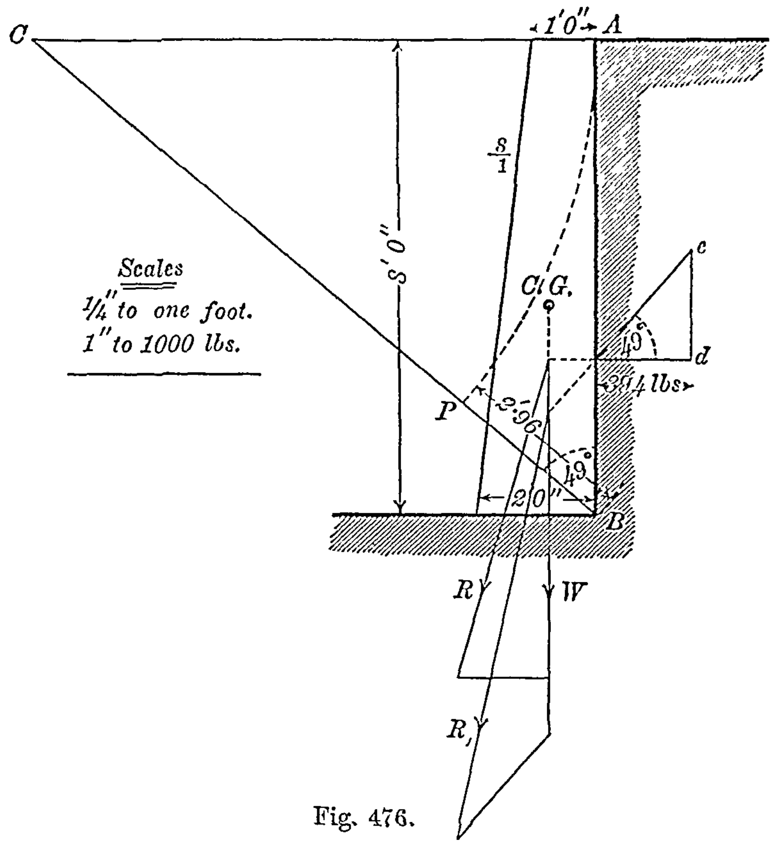


Fig. 476.

and the thrust, and it will be seen that R intersects the plane of the base sufficiently inside the wall. R_1, the resultant found according to Scheffler's method, intersects the base still more inside the wall. It is left as an exercise for the student to find the maximum intensity of compression at the edge.

The student should observe that an extreme case has been chosen for this example, the earth is very light and the angle of repose large. If the earth were not consolidated, the angle of repose would be 29°, and it will be found that the mean thickness (T_1) would then be 2.4 feet.

Retaining Wall for London Clay.

Example 49.—Same wall as in the previous example, but designed to retain London clay recently excavated and saturated with water (see Fig. 477).

In this case we find, on referring to Table XVII,

$$w = 120 \text{ lbs. per cubic foot, and}$$

$$K = 0.53$$

Hence (Equation 101, p. 240) $T = 0.53 \times 8 \sqrt{\frac{120}{112}} = 4.4$ feet,

$$T_1 = 0.80 \times 4.4 = 3.5 \text{ feet.}$$

Therefore a wall of the section shown in Fig. 477 is required.

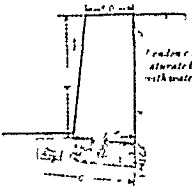


Fig. 477.

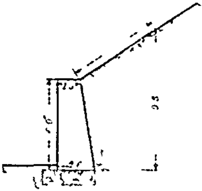


Fig. 478.

Surcharged Retaining Wall (Loamy Earth).

Example 50.—Find the dimensions of the surcharged retaining wall shown in Fig. 478. The earth is supposed to be of a loamy nature, the face of the wall is to be vertical, and the back to have a batter of about $\frac{5}{1}$, the wall to be built of rubble masonry.

Preliminaries.—From Table XVII we find

$$w = 80 \text{ lbs. per cubic foot,}$$

$$W = 140 \quad " \quad "$$

and

$$K = 0.33$$

On measuring H along the slope we find

$$H_1 = 9.3 \text{ feet,}$$

and therefore

$$T = 0.33 \times 9.3 \sqrt{\frac{80}{140}} = 2.3 \text{ feet.}$$

Hence from (3), p. 240,

$$T_1 = 2.3 \times 0.85,$$

$$= 2 \text{ feet nearly.}$$

The dimensions in Fig. 478 give a mean thickness of 2 feet and a batter of $\frac{5}{1}$ at the back.

Baker's practical Rules for Thickness of Retaining Walls.

Proportion of thickness to height.—"As a result of his own experience" Sir Benjamin Baker "makes the thickness of retaining walls in ground of an average character equal to $\frac{1}{3}$ of the height from the top of the footings."

He also says that "A wall quarter of the height in thickness, and battering 1 inch or 2 inches per foot on the face, possesses sufficient stability when the backing and foundation are both favourable." Also that "under no ordinary conditions of surcharge or heavy backing is it necessary to make a retaining wall on a solid foundation more than double the above, or $\frac{1}{2}$ of the height in thickness."

Equivalent fluid pressure.—He says further that "Experiment has shown the actual lateral thrust of good filling to be equivalent to that of a fluid weighing about 10 lbs. per cubic foot; and allowing, for variations in the ground, vibration, and contingencies, a factor of safety of 2, the wall should be able to sustain at least 20 lbs. fluid pressure, which will be the case if $\frac{1}{4}$ of the height in thickness."

These rules may be usefully applied to check the calculations made by other methods.

Foundations.

Ordinary firm earth will safely bear a pressure of about 1 to $1\frac{1}{2}$ ton per square foot, while moderately hard rock will bear as much as 9 tons.

It is therefore not worth while to make any calculations for the foundations of ordinary walls because the pressures they cause are so small.

The foundations of retaining walls are subject, however, to considerable pressures, which moreover are not uniformly distributed. They should therefore be of such a width that the maximum intensity of pressure is not greater than the soil can safely bear, and the centre of pressure should not be nearer the outside edge than $\frac{1}{3}$ the width of the foundation.

Example 50a.—*Pressure.*—As an example suppose Nc, Fig. 478*a* (the normal constituent of the resultant of the weight of the wall for 1 foot in length and the pressure of the earth upon it) to be equal to 2 tons.

Assume foundations such as those shown in Fig. 477, the centre of pressure c being at $\frac{1}{3}$ the width cf of the foundation from the outer edge, then the maximum intensity of pressure will be at e (see p. 219), and it will be equal to $2 \times \frac{2}{6} = \frac{2}{3}$ tons per square foot.

It is evident, therefore, that on firm earth, clay, or harder soils the maximum intensity of pressure would not be too great.

If the soil were very loose and unfit to bear weight then the concrete

foundation would have to be extended so that the maximum pressure upon it should not exceed what the soil could bear, and in extreme cases the foundation would have to be prepared by driving piles.

If it is very important for any reason that the pressure on the earth should be uniform, the toe of the *concrete must be extended so* that *c* should be the centre of the earth pressed upon.

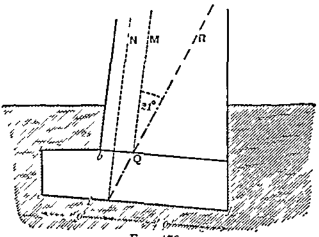


Fig 478*a*.

Concrete base.—Care must be taken that the concrete is made so thick that there is no danger of its breaking across.

In Fig. 478*a* *abde* is in the condition of a cantilever nearly uniformly loaded and tending to break off at *bd*.

Thus see Appendix VII.

$$\frac{wl}{2} = \frac{bd^2 \times f_o}{6}$$

taking $f_o = 100$ lbs. per square inch,[1]

$$wl = \frac{2 \times 12 (\times 18)^2 \times 100}{6 \times 24},$$

$$= 5400 \text{ lbs.}$$

$$= 2\tfrac{1}{2} \text{ tons nearly.}$$

Therefore the cantilever will bear $2\frac{1}{2}$ tons distributed or $1\frac{1}{4}$ ton per foot superficial without breaking.

Whereas the most intense pressure at the end is only $\frac{2}{3}$ ton per foot (*giving a factor of safety of* $\frac{2\frac{1}{2}}{\frac{2}{3}}$ = *nearly* 4), *and the mean pressure distributed* only $\frac{7}{12}$ ton.

The concrete therefore is amply thick enough. If made 12″ thick it would bear about $\frac{1}{2}$ ton per foot-superficial.

Sliding on concrete.—To prevent danger of the wall sliding forward on its concrete base, the tangent of the angle MQR (between the normal MQ to the base and the resultant pressure RQ) must not be greater than $\frac{4}{5} \times$ the coefficient of friction of brickwork with damp mortar, Table XV*a*, *i.e.* not greater than $\frac{4}{5} \times \cdot 74 = \cdot 59$.

The angle RQM measures 21°, the tangent of which is ·38, so that the condition is amply fulfilled and the wall quite safe against sliding.

Sliding on clay.—In the same way by using the proper coefficients from Table XV*a* it will be found that the wall would slide on wet clay but would be safe against sliding on dry clay.

[1] Baker.

CHAPTER XV.

ARCHES.

IN Parts I. and II. some preliminary remarks were made in connection with arches; the names of the different parts were given, and also some practical details as to the construction of brick arches.

It is proposed to show in this chapter how to ascertain the thickness of the arch ring in order that the arch may safely resist the load it has to bear.

The calculations in connection with the stability of the arch ring are amongst the most difficult that occur in engineering, but there is a simple graphic method by means of which the desired result may be obtained by a series of approximations, and it is this method which will be described.

It should, however, be observed that in the large majority of cases the arches used in building construction have but a small span, and are, moreover, only repetitions of what have been built before, so that there is ample experience to fall back upon. There is therefore no necessity, in such cases, to make any calculations, and the dimensions can be obtained by referring to a table such as that given in Molesworth's *Pocket-Book*,[1] or Table XVII*a*.

Graphic Method of determining the Stability of an Arch.

Let I, J, K, L be any voussoir[2] of an arch. The load on this voussoir consists of its own weight and of the portion of the weight carried by the arch included between the dotted lines (Fig. 479). Let w be the resultant of both these weights acting through their common C.G. The voussoir is also subjected to the pressure P of the voussoir above it, and to the reaction R of the voussoir below it. These are the only three forces acting on the voussoir, and if we know w and P we can find R and also its

[1] Page 108, 23d edition.

[2] The student is referred to Parts I. and II. for the meaning of the various technical terms employed.

point of application C, which is the centre of pressure for the joint I, J Now we know (see p 221) that for stability the point C must lie inside I, J, and that, moreover, to prevent crushing the material of the voussoir, C must not approach I or J nearer than a certain distance, depending on the value of R, and on the resistance to crushing of the material On this point Dr Scheffler has laid down a general rule that the point C should lie within the *middle half* of the joint but other authorities recommend that C should lie within the *inner third of the joint*[1] *in order that no tension be excited* (see p 221) Dr Scheffler's assumption is amply safe for all ordinary *cases and will therefore be accepted for these Notes*

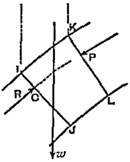


Fig 479

Now, for purposes of calculation, the arch can be considered as divided up into a number of *imaginary* voussoirs as shown in Fig 480, and we can suppose that the centre of pressure at each

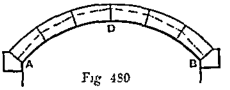


Fig 480

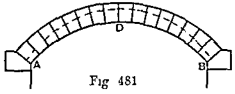


Fig 481

joint has been found, and that these centres of pressure are then joined together by a broken line If the arch had been divided up into a greater number of imaginary voussoirs, as in Fig 481, the line joining the centres of pressure would be almost a continuous curve, and if a still greater number of voussoirs had been assumed, the line would hardly be distinguishable from a curve This line is called the *line of resistance* and from it we can obtain the point of application (centre of pressure) and direction of the pressure at any section of the arch

Applying therefore, the results arrived at in connection with one voussoir, we see that for safety the line of resistance must lie within the middle half of the arch ring as shown in Fig 485

For mere stability, if the material of the arch ring were uncrushable the line of resistance might just touch the outside of the arch ring

The line of resistance also gives the direction of the resultant

[1] For important arches the line of resistance should always be kept within the middle *third* of the arch ring as in the Example Appendix XX

pressure on any joint of the arch ring, this direction being a tangent to the curve; and should this direction be too much inclined to the joint the arch would fail by sliding.[1] As a matter of fact, however, the inclination seldom, probably never, is sufficient to produce failure in this manner, except in straight arches (see Fig. 14, Part I.)

To find the line of resistance.—The problem therefore is to find the line of resistance, and herein lies all the difficulty.

On looking at Fig. 481 it will be seen that the line of resistance is horizontal at the point D; at this point therefore the pressure is horizontal.

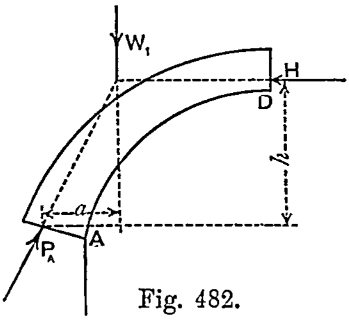


Fig. 482.

Suppose the portion DB of the arch is removed (Fig. 482), then we have the left-hand portion of the arch kept in equilibrium by—

(1) The portion of the load it has to support (W_1).

(2) The horizontal pressure H at D.

(3) The pressure P_A at the abutment A.

Now, a force is determined when we know (1) its direction, (2) its point of application, and (3) its magnitude. W_1 is, or can be, so determined. Of H we only know the direction; and as to P_A we know nothing except that it is equal and opposite to the resultant of W_1 and H. We therefore cannot solve the problem until we know the point of application and the magnitude of H. To obtain these we must resort to what is known as Moseley's *Principle of Least Resistance.*

Professor Rankine states this principle as follows :—

"If the forces, which balance each other in or upon a given body or structure, be distinguished into two systems, called respectively *active* and *passive*, which stand to each other in the relation of cause and effect, then will the passive forces be the *least* which are capable of balancing the active forces, consistently with the physical condition of the body or structure."

In the case under consideration W_1 represents the *active* forces and H and P_A the *passive* forces.

It therefore follows that, in the actual arch, the magnitude of H will be the least possible consistently with the physical condition of the arch. And it will also be seen that to each value of H there is a corresponding line of resistance.

Combining this with the condition relative to the line of resistance (p. 247) we arrive at the following result.

The value of H will be such that the corresponding line of resistance is just included *within* the inner half of the arch ring.

This line of resistance is called the *line of least resistance.*

Theory of graphic method.—To determine the true line of least resistance mathematically is most difficult, even in simple cases; but by the graphic method of trial and error already referred to, the line of least resistance can be found very expeditiously—only

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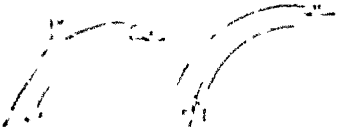
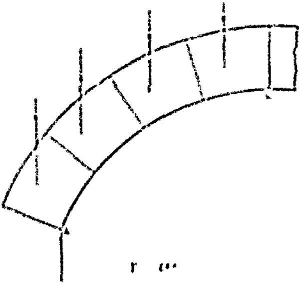
[1] "To insure stability of friction the normal to each joint must not make an angle greater than the angle of repose with a tangent to the line of pressures drawn through the centre of resistance of that joint" (Rankine, *Civil Engineering*).





of resistance to pass below K. H_1 is therefore the least admissible value of H, and the corresponding line of resistance is the line of least resistance.

Clearly, therefore, the next step is to establish a method of finding the line of resistance corresponding to any assumed value of H.

Suppose that the portion AD of the arch ring is divided up into a number of imaginary voussoirs, as shown in Fig. 486, and let the load on each of these voussoirs be found, namely w_1, w_2, w_3, etc.

Now the first voussoir is kept in equilibrium by three forces, as shown in Fig. 487. We know H and w_1, but P_1 has to be found. To do this, draw ab (Fig. 488) to represent H, and aa_1 to represent w_1; then (by the triangle of forces, p. 179) a_1b will represent P_1. We therefore have only to draw P_1 parallel to a_1b through the intersection of H and w_1 to find c_1.

Proceeding to the second voussoir we have forces as shown in Fig. 489; and

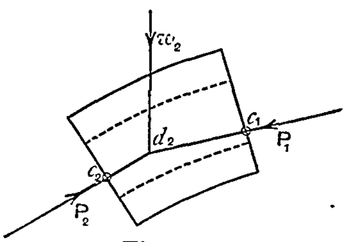


Fig. 489.

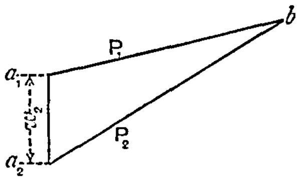


Fig. 490.

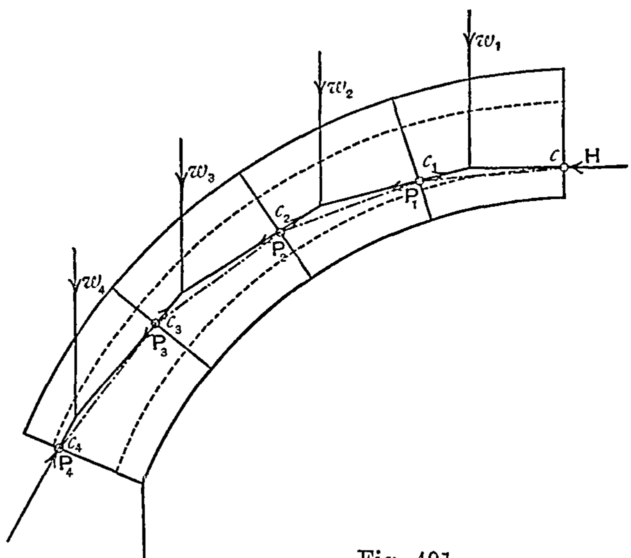


Fig. 491.

Fig. 490 gives the triangle of forces. c_2 therefore can be found as before. The same process can be repeated for each succeeding voussoir, and by this means the various centres of pressure can be found, and by joining them the line of resistance can at once be obtained (Fig. 491).

It will be observed that Figs. 488 and 490 can be combined to form one

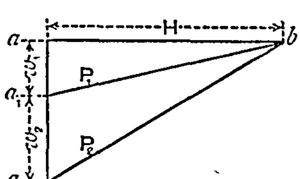


Fig. 492.

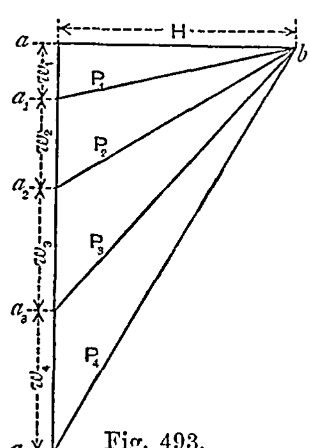


Fig. 493.

figure, as shown in Fig 492, and similarly the triangles of forces for all the succeeding joints can be combined into one figure (Fig 493), thus saving a considerable amount of drawing

It will also be observed that the broken line formed by the forces H, P_1, P_2, etc (Fig 491), approximates to the line of resistance, and if double the number of voussoirs had been taken, the approximation would be still closer, as shown in Figs. 494 and 495 With a very large number of voussoirs, the line of resistance and the broken line formed by the direction of the pressures at the joints practically coincide

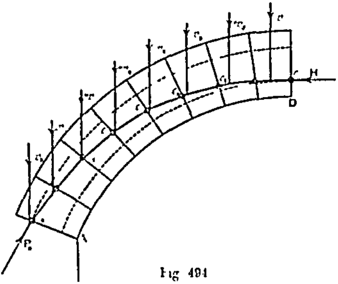


Fig 494

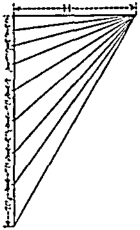


Fig 495

Practical Graphic Method of finding the Line of Resistance in an Arch with a Symmetrical Load.—The line of resistance corresponding to any particular value of H can therefore be found by the following very simple graphic method.

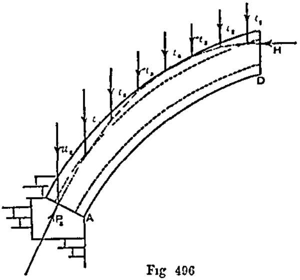


Fig 496

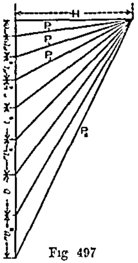


Fig 497

Trial line—Find the loads w_1, w_2, w_3 etc, acting on the arch

(Fig. 496), and draw the diagram of forces, Fig. 497, thus obtaining the forces P_1, P_2, P_3, P_4, etc. Then find the intersection of H and w_1, then the intersection of P_1 and w_2, and so on. Join these points of intersection, and the broken line so obtained will be approximately the line of resistance required.

It will be observed that this line of resistance (Fig. 496) is not included within the inner half of the arch ring. The reason is that the point of application of H has been chosen too high.

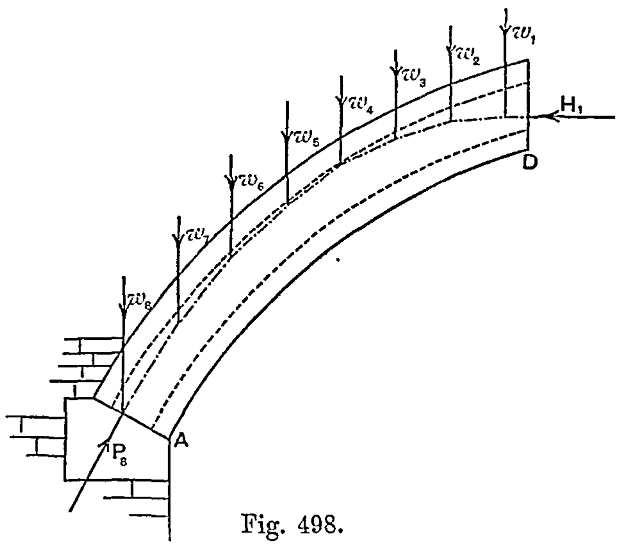


Fig. 498.

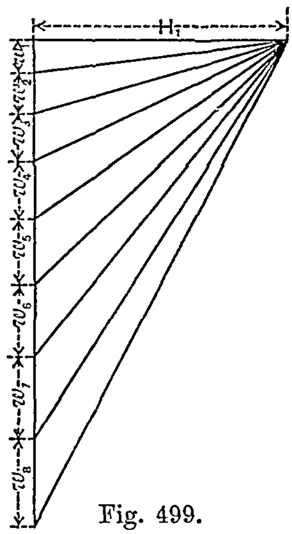


Fig. 499.

True line.—Lowering the point of application, which will also increase H to H_1 (Fig. 498), we obtain a new diagram of forces (Fig. 499) and a new line of resistance, which is now found to *just* fit in within the inner half of the arch ring, and is therefore the line of least resistance required.

To find where the line of resistance is horizontal.—On referring to p. 248 it will be seen that the section D was taken through the point where the line of resistance is horizontal, and we must now show how this point can be obtained.

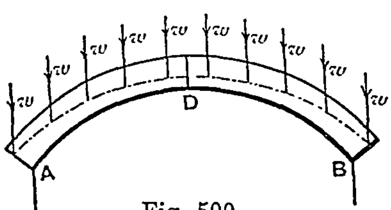


Fig. 500.

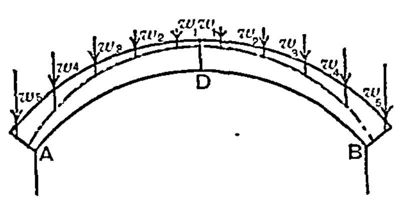


Fig. 501.

Taking the simplest possible case, when the load is uniformly distributed, as in Fig. 500, it is clear from symmetry that D will be situated at the centre of the arch ring.

Or, again, if the load is symmetrically distributed D will still be at the centre of the arch ring (Fig 501).

In both these cases the line of least resistance will be symmetrical about the point D.

Now let us investigate the effect a concentrated load,[1] placed at any point on the arch ring, has on the position of the point D, as shown in Fig 502. Let $\overline{W}$ be this concentrated load, and let

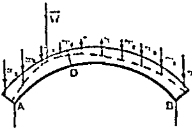


Fig 502

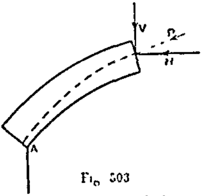


Fig 503

it be applied at the point P. If we take *any* section of the arch, as in Fig 503, the thrust at the section can be represented by the horizontal thrust H and a vertical force V What we want to find is the section at which V is nothing If we regard V in the light of a shearing stress we see on referring to p 56 that since, as a consequence of Rule 2, the shearing stress is nothing under the resultant of all the loads, therefore the section of the arch we require will be under the resultant of $\overline{W}$ and w_4, w_5 w_3, w_4

In any case, therefore, the point D can be found by determining the resultant load on the arch, and this can be done very simply, by finding the centre of gravity of all the loads, as explained in Appendix VI.

Thickness of the arch ring—The thickness of the arch ring should be sufficient to allow of the intensity of pressure being not greater than what the material is able to bear safely Molesworth gives a rule, based on Rankine's, for finding the thickness of the arch at the crown, or, in other words, the depth of the keystone It is as follows:—

$$\text{Depth of keystone} = n\sqrt{\text{radius at crown}} \qquad \textbf{(103)},$$

where $n = 0{\cdot}3$ for blockstone,
$= 0{\cdot}4$ for brickwork,
$= 0{\cdot}45$ for rubble stonework

The depth thus found is expressed in feet.

[1] A different method of treating an arch with an unsymmetrical load will be found in Appendix XX

The actual intensity of pressure at any section of the arch ring can, however, be found, as soon as the line of least resistance is known, by applying Equation 95.

Semicircular Brick Arch.

Example 51.—Find the line of least resistance in the symmetrically loaded semicircular brick arch shown in Fig. 504 when loaded with a uniform load of 1 ton per foot run, including its own weight.

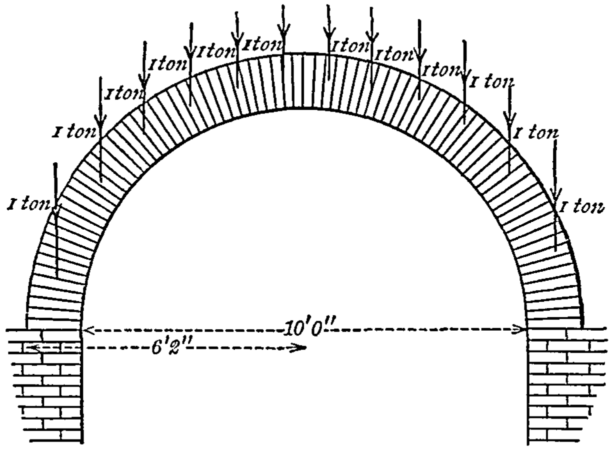


Fig. 504.

Preliminaries.—It will be found, by applying Equation 103, that the thickness of the arch ring at the crown is

$$= 0{\cdot}4\sqrt{5} = 0{\cdot}9 \text{ foot nearly.}$$

14″ is therefore the nearest brick dimension, and we will call this 1·2 foot.

The arch being uniformly loaded, we need only consider one half of the arch, and the thrust at the crown will be horizontal, as shown in Fig. 505.

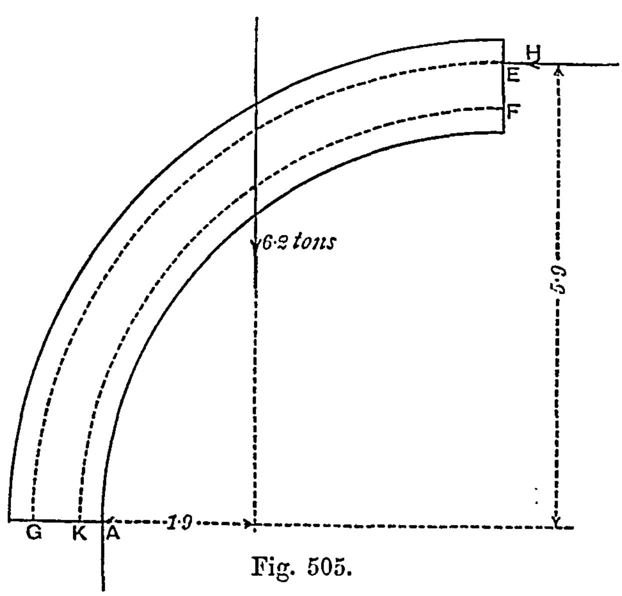


Fig. 505.

The total load on the half arch is 6·2 tons, and its direction is at a distance

of 1·9 foot from the point A. Now, as seen at p 249, H will be a minimum when applied at the point E, and its lever arm will then be

$$5 + (\tfrac{3}{4} \times 1{\cdot}2) = 5{\cdot}9 \text{ feet.}$$

The lever arm of the load is

$$1{\cdot}9 + (\tfrac{1}{4} \times 1{\cdot}2) = 2{\cdot}2 \text{ feet}$$

Hence

$$H_{(\text{min.})} = \frac{2{\cdot}2}{5{\cdot}9} \times 6{\cdot}2,$$
$$= 2{\cdot}3 \text{ tons}$$

Again, H will be a maximum when applied at F, and when moments are taken about G In this case the lever arm of H is

$$5 + (\tfrac{1}{4} \times 1{\cdot}2) = 5{\cdot}3 \text{ feet,}$$

and of the load

$$1{\cdot}9 + (\tfrac{3}{4} \times 1{\cdot}2) = 2{\cdot}8 \text{ feet.}$$

Hence

$$H_{(\text{max.})} = \frac{2{\cdot}8}{5{\cdot}3} \times 6{\cdot}2,$$
$$= 3{\cdot}26 \text{ tons.}$$

The true value of H, giving the line of *least* resistance, must lie between these two values, provided of course the arch ring is able to withstand the load put upon it.

As a first trial draw a line of resistance through E. Following the process explained at p 251 we obtain the line of resistance given in Fig 506,

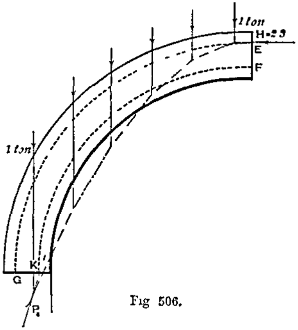


Fig 506.

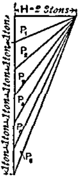


Fig 507

that is, draw H = 2·3 tons *horizontally* through E to intersect the first load of 1 ton, from this intersection draw a line parallel to P_1 (Fig 507) to meet the second load of 1 ton, then from this intersection a line parallel to P_2 to meet the third load, and so on until a line parallel to P_5 is drawn to meet the last load From this point of intersection a line parallel to P_6 is drawn and gives the direction of the pressure at the springing, P_6 in Fig 507 being

the magnitude. This line of resistance passes below the soffit of the arch, and therefore the value of H must be increased. It should be observed that the direction of P_6 passes through K, the point about which moments were taken, and this forms a valuable check on the accuracy of the drawing.

As a second trial increase H to $H_{(max.)}$, namely, 3·26 tons. The point of application of H will in this case be F, and the line of resistance shown in Fig. 508 will be obtained, which also cuts through the arch ring at the soffit.

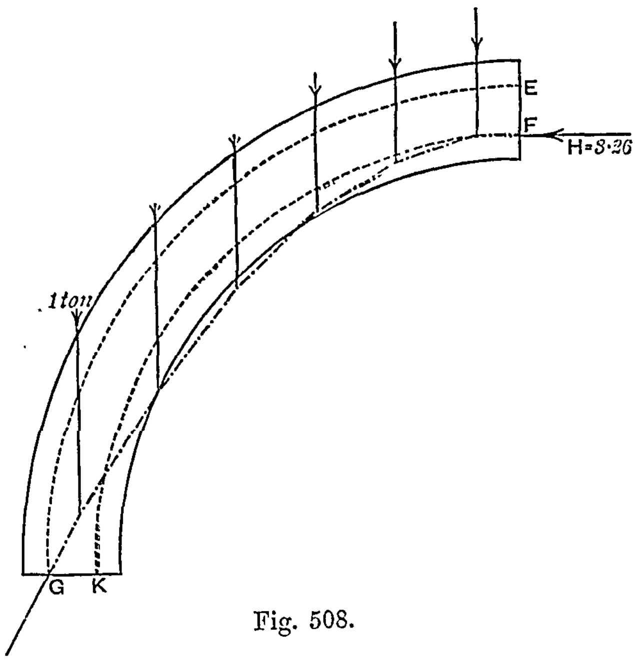


Fig. 508.

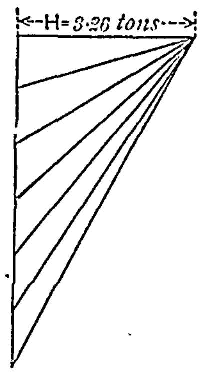


Fig. 509.

As a last trial apply H at E and take moments about G, so that the line of resistance will pass through G. We find

$$H=\frac{2\cdot 8}{5\cdot 9}\times 6\cdot 2=2\cdot 95 \text{ tons},$$

and the corresponding line of resistance shown in Fig. 510 is obtained. This line of resistance is within the arch ring at all points, but at the haunches it is outside the inner half of the arch ring, and therefore does not comply with Dr. Scheffler's rule.

To raise the line of resistance at the haunches so that it will be within the inner half of the arch ring, we can either *increase* the load above the haunches or increase the thickness of the arch ring at the springing. The first alternative is left as an exercise for the student.

Thickening arch.—Supposing that the thickness of the arch ring is increased to 1′ 6″ at the springing, as shown in Fig. 511, the point G is now at a distance of $\frac{3}{4}\times 1\cdot 5=1\cdot 13$ from the inner point of the springing, and the lever arm of the load becomes $1\cdot 9\times 1\cdot 13=3\cdot 0$ nearly. Hence, if H is applied at E,

$$H=\frac{3\cdot 0}{5\cdot 9}\times 6\cdot 2=3\cdot 15 \text{ tons, and the}$$

True line of resistance shown in Fig. 511 is obtained. This line of resistance is practically everywhere included within the inner half of the arch ring, and is therefore the line of least resistance required. Practically, however, a brick

arch would not be built of varying depth owing to the expense of cutting the bricks, and the arch ring would therefore be made 1' 6" deep throughout. This expedient could, however, be adopted if the arch were built in masonry.

Thickness to resist crushing.—The thickness of the arch ring must be sufficient to prevent the intensity of pressure being greater than the pressure the material can safely bear. Now on inspection it would appear that the greatest intensity of pressure occurs either at the point M (Fig. 513) or at

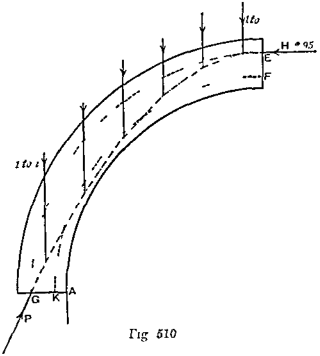


Fig. 510

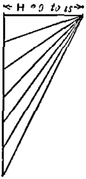


Fig. 510a

the abutment, and to find the required thickness it will be necessary to ascertain at which of these points the intensity of pressure is greatest. At the point M the line of least resistance is practically perpendicular to the joint between the voussoirs, and on referring to Fig. 512 it will be found that the pressure at the point M is 5.1 tons. We must now employ Equation 95, p. 219, and we can write

$$Y = 5.1 \text{ tons}$$

$$GC = \tfrac{14}{4} \text{ inches}$$

$$AE = b \text{ (the width of the arch ring)}$$

Hence
$$p_{\text{max.}} = \frac{2 \times 5.1 \times 4}{3 \times 14 \times b} = \frac{0.97}{b},$$

or
$$b = \frac{0.97}{p_{\text{max.}}}$$

Now proceeding to the abutment we find from Fig. 512 that the pressure is 6.8 tons, but this pressure is inclined to the joint, and we must resolve perpendicularly to the joint. We thus find (Fig. 514)

$$Y = 6 \text{ tons},$$

also
$$GC = \tfrac{18}{4} \text{ inches}$$

Hence
$$p_{(\text{max.})} = \frac{2 \times 6 \times 4}{3 \times 18 \times b_1} = \frac{0.89}{b_1},$$

or
$$b_1 = \frac{0\cdot 89}{p_{(\text{max.})}}.$$

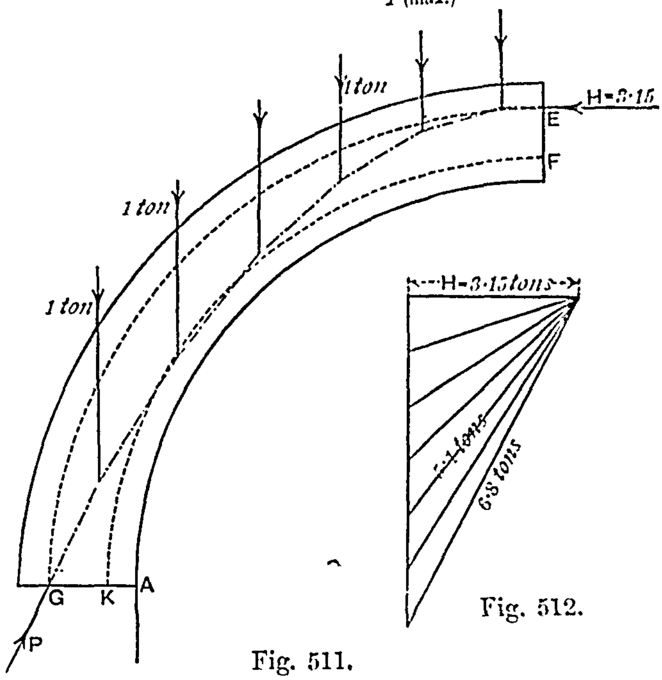


Fig. 511.

Fig. 512.

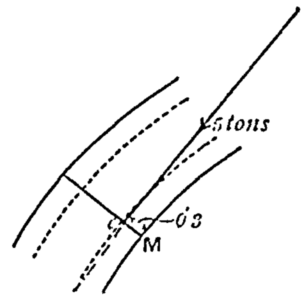


Fig. 513.

Therefore b is greater than b_1, or, in other words, the maximum intensity of pressure is greater at M than at the abutment.

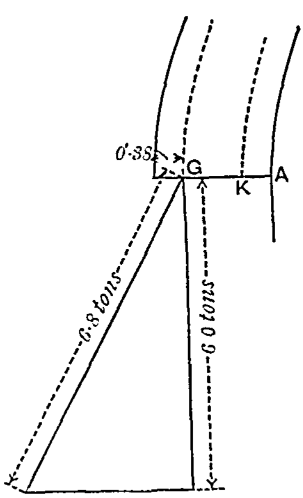


Fig. 514.

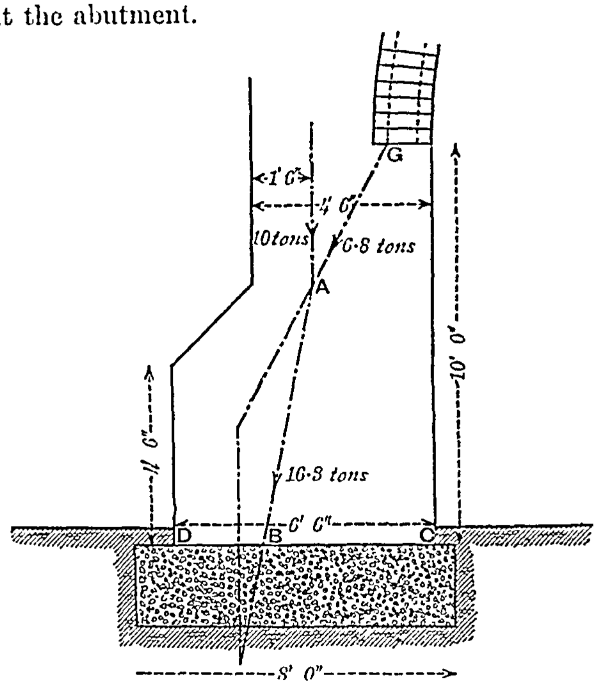


Fig. 515.

If the arch is built in ordinary brickwork in mortar, p can be taken at about 0·5 cwt. per square inch. Hence
$$b = \frac{0\cdot 97 \times 20}{0\cdot 5} = 38 \text{ inches.}$$

But if the arch were built in cement p can be taken at 0.8 cwts per square inch and then

$$b = \frac{0.97 \times 20}{0.8} = 24 \text{ inches.}$$

Arch with unsymmetrical Load (See Appendix XX.)

Abutment for Arch

Example 51a.—As an example it will be assumed that the arch is situated at one end of the wall of a building as shown in Fig. 515. The forces acting on the abutment are the thrust of the arch, namely, 6.8 tons as found above and the weight of the brick work in the wall immediately over the wall including the weight of the abutment itself. It will be assumed that this weight is 10 tons acting as shown in Fig. 515. These two forces intersect at A and completing the triangle of forces we obtain B as the centre of pressure at the joint DC. In order that no tension be excited at C we must make $DB = \frac{1}{3}BC$. It must also be ascertained that the intensity of pressure is not too great. *The vertical pressure on the joint DC* is 16 tons. Hence taking the width of the arch as 23″ we have from (95)

$$p_{(\max.)} = 2 \times \frac{16 \times 20}{6'6 \times 23},$$

$$= 0.36 \text{ cwt. per square inch nearly.}$$

It will be seen from Table IX that 0.5 cwt. is safe.

The maximum pressure at the joint at A will, however, be somewhat greater, the vertical pressure will be reduced by about 1 ton (viz. the weight of the abutment below A), and as A is (according to the dimensions taken) at $\frac{1}{3}$ the width of the joint from the outer edge

$$p_{(\max.)} = 2 \times \frac{15 \times 20}{4'6 \times 23}$$

$$= 0.48 \text{ cwt. per square inch nearly.}$$

Resistance to sliding can be found as before.

The concrete foundations to the abutment can be designed on the same principles as above, taking care that the maximum intensity of pressure is not greater than the soil can withstand with safety.

Table of thickness of arches.—The arches used in building construction are not generally of very great span and they do not as a rule require to be calculated. Their thickness is often governed by appearance or it may be found in Tables founded upon experience. One of these is given at p. 343, and others in Molesworth's and Hurst's *Pocket Books*.

Chapter XVI.

HYDRAULICS[1]

(*As applied to Building Construction*).

HYDRAULICS is a subject which is related to building construction only to a limited extent, and in fact only two subdivisions of the subject need be considered, namely—

1. The motion of liquids through pipes in connection with water supply and disposal of sewage.

2. The delivery of water from a jet in connection with standpipes.

We must first consider and define some of the terms in use.

Pressure.—If we imagine a small cube immersed in a liquid as shown in Fig. 516, the pressure exerted by the liquid on each of the six faces of the cube will be normal to that face, and if the cube is supposed to be very small indeed, all these pressures will be almost equal. If the cube be now supposed to diminish without limit it will become a point; the pressures will become exactly equal, and will then be the *pressure at a point*, as at P, for instance, in Fig. 517. It is shown in works on Hydrostatics that the pressure at a point in a liquid is proportional to the depth below the surface of the liquid, and in fact that the intensity pressure, that is, the pressure per unit of area (a square inch, for instance), is equal to the weight of a column of liquid of a height equal to the depth below the surface and one unit of area in cross section. Thus if D is 50 feet and the liquid is water, a cubic foot of which weighs almost exactly 62·4 lbs.

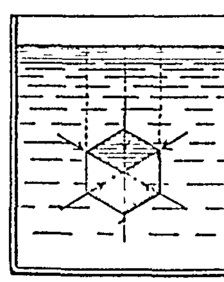


Fig. 516.

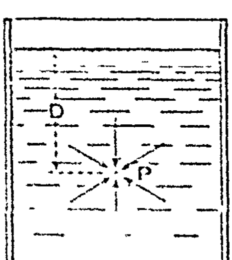


Fig. 517.

[1] For *Practical Formulæ*, see pp. 267, 281, and App. XXI.

at a temperature of 52°·3 Fahr the pressure on a square inch at the point P would be

$$50 \times \frac{1}{144} \times 62 \cdot 4 = 21 \cdot 67 \text{ lbs}$$

The *intensity of pressure at the point* P is therefore 21·67 lbs or as it is usually expressed 21·67 lbs per square inch

Head of pressure.—The depth of the point P (Fig 517) is also called the head of pressure at the point P or simply the head and is generally expressed in feet

Head of elevation.—The height of the point P (Fig 517) above some datum level is called the head of elevation of the point P

Loss of head.—When a liquid is in motion each particle is constantly moving from a place of greater head to a place of lesser head and the difference between the two heads is called the loss of head This loss of head may be entirely a loss of head of pressure or entirely a loss of head of elevation or again partly a loss of head of pressure and the remainder a loss of head of elevation

The following examples will assist to illustrate the above—

AB (Fig 518) is a pipe connected to a tank T at A there is

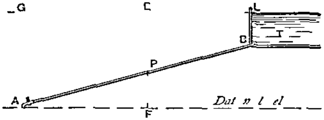


Fig 518

a tap When the tap A is closed the head at the point P will be

Head of pressure EP

Head of elevation FP

At A the head of pressure is GA and there is no head of elevation with reference to the datum chosen

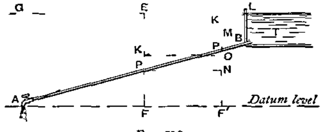


Fig 519

When the tap is opened however the pressure at A is reduced

to zero, and if the resistance of the pipe to the flow of the water is uniform, the pressure at each point of the pipe can be represented by the straight line AL, as shown in Fig. 519. Therefore the head at P will now be:

Head of pressure, KP;
Head of elevation, FP.

It will be observed that the head of elevation is not altered, but that the head of pressure is reduced by the amount KE. With reference to a second point P′, the head of pressure is K′P′, and the head of elevation F′P′; but the "loss of head" between the two points is K′O. This loss of head is made up of K′M, the loss of the head of pressure (KM is drawn parallel to PP′), and of MO, equal to P′N, which is the loss of the head of elevation

If the pipe is horizontal, as in Fig. 520, it is clear that the

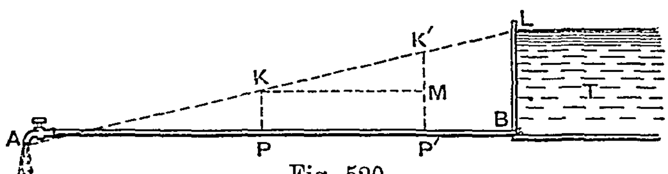


Fig. 520.

loss of head between the two points P and P′ is entirely loss of head of pressure, and there is no loss of head of elevation.

On the other hand, if water is flowing in an open channel,

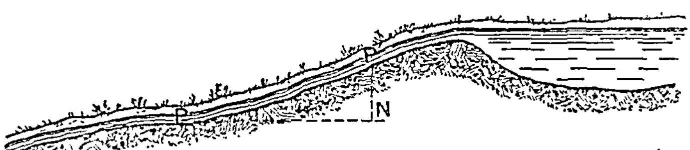


Fig. 521.

as in Fig. 521, the loss of head is entirely loss of head of elevation; in fact, the water is not flowing under pressure.

Another way of looking at the matter is as follows:—The water, when flowing in the pipe or in the channel, has to overcome the resistance of the pipe or of the channel, which means that the water has to do a certain amount of *work*, and this *work* is proportional to the loss of head. Supposing, for instance, that 1 lb. of water, in flowing through a pipe, loses 10 feet of head, then the work done by that pound of water is 10 foot-lbs.

Wetted perimeter.—In an open channel, or in the case of a pipe not flowing full, the portion of the cross section of the channel, or of the pipe, wetted by the liquid (from E to F, Figs. 522 and 523) is called the wetted perimeter. It is also called the "*border.*"

Hydraulic mean depth.—The quotient of the area of the

cross section of the liquid divided by the wetted perimeter is called the hydraulic mean depth and will be denoted by R.

Thus in Figs. 522 and 523, $\dfrac{\text{The area of EBF}}{\text{The length of EBF}} = R$

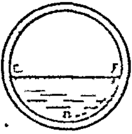


Fig. 522

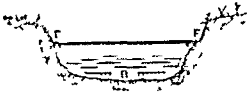


Fig. 523

MOTION OF LIQUIDS IN PIPES

The motion of a liquid in a pipe is due to gravity and if the pipe were perfectly smooth and offered no resistance, the velocity would increase without limit. But as already mentioned pipes do offer a resistance depending on their diameter and on the state of their inner surfaces so that the velocity, instead of increasing continually, is limited. Moreover the velocity of the liquid is not uniform throughout the cross section of the pipe, but is greatest in the centre diminishing gradually at first and then rapidly towards the sides. Close to the sides the liquid does not flow freely, but is disturbed by numerous small eddies. It was at one time thought that neither the diameter nor the state of the surfaces had any influence on the resistance, and to account for this it was supposed that these eddies formed, as it were, a liquid lining to the pipe.

Darcy has, however, shown by means of an elaborate series of experiments that this view, due to De Prony, is erroneous and has also shown that the mistake arose from combining experiments on pipes of large diameter but with rough surfaces, and experiments on smooth pipes but of small diameter, and that the resistance offered by the rough surface in the one set happened to exactly equal the resistance due to the small diameter in the other set, so that it appeared that neither the state of the surface nor the diameter had any influence on the resistance.

Although this supposition is now known to be erroneous yet in many of the formulae in general use for obtaining the discharge of pipes, the effect of the diameter on the resistance is not taken into account, nor is any notice taken of the degree of roughness of the surface; practically this latter is of little moment, for whatever the surface may have been originally it will speedily be covered with sediment. These formulae are empirical and are

based on numerous experiments that have been made on the flow of liquids in pipes. It should, however, be observed that these experiments, made at various times by different observers, are neither numerous enough nor were they sufficiently systematically carried out to obtain results of scientific accuracy, although they may be good enough for practical purposes.

We have to consider two cases of the flow of liquids through pipes, namely—

1. When the pipe is flowing full and the liquid is therefore impelled to move by the pressure of the head of liquid.

2. When the pipe is flowing partially full. In this case the liquid is not under pressure, and simply flows down, owing to the slope of the pipe, as it would in an open channel.

We will consider each of these cases separately.

DISCHARGE FROM A PIPE FLOWING FULL UNDER PRESSURE.

As already mentioned, numerous formulæ have been proposed to find the discharge from a pipe, but we will only consider four and then select for future use the one best adapted for our purpose.

One of these, known as *Eytelwein's formula*, is

$$V = 108\sqrt{\frac{D}{4} \times \frac{H}{L} - 0{\cdot}13} \qquad (104),$$

where

$$V=\frac{(\frac{1}{4}\times\frac{1}{12})^{0\cdot657}\times(\frac{1}{100})^{\frac{1}{1\cdot80}}}{0\cdot004787},$$

$$=\frac{0\cdot0786}{0\cdot004787\times1\cdot29},$$

$$=1\cdot27 \text{ feet per second.}$$

Example 53.—Find the velocity in a pipe 1 inch in diameter, 100 feet long, the effective head being 10 feet.

By Equation 104,

$$V=108\sqrt{\tfrac{1}{4}\times\tfrac{1}{12}\times\tfrac{10}{100}}-0\cdot13,$$

$$=4\cdot8 \text{ feet per second.}$$

By Equation 105,

$$V=140\sqrt{\tfrac{1}{4}\times\tfrac{1}{12}\times\tfrac{10}{100}}-11\sqrt[3]{\tfrac{1}{4}\times\tfrac{1}{12}\times\tfrac{10}{100}},$$

$$=4\cdot99 \text{ feet per second.}$$

By Equation 110,

$$V=80\sqrt{\tfrac{1}{4}\times\tfrac{1}{12}\times\tfrac{10}{100}},$$

$$=3\cdot65 \text{ feet per second.}$$

By Thrupp's formula (Equation 108),

$$V=\frac{(\frac{1}{4}\times\frac{1}{12})^{0\cdot657}\times(\frac{10}{100})^{\frac{1}{1\cdot80}}}{0\cdot004787},$$

$$=\frac{0\cdot0786}{0\cdot004787\times3\cdot594},$$

$$=4\cdot59 \text{ feet per second.}$$

Example 54.—Find the velocity in a pipe 4 inches in diameter, 100 feet long, the effective head being 1 foot.

By Equation 104,

$$V=108\sqrt{\tfrac{1}{4}\times\tfrac{4}{12}\times\tfrac{1}{100}}-0\cdot13,$$

$$=2\cdot99 \text{ feet per second.}$$

By Equation 105,

$$V=140\sqrt{\tfrac{1}{4}\times\tfrac{4}{12}\times\tfrac{1}{100}}-11\sqrt[3]{\tfrac{1}{4}\times\tfrac{4}{12}\times\tfrac{1}{100}},$$

$$=3\cdot01 \text{ feet per second.}$$

By Equation 110, taking $C=102$,

$$V=102\sqrt{\tfrac{1}{4}\times\tfrac{4}{12}\times\tfrac{1}{100}},$$

$$=2\cdot94 \text{ feet per second.}$$

By Thrupp's formula (for cast iron, Equation 109),

$$V=\frac{(\frac{1}{4}\times\frac{4}{12})^{0\cdot63}}{0\cdot006752}\times(\tfrac{1}{100})^{\frac{1}{2}},$$

$$=\frac{0\cdot209}{0\cdot006752\times10},$$

$$=3\cdot09 \text{ feet per second.}$$

Example 55.—Find the velocity in a pipe 4 inches in diameter, 100 feet long, the available head being 10 feet.

By Equation 104,

$$V=108\sqrt{\tfrac{1}{4}\times\tfrac{4}{12}\times\tfrac{10}{100}}-0\cdot13,$$

$$=9\cdot73 \text{ feet per second.}$$

By Equation 105,

$$V = 140\sqrt{\tfrac{1}{4} \times \tfrac{4}{12} \times \tfrac{10}{100}} - 11\sqrt[3]{\tfrac{1}{4} \times \tfrac{4}{12} \times \tfrac{10}{100}},$$
$$= 10.53 \text{ feet per second}$$

By Equation 110,

$$V = 102\sqrt{\tfrac{1}{4} \times \tfrac{4}{12} \times \tfrac{10}{100}},$$
$$= 9.31 \text{ feet per second}$$

By Thrupp's formula (for cast iron, equation 109),

$$V = \frac{(\tfrac{1}{4} \times \tfrac{4}{12})^{0.63}}{0.006752} \times (\tfrac{10}{100})^{\frac{1}{2}}$$
$$= \frac{0.209}{0.006752 \times 10^{\frac{1}{2}}},$$
$$= 9.79 \text{ feet per second}$$

Practical Formula (Darcy's).

It will be observed that in all cases Darcy's formula gives the lowest velocity, and practically it is better to under estimate the velocity than *to over-estimate it.* *It is also a simple formula* to use, so that on the whole it is to be preferred to Thrupp's in cases where great accuracy is not essential, and it will therefore be used to work out any further examples in this book.

The discharge of the pipe can of course easily be found as soon as the *mean* velocity is known, thus

$$\text{Discharge in cubic feet per second} = V \times \frac{\pi D^2}{4}$$

or $$F = 0.785D^2V \qquad \textbf{(111)}$$

$$\text{Discharge in gallons per minute} = V \times \frac{\pi D^2}{4} \times 6.25 \times 60,$$

or $$G = 294D^2V \qquad \textbf{(112)}$$

In practice, however, the question is generally to find what diameter of pipe is required for a given discharge, the effective head and length of pipe being known. Combining equations 110 and 112 together we get

$$G = 294D^2 \times C\sqrt{\frac{D}{4} \times \frac{H}{L}}$$

whence, by squaring and simplifying,

$$D = \frac{1}{7.37}\left(\frac{L}{H}\,\frac{G^2}{C^2}\right)^{\frac{1}{5}} \qquad \textbf{(113)}$$

D being expressed in *feet*

Or $$d = 1.63\left(\frac{L}{H}\,\frac{G^2}{C^2}\right)^{\frac{1}{5}} \qquad \textbf{(114)}$$

when d is expressed in inches

To obtain the fifth root of the expression in brackets, either logarithms can be used, or else a table of fifth roots, such as published in Molesworth's *Pocket-Book.*

Equation 114 can, however, be written

$$\frac{L}{H} \cdot G^2 = C^2\left(\frac{d}{1{\cdot}63}\right)^5 \quad . \quad . \quad (115).$$

Now $C^2\left(\frac{d}{1{\cdot}63}\right)^5$ depends only on d, or, in other words, for every value of d there is a corresponding value of the expression $\frac{L}{H} \cdot G^2$.

Table XVIII. gives the value of $\frac{L}{H} \cdot G^2$ for various sizes of pipes, and also shows the diameter of pipe that should be practically adopted to allow for the incrustation which takes place in water-supply pipes.

Table XIX. is a similar table for larger pipes, the discharge being reckoned in cubic feet per second, and is obtained from the formula

$$\frac{L}{H} \cdot F^2 = C^2 \cdot \frac{\pi^2}{64} \cdot \left(\frac{d}{12}\right)^5 \quad . \quad . \quad (116),$$

where F is the discharge in cubic feet per second.

These Tables simplify the calculations very considerably, as will be shown farther on.

Loss of Head.

On referring to p. 264 it will be seen that H is defined as being the effective head. In Fig. 524 A′G is the total head, and the

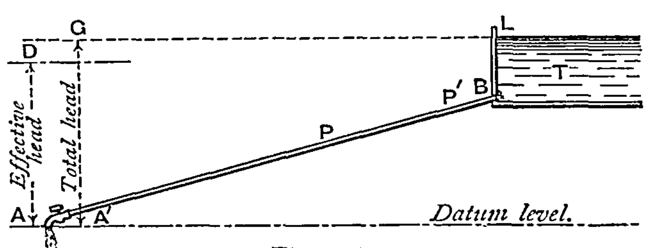


Fig. 524.

effective head AD is somewhat less, the loss being due to certain resistances which will be mentioned later on. It will also be seen that the effective head is equal to the loss of head between A and B, due to the resistance of the pipe; in fact, the flow of water in the pipe is such as to produce this equality.

Further, if we know the loss of head between two points P P in the pipe and the length of pipe between these two points we can find the velocity and the discharge.

The resistance of the pipe causes by far the greatest loss of head, but there are some minor losses produced as follows :—

Loss of head due to orifice of entry.—The orifice of entry obstructs to a certain extent the flow of water into the pipe causing therefore a loss of head. This loss depends on the form

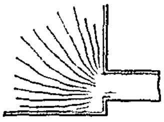


Fig. 525

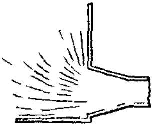


Fig. 526

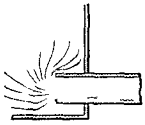


Fig. 527

of the orifice, as will readily be seen from Figs. 525, 526 and 527, and it can be found from the formula

$$H_o = V^2 \times C \qquad (117)$$

where H_o is the loss of head due to orifice.

$C = 0.007849$ for round orifices such as the end of the pipe Fig. 525.

$= 0.000114$ ditto when splayed or bell mouthed Fig. 526.

$= 0.014846$ when the pipe projects into the cistern (diameter uniform) Fig. 527.

Example 56.—Find the loss of head due to a round orifice projecting into a cistern when the mean velocity in the pipe is 2 feet per second.

From the above we have

$$H_o = 2^2 \times 0.014846 \text{ feet}$$
$$= 0.06 \text{ feet.}$$

Loss of head due to velocity.—The water in the reservoir or cistern is at rest, and a certain amount of head is lost in causing the water to take up the velocity in the pipe, or as it would be more scientifically expressed, a certain amount of energy of position (which is measured by the head) has to be converted into energy of motion. The energy of motion is *measured* by

$$\frac{V^2}{64.4}$$

Hence if H_v be the loss of head due to velocity

$$H_v = \frac{V^2}{64.4} = V^2 \times 0.0155 \qquad (118)$$

Example 57.—Find the loss of head due to a velocity of 2 feet per second.

From the above $H_v = 2^2 \times 0{\cdot}0155$ feet,
$= 0{\cdot}062$ feet.

Loss of head due to bends and elbows.—Every bend in a pipe causes a resistance and a consequent loss of head, and elbows do so to a still greater degree.

These losses can be found from the following formulæ, somewhat modified from those given in Molesworth's *Pocket-Book* and in Hurst's *Pocket-Book*.

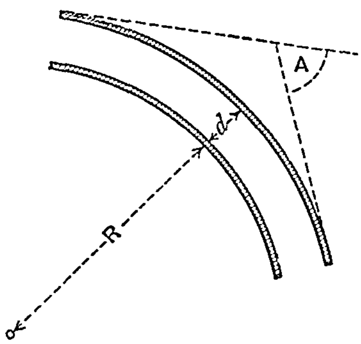


Fig. 528.

Calling the loss of head due to bends H_B we have

For bends $H_B = bAV^2$. **(119)**,

where b is a coefficient depending on the ratio, $\frac{R}{d}$, of the radius of the bend to the internal diameter of the pipe, and A is the change in direction caused by the bend measured in degrees (see Fig. 528). Table XXI. will be found to give values of b for various values of $\frac{R}{d}$.

Example 58.—Find the loss of head due to a bend of 30° in a pipe 2″ diameter, the radius of the bend being $3\frac{1}{2}$ inches, the velocity being 2 feet per second.

We have $\frac{R}{d} = \frac{3{\cdot}5}{2} = 1{\cdot}75$.

In Table XXI. we find for

$\frac{R}{d} = 1{\cdot}55$, $b = 0{\cdot}000015$,

and for $\frac{R}{d} = 2$, $b = 0{\cdot}000013$.

We may therefore take $b = 0{\cdot}000014$.

Hence $H_B = 0{\cdot}000014 \times 30 \times 2^2$,
$= 0{\cdot}0017$ foot.

For elbows $H_E = c.V^2$ **(120)**,

where c is a coefficient depending on the angle A of the elbow (see Fig. 529), and Table XXII. gives values of c for certain values of A.

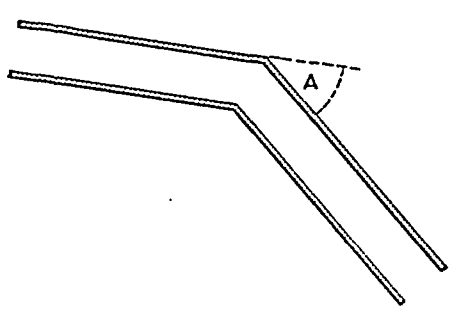


Fig. 529.

Example 59.—Find the loss of head due to an elbow of 30° when the velocity is 2 feet per second.

From Table XXII. we find $c = 0{\cdot}0011$.

Hence

$$H_e = 0.0011 \times 2^2,$$
$$= 0.0044 \text{ foot}$$

The above constitute all the minor losses of head, and as will be seen by the following example, these losses, unless the bends and elbows are very numerous, are small in comparison with the loss due to the resistance of the pipe, and can practically be neglected

Example 60—Water is flowing at the rate of 2 feet per second in a pipe 2" diameter, find the loss of head due to the resistance of the pipe in 100 feet length

From Equation 110,

$$V = 93\sqrt{\frac{D}{4}\,\frac{H}{L}},$$

and substituting,

$$2 = 93\sqrt{\tfrac{1}{4} \times \tfrac{2}{12} \times \frac{H}{100}},$$

squaring, etc,

$$H = \frac{4 \times 4 \times 12 \times 100}{93^2 \times 2},$$
$$= 1.11 \text{ feet}$$

It is thus clearly shown that the losses due to entry, velocity, etc., are in ordinary cases insignificant in comparison with the loss due to the resistance of the pipe Of course if there are a great number of bends and elbows it might be necessary to allow for the loss of head due to them

PRACTICAL EXAMPLES

We will now proceed to work out some practical examples

Water Supply to a House (Intermittent Service)

Example 61.—A house is supplied with water on the intermittent system 400 gallons is the quantity of water used per diem, and the water is *turned on for four hours every day, and flows into a supply cistern* The ball cock in the cistern is 30 feet above the main in the road, and the service pipe is 120 feet long The head in the main, at the junction with the service pipe, is 40 feet Find the diameter of the service pipe

At the tap the available head, *i e* the head of pressure less the head of *elevation, is*

$$40 - 30 = 10 \text{ feet},$$

and the required discharge into the cistern to fill it in four hours,

$$\frac{400}{4 \times 60} = 1.67 \text{ gallons per minute}$$

Diameter of pipe by Darcy's formula—Hence substituting in Equation 113, and taking $C = 65$ since the pipe will be a small one,

$$D = \frac{1}{7 \cdot 37} \left(\frac{120}{10} \cdot \frac{1 \cdot 67^2}{65^2} \right)^{\frac{1}{5}},$$

$$= \frac{1}{7 \cdot 37} \left(\frac{1}{126 \cdot 8} \right)^{\frac{1}{5}},$$

$$= \frac{1}{7 \cdot 37 \times 2 \cdot 63},$$

$$= 0 \cdot 0517 \text{ foot},$$

or $$d = 0 \cdot 62 \text{ inch}.$$

Strictly the value $C = 65$ is for a $\frac{1}{2}''$ pipe, and for a pipe 0·62 inch diameter $C = 70$ would be a more correct value. It will be found by repeating the above calculation with the new value of C that the diameter of the pipe is 0·052 foot, so that there is practically no difference. Adding $\frac{1}{6}$ to allow for incrustation,

$$d = 0 \cdot 72 \text{ inch},$$

so that a $\frac{3}{4}''$ pipe would practically do.

Diameter of pipe by Table.—This result may be obtained more readily by the use of Table XVIII. We have

$$\frac{L}{H} \cdot G^2 = \frac{120}{10} \times 1 \cdot 67^2,$$

$$= 33 \cdot 3.$$

And on referring to the Table it will be seen that $\frac{3}{4}''$ is the nearest market size of pipe, allowing for incrustation.

TAP.—It has tacitly been considered in the above that the tap in the cistern has the same bore as the pipe—a small reduction in the bore does not affect the flow very much, but if the reduction is at all considerable the flow is much impeded, as will be seen by the following.

The data being the same, find the diameter required for the delivery pipe :

1. When the bore of the tap is $\frac{1}{2}''$.
2. When the bore of the tap is $\frac{1}{4}''$.
3. When the bore of the tap is $\frac{3}{16}''$.

As in each case the discharge must be 1·67 gallon per minute in order that the cistern may be filled in four hours, it follows that the smaller the diameter of the tap, the greater must be the velocity of the issuing stream. If V is this velocity expressed in feet per second, the volume of water issuing in one second is

$$V \times \frac{D^2}{4} \cdot \pi \text{ cubic feet},$$

or $$V \times \frac{D^2}{4} \cdot \pi \times 6 \cdot 24 \text{ gallons}.$$

And since G is the discharge in gallons per minute we have

$$G = V \times \frac{D^2}{4} \times \pi \times 6 \cdot 24 \times 60,$$

whence by reduction

$$V = 0 \cdot 0034 \times \frac{G}{D^2} \quad . \quad . \quad . \quad . \quad . \quad (121),$$

where D is in feet, or

$$V = 0 \cdot 49 \times \frac{G}{d^2} \quad . \quad . \quad . \quad . \quad . \quad . \quad (122),$$

where d is expressed in inches.

Applying Equation 122 to the first case under consideration we have

$$V = 0\ 49 \times \frac{1\ 67}{(\frac{1}{2})^2} = 3\ 3 \text{ feet per second}$$

There must therefore be sufficient head at the tap to produce a velocity of 3 3 feet per second The required head can be found from Equation 118, thus

$$H_v = \frac{3\ 3^2}{64\ 4} = 0\ 17 \text{ foot nearly}$$

The available head to force the water through the supply pipe instead of being 10 feet, as before, is reduced to

$$10 - 0\ 17 = 9\ 83 \text{ feet},$$

but clearly this reduction in the head is not sufficient to make any practical difference in the size of the pipe required

In the second case, when the diameter of the tap is only $\frac{1}{4}$ inch, we have

$$V = 0\ 49 \times \frac{1\ 67}{(\frac{1}{4})^2},$$
$$= 13\ 1 \text{ feet per second}$$

Hence

$$H_v = \frac{13\ 1^2}{64\ 4} = 2\ 7 \text{ feet.}$$

Let us see whether this greater reduction will make any difference in the size of the pipe required We have, using Table XVIII

$$\frac{L}{H}\ G^2 = \frac{120}{10 - 2\ 7} \times 1\ 67^2,$$
$$= 46,$$

so that a $\frac{3}{4}$ pipe is still sufficient, allowing for incrustation

In the third case we have

$$V = 0\ 49 \times \frac{1\ 67}{(\frac{3}{16})^2} = 23\ 3 \text{ feet per second.}$$

$$H_v = \frac{23\ 3^2}{64\ 4} = 8\ 4 \text{ feet}$$

The available head is therefore

$$10 - 8\ 4 = 1\ 6 \text{ feet}$$

Hence

$$\frac{L}{H}\ G^2 = \frac{120}{1\ 6} \times 1\ 67^2 = 209$$

This number corresponds in Table XVIII to a 1 pipe

BENDS —Find the effect of 10 bends of 90° each and 4 inches radius

On reference to Equation 119 it will be seen that the smaller the diameter of the pipe, the greater is the resistance of a bend in it Therefore to be on *the safe side we ought to reckon the resistance of bends when the diameter of* the pipe has been diminished by incrustation, or, in other words resistance of the bends should be reckoned on the diameter found to be required before incrustation has been allowed for From Table XVIII and also from p 271 it will be seen that in the present instance this diameter is 0 6 inch nearly

Referring to Equation 119 we see that we must first find the value of *b*

Now $$\frac{R}{d} = \frac{4}{0{\cdot}6} = 7 \text{ nearly.}$$

Hence from Table XXI.

$$b = 0{\cdot}000011.$$

We next require V; from Equation 122

$$V = 0{\cdot}49 \times \frac{1{\cdot}67}{(0{\cdot}6)^2},$$
$$= 2{\cdot}27 \text{ feet per second.}$$

Hence from Equation 119

$$H_B = 0{\cdot}000011 \times 90 \times 2{\cdot}27^2,$$
$$= 0{\cdot}005 \text{ foot.}$$

And for ten bends the loss of head will be only 0·05 foot. This loss of head is much too small to have any practical effect on the size of the pipe, and thus the opinion expressed at p. 271 is confirmed.

Water Supply to a House (Constant Service).

Example 62.—*The water supply to a house* is represented in Fig. 530. Find the diameter of the pipes in order that the tap at A may discharge three

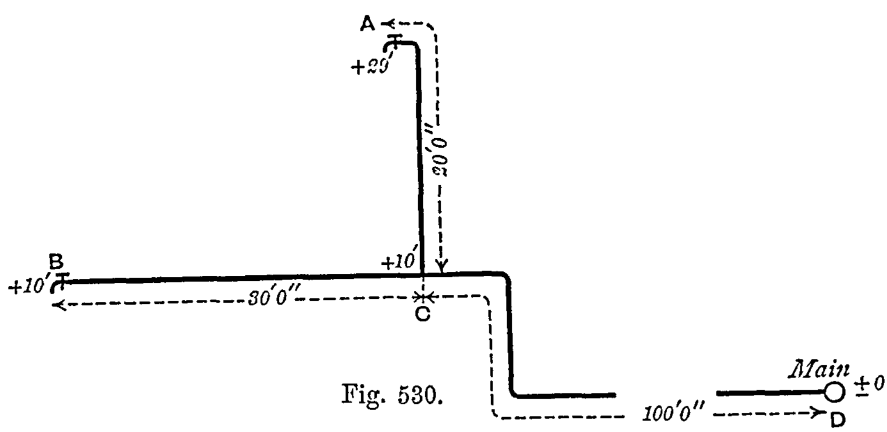


Fig. 530.

gallons per minute, and the tap at B two gallons per minute, when both taps are open, allowing for incrustation. Find also the discharge from each tap when the other is closed. The available head in the main is 90 feet. The levels are shown thus + 10 with reference to the main.

To find size of BC and AC.—The best way to proceed is to assume the size of one of the branches. For instance, supposing BC is taken as $\frac{1}{2}''$ diameter.

Then, from Table XVIII., $$\frac{L}{H} \cdot G^2 = 5.$$

Hence if we take $L = 30$ feet and $G = 2$ gallons, H will be the loss of head from C to B, thus

$$H = \frac{30 \times 2^2}{5},$$
$$= 24 \text{ feet.}$$

Now since B and C are at the same level the available head at C ought to be 24 feet.

In the pipe AC the water has to rise 19 feet, hence the available head is

$$24 - 19 = 5 \text{ feet.}$$

Hence $$\frac{L}{H}\ G^2 = \frac{20}{5} \times 3^2 = 36$$

So that (Table XVIII) a $\frac{5}{8}$' pipe is too small and a $\frac{3}{4}$' is too big. We have, however, made no allowance for the various minor losses of head. It will be well to see whether in this case they are of sufficient magnitude to take into account.

Minor losses of head.—In the first place, if the pipe AC is connected to BD by means of a T piece, as shown in Fig 531, the general flow of the water will be along DB, as shown by the arrows. Consequently in entering the pipe CA there will be a loss of head due to the change in the direction of the flow, a part of which may be taken as the loss of head required to impart the velocity in the pipe AC, but, in addition, eddies are formed (called gurglitation) which cause another loss of head, generally taken as *twice* that due to velocity. On the whole it is usual to reckon the loss as *three* times that due to velocity.[1] We must therefore first find the velocity.—Supposing a $\frac{3}{4}$' pipe is provisionally decided upon, then we must reckon the velocity when the diameter of the pipe has been diminished by incrustation. From Table XVIII it will be seen that the diminished diameter is 0.64 inch. Hence from Equation 122

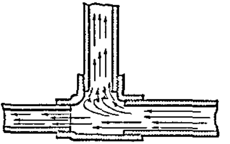


Fig 531.

$$V = 0.49 \times \frac{3}{(0.64)},$$
$$= 3.6 \text{ feet per second},$$

and $$H_v = \frac{3.6^2}{64.4} = 0.2 \text{ foot}$$

So that the loss of head is

$$3 \times 0.2 = 0.6 \text{ foot}$$

BENDS.—We will neglect the loss due to bends, it must be small owing to the low velocity.

TAP.—The tap, if smaller than the pipe, will cause a slight loss of head. Supposing that the bore of the tap is $\frac{1}{2}$', or 0.48 inch, making a slight allowance for incrustation, then to discharge three gallons per minute the issuing velocity must be

$$V = 0.49 \times \frac{3}{(0.48)},$$
$$= 6.4 \text{ feet per second}$$

or $$H = \frac{6.4^2}{64.4} = 0.63 \text{ foot.}$$

But the velocity in the pipe is 3.6 feet per second, which represents a head of 0.2 foot as already seen. Hence the additional head required to produce the issuing velocity, or, in other words, the loss of head due to issue, is

$$0.63 - 0.2 = 0.43 \text{ foot.}$$

TOTAL MINOR LOSS OF HEAD.—Altogether, therefore, we must reckon on a loss of head of $0.6 + 0.43$,

[1] See Hurst's *Architectural and Surveyor's Handbook*.

$$= 1{\cdot}03 \text{ foot,}$$

or say 1 foot.

The available head is therefore reduced to 4 feet; we thus have

$$\frac{L}{H} \cdot G^2 = \frac{20}{4} \times 3^2 = 45.$$

It thus appears (Table XVIII.) that a $\frac{3}{4}''$ pipe is the proper size.

To FIND SIZE OF CD.—We now have to find the diameter of the pipe CD. We found that the available head at C ought to be 24 feet, and there is a loss of head in the pipe CD of 10 feet, due to difference of level. Hence the available head is

$$90 - 10 - 24 = 56 \text{ feet.}$$

Therefore $$\frac{L}{H} \cdot G^2 = \frac{100}{56} \times (2+3)^2 = 45,$$

which corresponds to a $\frac{3}{4}''$ pipe.

Since the pipes AC and CD are both of the same diameter, the joint at C would be made as shown in Fig. 532, instead of as in Fig. 531, and this would prevent the loss of head of 0·6 foot, and the loss would occur in the $\frac{1}{2}''$ pipe.

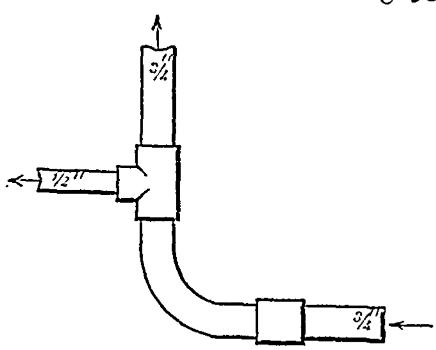


Fig. 532.

Referring back to p. 274 it will be seen that the diameter of the pipe BC was *assumed* as $\frac{1}{2}''$. Had a larger diameter been assumed the loss of head in BC would have been less, so that the available head at C producing the discharge in AC would also have been less, and consequently this pipe would have to be made bigger. On the other hand, the available head in CD would have been greater, hence this pipe would have been smaller; that is, the service pipe would be smaller than the branches, which is not advisable. Thus the arrangement worked out is practically the best.

To find the discharge from one tap when the other is closed.—First, when tap A is open, it is clear that the available head is $90 - 29 = 61$ feet, or, allowing for the various minor losses of head, say 60 feet.

Hence from Table XVIII.

$$\frac{L}{H} \cdot G^2 = \frac{100+20}{60} \times G^2 = 48.$$

Thus $$G^2 = \frac{48 \times 60}{120} = 24,$$

or $G = 4{\cdot}9$ gallons per minute.

When tap B is open and tap A closed, we have to deal with two sizes of pipe. Let H_1 be the head at the point C, then $90 - 10 - H_1$ will be the available head in the portion CD of the pipe. Hence for the pipe BC

$$\frac{L}{H} \cdot G^2 = \frac{30}{H_1} \cdot G^2 = 5 \quad . \quad . \quad . \quad . \quad (a),$$

and similarly for the pipe CD

$$\frac{L}{H} \cdot G^2 = \frac{100}{80 - H_1} \cdot G^2 = 48 \quad . \quad . \quad . \quad (b).$$

Now the discharge G is the same in both pipes. Hence, dividing Equation (*a*) by Equation (*b*),

$$\frac{30}{100}=\frac{5H_1}{48(80-H_1)},$$

or
$$3\times 48(80-H_1)=50H_1,$$

whence
$$H_1=\frac{3\times 48\times 80}{194},$$
$$=59.5 \text{ feet.}$$

Inserting this value in Equation (*a*),

$$G^2=\frac{5\times 59.5}{30}=9.9,$$

or $G=3.15$ gallons per minute

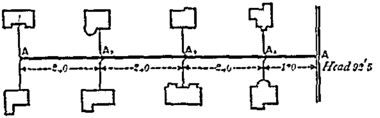
Of course we ought to get the same result from Equation (*b*), namely—

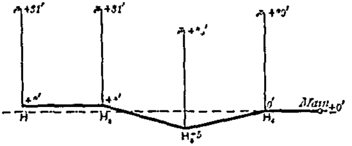
$$G^2=\frac{48(80-59.5)}{100}=9.9$$

Water Supply to Eight Houses in a Street.

Example 63.—*Eight houses, as shown on plan in Fig. 533, are to be supplied with water from a branch main connected to the main at the point*

Fig. 533





Vertical scale 10 times horizontal scale

Fig. 534.

A. The levels of the branch main are shown in Fig. 534, and each house is fitted up as in Example 62. Find the diameter of the branch main capable of supplying each house approximately with 5 gallons of water per minute.

On referring to Example 62 it will be seen that the highest tap was 29 feet above the main, that it was connected to the main by 120 feet of $\frac{3}{4}''$ pipe, and that the discharge from it was 4.9 gallons per minute when the head in the main was 90 feet. It will also be seen that the discharge is almost the same when both taps are running. We will therefore ignore the lower tap in the present example.

SIZE OF MAIN.—*Half the taps opened simultaneously*.—It is very unlikely that all the houses will require water at the same time, and a very

safe assumption to make is that half the number of houses require the full amount of water at the same time. Supposing, therefore, that the upper taps in houses 1, 2, 1′, 2′ are opened simultaneously and that an average of 4·9 gallons is running out of each, then, from Example 62, the head at A_2 must be 61 feet more than the height of the tap, that is

$$H_2 = 61 + 31 = 92 \text{ feet.}$$

Hence the available head in the portion A_2A of the branch main is

$$92{\cdot}5 - 92 = 0{\cdot}5 \text{ foot.}$$

Again, the discharge at A_2 must be

$$4 \times 4{\cdot}9 = 19{\cdot}6 \text{ gallons per minute.}$$

Hence the value of the expression $\frac{L}{H} \cdot G^2$ is

$$\frac{600}{0{\cdot}5} \times 19{\cdot}6^2 = 460{,}000 \text{ nearly.}$$

And referring to Table XVIII. it will be seen that a 4″ pipe is the nearest market size, allowing for incrustation.

Theoretically the branch main ought to be made to diminish after the junction to each house, but the practice is to maintain the same diameter, for some distance at any rate.

All upper taps opened simultaneously.—As an exercise let us inquire what the discharge would be supposing all the upper taps were open. Although it may appear paradoxical, the discharge from each tap will not be very much diminished. The direct solution of the question, however, leads to several cumbersome quadratic equations, and the best way is to work by approximation.

Thus as a first approximation let us suppose that the average discharge per tap is 4·0 gallons per minute when all the taps are open, then the discharge at A_4 must be

$$8 \times 4 = 32 \text{ gallons per minute.}$$

Now from Table XVIII. we have for a 4″ pipe

$$\frac{L}{H} \cdot G^2 = 420{,}000.$$

Hence

$$H = \frac{120 \times 32^2}{420{,}000},$$
$$= 0{\cdot}29 \text{ feet,}$$

which is the loss of head between A and A_4. Hence H_4, the head at A_4, is

$$92{\cdot}5 - 0{\cdot}29 = 92{\cdot}21 \text{ feet}$$

since there is no difference in level.

The discharge from the tap in house No. 4 will therefore be

$$G_4{}^2 = \frac{48 \times (92{\cdot}21 - 29)}{120},$$

or

$$G_4 = 5 \text{ gallons per minute.}$$

It follows that the discharge at A_3 will be

$$32 - 2 \times 5 = 22 \text{ gallons.}$$

Hence

$$H = \frac{240 \times 22^2}{420{,}000} = 0{\cdot}28,$$

or

$$H_3 = 92{\cdot}21 + 5 - 0{\cdot}28,$$
$$= 96{\cdot}93.$$

Therefore discharge in house No 3 is

$$G_3^2 = \frac{48(96\ 93 - 24)}{120},$$

or $$G_3 = 5\ 4 \text{ gallons per minute}$$

The discharge at A_2 will therefore be

$$22 - 2 \times 5\ 4 = 11\ 2$$

Hence

$$H = \frac{240 \times 11\ 2^2}{420{,}000} = 0\ 07,$$

or $$H_2 = 96{\cdot}93 - 7 - 0\ 07, \\ = 89\ 86$$

Hence discharge in house No 2 will be

$$G_2^2 = \frac{48(89\ 86 - 31)}{120},$$

$$G_2 = 4\ 8 \text{ nearly}$$

We still have the discharge from house No 1, so that clearly the original assumption of 4 gallons per minute is considerably too small

As a second trial assume 4 9 gallons as the average discharge of each tap per minute, then repeating the above calculations we will find

$$H_4 = 92\ 5 - 0\ 44 = 92\ 06,$$
$$G_4 = 5 \text{ gallons}$$
$$H_3 = 92\ 06 + 5 - 0\ 48 = 96\ 58,$$
$$G_3 = 5\ 4 \text{ gallons}$$
$$H_2 = 96\ 58 - 7 - 0\ 19 = 89\ 39,$$
$$G_2 = 4\ 8 \text{ gallons}$$
$$H_1 = 89\ 39 - 0\ 04 = 89\ 35,$$
$$G_1 = 4\ 8 \text{ gallons}$$

Practically, therefore, we get the same values as we did before, when only half the number of taps were running, and the reason is, that a small alteration in the head produces a considerable alteration in the discharge of a 4 pipe, but no practical difference in the discharge of a $\frac{3}{4}$ pipe

As a further exercise let us find the discharge in the pipe connecting house No 3, when only the corresponding tap is opened We have for the $\frac{3}{4}$' pipe

$$G_3^2 = \frac{48(H_3 - 24)}{120},$$

and for the 4 pipe

$$G_3^2 = \frac{420{,}000(92\ 5 + 5 - H_3)}{240 + 120}$$

Therefore

$$48(H_3 - 24) = \frac{420{,}000(97\ 5 - H_3)}{3},$$

$$H_3\left(48 + \frac{420\ 000}{3}\right) = \frac{420{,}000 \times 97\ 5}{3} + 48 \times 24$$

whence

$$H_3 = 97\ 5 \text{ very nearly},$$

that is to say, that the flow of water from A to H_3 is so little, that there is no appreciable loss of head Therefore

$$G_3^2 = \frac{48(97\ 5 - 24)}{120},$$

$$G_3 = 5{\cdot}42 \text{ gallons per minute.}$$

When the pipes are new, the discharge will of course be greater, for instance in the last case, since the corresponding value from Table XVIII. for a pipe 0·75 inch diameter is 117.

$$G'^2_3 = \frac{117(97{\cdot}5 - 24)}{120},$$

$$G'_3 = 8{\cdot}4 \text{ gallons per minute.}$$

Discharge from Pipes flowing partially full.

It has been shown by experiment that the mean velocity of flow of a liquid in a pipe is *roughly* proportionate to

$$\sqrt{RS},$$

where R is the mean radius or hydraulic mean depth (see p. 262), and S is the slope of the pipe (see p. 264).

An approximate formula would therefore be

$$V = C\sqrt{RS} \quad . \quad . \quad . \quad \mathbf{(123)},$$

the units of measurement being the foot and the second, and C being a constant. Different values have been ascribed to C by various experimenters as follows:—

Beardmore . . .	95,
Downing	100.

But Darcy makes C vary according to the diameter of the pipe, as in the Table given at p. 265.

Neville's formula—namely,

$$V = 140\sqrt{RS} - 11\sqrt[3]{RS} \quad . \quad . \quad . \quad (124)$$

—is considered to be more accurate, but as in practice the choice is limited to certain sizes of pipes, the additional accuracy is not worth the considerable complication introduced into the calculations.

The student will observe that these formulæ are similar in form to those given for pipes running full under pressure. Now for a pipe running full

$$R = \frac{\pi D^2}{4} \times \frac{1}{\pi D} = \frac{D}{4},$$

and

$$S = \frac{H}{L}.$$

So that

$$RS = \frac{D}{4} \cdot \frac{H}{L},$$

from which it will be seen that the formulæ for pipes running full are a special case of those now given. A distinction must, however, be made

between $\frac{H}{L}$ and S, the former is the head of water divided by the length of the pipe, whereas S is the slope of the pipe, and may vary at different points Thus in Fig 535 if the pipe were running full we would have to substitute

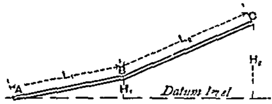


Fig 535

$\frac{H_2}{L_1+L_2}$ for $\frac{H}{L}$ in Equation 110, and the velocity of the water would be the same throughout the pipe, but if the pipe were only partially full, then for the portion AB

$$S=\frac{L_1}{H_1}$$

and for the portion BC

$$S=\frac{H_2-H_1}{L_2},$$

and the velocity would be greater in BC than in AB

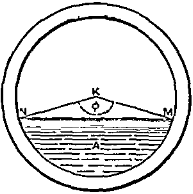


Fig 536

Darcy's formula—For the same reasons as those given at p 267, Darcy's formula is preferred, and will be used in working out examples

To find the discharge we must multiply the velocity by the area of discharge Let A be this area, then

$$F=A\ C\sqrt{RS} \qquad (125)$$

To find A in circular pipes we must subtract the area of the triangle NKM (Fig 536) from the area of the sector NKM, thus—

$$A=\frac{\phi}{360}\times\pi\frac{D^2}{4}-\frac{D^2}{8}\sin\phi \qquad (126),$$

whence

$$R=\frac{\frac{1}{4}\left(\frac{\pi\phi}{360}-\frac{\sin\phi}{2}\right)D^2}{\frac{\pi\phi}{360}\ D} \qquad (127)$$

These expressions are given to show how the discharge could be calculated if necessary, but in practice what we require to know in connection with drain pipes is whether, when conveying the average quantity of sewage, the velocity is sufficient to keep the pipe clean, without flushing[1] Of course the size of the drain pipe depends on the maximum quantity of liquid to be conveyed

Velocity—The question therefore is, knowing the diameter of the pipe and the discharge, to find the velocity We have from (123)

[1] See p 283

$$V = C\sqrt{RS}.$$

But $$R = \frac{A}{P},$$

where P is the wetted perimeter expressed in feet. Further,

$$A = \frac{F}{V}.$$

Hence $$V^2 = C^2 . \frac{F}{PV} . S,$$

or $$V^3 = \frac{C^2F . S}{P} \quad . \quad . \quad . \quad (128).$$

Now we do not know P, and we must therefore assume a trial value P_1, and thus we obtain an approximation (V_1) to V. But

$$F_1 = A_1V_1,$$

where A_1 is the corresponding value to P_1 obtained from Table XXIII.

If F_1 is greater than F, then V_1 is also too great or P_1 is too large, and *vice versâ*. Two or three trials ought to give a sufficiently accurate value of V.

Example 64.—A 15-inch drain pipe, laid at a slope of $\frac{1}{200}$, is discharging 0·2 cubic foot per second. Find the velocity.

Assume $$P_1 = 0·6 \times D, \quad = 0·75 \text{ foot}.$$

Then from Equation 128 $$V_1{}^3 = \frac{110^2 \times 0·2}{0·75 \times 200},$$

or $$V_1 = 2·5 \text{ feet per second}.$$

But from Table XXIII. the corresponding value of A_1 when $\frac{P}{D} = 0·6$ is

$$0·034 \times D^2, \quad = 0·053 \text{ square foot}.$$

Hence $$F_1 = 0·053 \times 2·5, \quad = 0·132 \text{ cubic foot per second},$$

so that clearly P_1 is too small.

Assume as a second trial

$$P_2 = 0·7 \times D, \quad = 0·87 \text{ foot}.$$

Then $$V_2{}^3 = \frac{110^2 \times 0·2}{0·87 \times 200},$$

or $$V_2 = 2·4 \text{ feet per second}.$$

But $$A_2 = 0·052 \times D^2, \quad = 0·0813 \text{ square foot}.$$

Hence $$F_2 = 0·195 \text{ cubic foot}.$$

V_2 is therefore a sufficiently near approximation. We will, however, try the next value of P from Table XXIII., namely—

Find what size of pipes should be used, and whether any arrangement for periodical flushing ought to be provided.

We will work out this example by means of Tables XIX. and XXIII.

Pipe BC.—For the maximum discharge we have

$$F = 4 \text{ cubic feet per second.}$$
$$H = 4\cdot7 - 1\cdot6 = 3\cdot1 \text{ feet.}$$
$$L = 320 \text{ feet.}$$

Hence

$$\frac{L}{H} \cdot F^2 = \frac{320}{3\cdot1} \times 4^2,$$
$$= 1650.$$

On referring to Table XIX. it will be seen that this number corresponds to a 12-inch pipe, and that this pipe will run very nearly full when discharging the maximum quantity.

To find the velocity of discharge we may consider that the pipe is running full. The discharge in that case can be found from

$$\frac{L}{H} \cdot F^2 = 1850,$$
$$F^2 = \frac{1850 \times 3\cdot1}{320},$$

or $\quad F = 4\cdot23$ cubic feet per second.

Hence $\quad V . \dfrac{\pi D^2}{4} = 4\cdot23,$

or, since $\quad D = 1$ foot,

$$V = \frac{4\cdot23 \times 4}{\pi \times 1^2},$$
$$= 5\cdot4 \text{ feet per second.}$$

The velocity is therefore well above the limit for flushing.

Next, to find the velocity when the average quantity of sewage is being discharged. Proceeding as in Example 64, assume

$$P = 0\cdot8D,$$

then, since (Table J.) $\quad C = 109\cdot5$ for a 12″ pipe,

from Equation 128 $\quad V_1{}^3 = \dfrac{109\cdot5^2 \times 0\cdot2 \times 3\cdot1}{0\cdot8 \times 1 \times 320},$

$$V_1 = 3\cdot07 \text{ feet per second.}$$

Hence, since (Table XXIII.) $\quad A_1 = 0\cdot075D^2,$

$$F_1 = 0\cdot075 \times 1^2 \times 3\cdot07,$$
$$= 0\cdot23 \text{ cubic foot.}$$

This is a sufficiently near approximation, and since the velocity is a little over 3 feet per second, there is no necessity to make any arrangements for flushing.

Pipe BD.—For the maximum discharge we have

$$F = 3 \text{ cubic feet per second.}$$
$$H = 13\cdot8 - 1\cdot6 = 12\cdot2 \text{ feet.}$$
$$L = 416 \text{ feet.}$$

Hence

$$\frac{L}{H} \cdot F^2 = \frac{416}{12\cdot2} \times 3^2 = 307.$$

On referring to Table XIX. it will be seen that a 9″ pipe will do.

then $$N = \frac{K}{3},\ M = \frac{2K}{3},\ O = K.$$

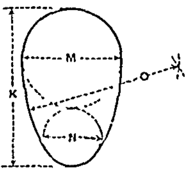


Fig 541.

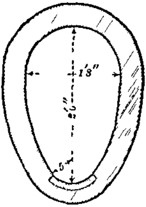


Fig 542

In the present case we must, in the first place, ensure that the sewer is large enough to carry the maximum discharge without running full. We have already seen that a circular pipe of 24 inches is required, and it will be near enough for practical purposes if we make the area of the egg shaped sewer equal to that of the circular sewer. Now the area of an egg shaped sewer of the form given above can be found from the formula

$$\text{Area} = 0.525 \times K^2 \qquad (129)$$

And the area of a circle 2 feet in diameter is

$$3.14 \text{ square feet.}$$

Hence

$$0.525 \times K^2 = 3.14,$$
$$K = 2.45 \text{ feet}$$

Therefore the depth can be taken as 2′ 6′. Hence N = 10″, M = 1′ 8″, and applying these dimensions we obtain the sewer shown in Fig 542.

Let us now see to what extent the velocity is greater than with the 2 foot drain pipe when the average discharge is flowing. The quantity flowing being small, it will probably not rise above the invert, so that we may regard it as flowing in a 10″ pipe. Supposing that

$$P_1 = 1.2 \times \frac{10}{12},$$

then $$V_1^3 = \frac{109^2 \times 0.22 \times 1.6 \times 12}{1.2 \times 10 \times 1283},$$

$$V_1 = 1.5,$$

and $$F_1 = 1.5 \times 0.203 \times \frac{10^2}{12^2},$$
$$= 0.213,$$

which is a very close approximation, and thus we see that the velocity is only very slightly increased.

Check by Neville's formula.—To check this result by using Neville's formula we have

$$R = \frac{.203 \times \frac{10^2}{12^2}}{1.2 \times \frac{10}{12}} = 0.14$$

Hence

$$V = 140\sqrt{0{\cdot}14 \times \frac{1{\cdot}6}{1283}} - 11\sqrt[3]{0{\cdot}14 \times \frac{1{\cdot}6}{1283}},$$
$$= 1{\cdot}85 - 0{\cdot}62,$$
$$= 1{\cdot}23 \text{ feet per second.}$$

The results therefore agree fairly well.

It appears therefore that, at any rate in this case, an egg-shaped sewer does not increase the velocity to any material extent.

JETS.

It is sometimes necessary to know, in connection with fountains and hydrants, the height to which a jet of water will rise.

When a stone is thrown vertically into the air it rises to a height depending on the velocity with which it was thrown. If the stone is not thrown vertically, but at an angle, it will describe a curve depending on the velocity and on the angle of elevation, *i.e.* the angle at which the stone is thrown. In the same way a vertical jet of water will rise to a height depending on the velocity with which it issues from the orifice (or nozzle), and an inclined jet will describe a curve depending on the velocity and the angle of elevation.

Now it is shown in books on dynamics that in the case of a stone thrown vertically, and neglecting the resistance of the air, the height to which it will rise is given by

$$h = \frac{V^2}{64{\cdot}4} \quad . \quad . \quad . \quad . \quad . \quad (130).$$

If the stone is thrown at an angle, then the curve described by it is represented by the equation

$$y = x \tan\theta - 16{\cdot}1 \frac{x^2}{V^2 \cos^2\theta},$$

where V is the initial velocity in feet per second, and θ is the angle of elevation, as shown in Fig. 543.

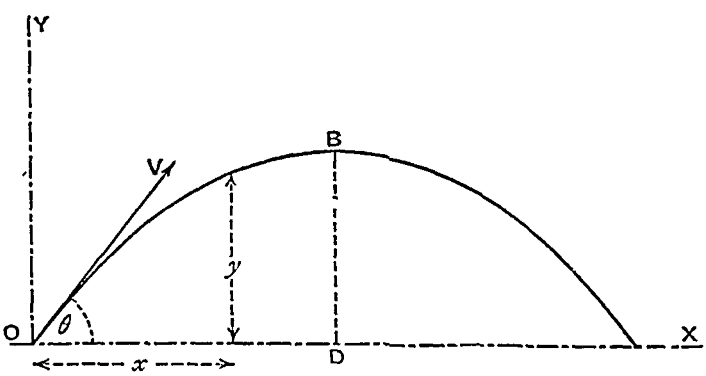


Fig. 543.

To find the range, or the distance from O where the stone strikes the horizontal plane, that is OA = R, put $y = 0$, then

$$\tan\theta = 16.1 \frac{R}{V^2 \cos^2\theta},$$

or
$$R = \frac{V^2}{16.1} \sin\theta\cos\theta \tag{131}$$

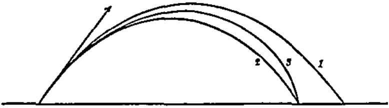
Further, to find the greatest height the stone rises to, or BD, put $y = \frac{R}{2}$, then

$$BD = h_{(max)} = \frac{V^2}{32.2} \tan\theta \sin\theta\cos\theta - \frac{16.1}{V^2\cos^2\theta} \cdot \frac{V^4}{32.2^2} \sin^2\theta\cos^2\theta,$$

or
$$h_{(max.)} = \frac{V^2 \sin^2\theta}{64.4} \tag{132}$$

These formulæ are applicable to jets projected *in vacuo*, and to make allowance for the resistance of the air the simplest way is to multiply the initial velocity by a reducing factor, or, in other words, to calculate the path for a less initial velocity than the actual.

The result is not quite accurate (see Fig. 544), but is near enough for practical purposes. The reducing factor dimin-



1 *Full velocity*
2 *Reduced velocity*
3 *Actual path*

Fig. 544

ishes as the ratio of the diameter of the jet to the head increases, and can be found from Table XXIV., which is based on experiment.

Example 67.—A jet of water is issuing from a $\frac{1}{2}$ inch nozzle with a velocity of 60 feet per second. Find

(a) *The height* to which the jet will rise when it is thrown vertically.

(b) *The distance* to which the jet will reach, and the *greatest height* it will attain, when thrown at the angles of 15° and 55° respectively.

A velocity of 60 feet per second corresponds to a head of

$$H = \frac{60^2}{64.4} = 56 \text{ feet}$$

Hence d being diam. of nozzle $\frac{H}{d} = \frac{56 \times 12}{\frac{1}{2}} = 1350$

We can therefore take (see Table XXIV.)

$$J = 0.91$$

Consequently for case (a), from Equation 130,

$$h = \frac{(0.91 \times 60)^2}{64.4} = 46 \text{ feet.}$$

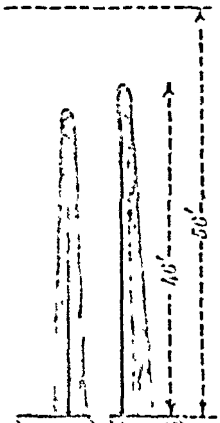


Fig. 545.

It should be observed that if the jet is perfectly vertical, and there is no wind, the water falls back on the jet, and thus reduces the height as shown in Fig. 545.

Again for case (*b*), when the angle of elevation is 15°, we have from Equation 131

$$R = \frac{(0{\cdot}91 \times 60)^2}{16{\cdot}1} \sin 15^\circ \times \cos 15^\circ,$$
$$= 46 \text{ feet};$$

and from Equation 132

$$h_{(\text{max.})} = \frac{(0{\cdot}91 \times 60)^2 \sin^2 15^\circ}{64{\cdot}4},$$
$$= 3{\cdot}1 \text{ feet}.$$

Lastly, when the angle inclination is 55°, we have from Equation 131

$$R = \frac{(0{\cdot}91 \times 60)^2}{16{\cdot}1} \sin 55^\circ \times \cos 55^\circ,$$
$$= 87 \text{ feet};$$

and from Equation 132

$$h_{(\text{max.})} = \frac{(0{\cdot}91 \times 60)^2}{64{\cdot}4} \times \sin^2 55^\circ,$$
$$= 31 \text{ feet}.$$

Issuing velocity of a Jet.—We must now see how to obtain the issuing velocity of a jet. If the nozzle offered no obstruction to the issuing stream of water, it is clear that the velocity would be that due to the head at the nozzle; but the nozzle does offer an obstruction, or, in other words, causes a loss of head, the amount of which depends upon the shape of the nozzle.

The reduction in velocity caused by the nozzle has been ascertained by experiment as follows:—

A well-shaped nozzle, such as the one shown in Fig. 546, reduces the velocity to

0·97 × velocity due to the head.

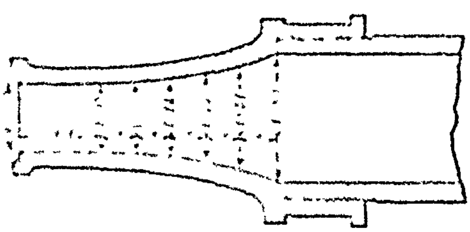


Fig. 546.

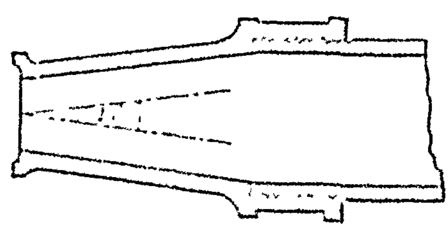


Fig. 547.

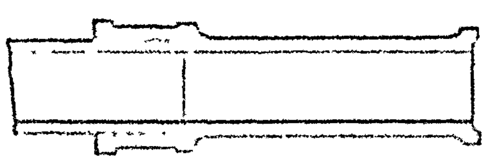


Fig. 548.

A converging nozzle, as in Fig 547, reduces the velocity to

$$0\ 94 \times \text{velocity due to head}$$

And a cylindrical nozzle, as in Fig 548, causes the following reductions —

When the diameter is

$\frac{1}{2}$ to $\frac{1}{3}$ length	0 81	}	Multiplied by velocity due to head
$\frac{1}{4}$ „ $\frac{1}{12}$ „	0 77		
$\frac{1}{11}$ „ $\frac{1}{24}$ „	0 73		
$\frac{1}{25}$ „ $\frac{1}{36}$ „	0 68		

The reduced velocity obtained by applying the above is of course the initial velocity of the jet to be used in finding the height or the range

Example 68 —A $\frac{1}{4}''$ nozzle for a vertical jet is connected to the end of a 1″ pipe, 300 feet long The head at the farther extremity of the one inch pipe is 100 feet, and the nozzle is 10 feet higher than this point Find the height of the jet and the quantity of water that will be discharged

Let V feet per second be the mean velocity of the issuing jet of water, then if G is the discharge in gallons per minute, we have from Equation 122

$$G = \frac{Vd^2}{49},$$

$$= 2\ 04 \times \tfrac{1}{16} \times V,$$

$$= 0\ 127 \times V$$

Now if the nozzle is of a good shape (see Fig 546) the velocity *due* to the head at the nozzle will only be $\frac{1}{0\ 97}$ greater than V Hence if H is the head at the nozzle

$$\left(\frac{V}{0\ 97}\right)^2 = 64\ 4H,$$

or

$$V^2 = 60\ 7H$$

Hence

$$\left(\frac{G}{0\ 127}\right)^2 = 60\ 7H,$$

or

$$G^2 = 0\ 97H \text{ nearly}$$

Again, we have from Table XVIII for a 1 pipe, allowing for incrustation and remembering that the nozzle is 10 feet *above* the farther extremity of the 1′ pipe,

$$G^2 = 250 \times \frac{100 - 10 - H}{300}$$

Hence

$$0\ 97H = \frac{250}{300} \times 90 - \frac{250}{300}\ H,$$

or

$$H = 41\ 7 \text{ feet}$$

The ratio of H to the diameter of the nozzle is therefore

$$\frac{41\ 7 \times 12}{\frac{1}{4}} = 2000$$

Hence by interpolation from Table XXIV the factor J is 0 82 And since from the above equation

$$V^2 = 60{\cdot}7 \times 41{\cdot}7,$$

we have $$h = \frac{(0{\cdot}82)^2 \times 60{\cdot}7 \times 41{\cdot}7}{64{\cdot}4},$$
$$= 26{\cdot}4 \text{ feet.}$$

Lastly for the discharge we have
$$G^2 = 0{\cdot}97 \times 41{\cdot}7,$$
or $$G = 6{\cdot}3 \text{ gallons per minute.}$$

Water Supply for House and Jet.

Example 69.—A jet, to rise to a height of 30 feet above the nozzle, is required for an ornamental piece of water. The water is to be obtained from a reservoir which is also to supply the house, the arrangements for which are the same as in Example 62. The various lengths and levels are given in Fig. 549. Find the diameters of the pipes that will be required, and also how much higher the jet will rise when no water is being supplied to the house.

We must first find the head required to produce a jet 30 feet high. To do this we require to know the issuing velocity.

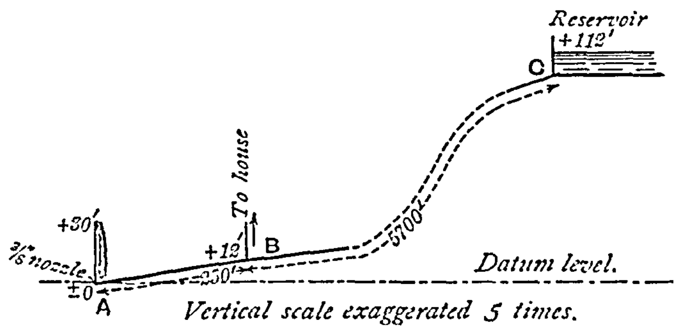


Fig. 549.

The head required at the nozzle will be about 40 feet, hence the ratio of the head to the $\frac{3}{8}''$ diameter of the nozzle is

$$\frac{H}{d} = \frac{40 \times 12 \times 8}{3} \text{ nearly,}$$
$$= 1280.$$

So that, referring to Table XXIV., it will be seen that the factor J can be taken at 0·90. Hence if V is the issuing velocity by Equation 130

$$30 = \frac{(0{\cdot}90 \times V)^2}{64{\cdot}4},$$

or $$V = 49 \text{ feet per second nearly.}$$

If the nozzle is well shaped, the velocity due to the head will be

$$\frac{49}{0{\cdot}97} = 50{\cdot}5 \text{ feet per second.}$$

Hence the head required to produce a jet of 30 feet with a good nozzle is

$$\frac{50{\cdot}5^2}{64{\cdot}4} = 39{\cdot}6 \text{ feet.}$$

It is to be observed that the head assumed (namely, 40 feet) for the purpose of calculating the factor J is very nearly the calculated head. Had

there been any considerable difference the calculated head would have been looked upon as a first approximation, from which a more accurate value of J could have been found

The next step is to find the discharge from the jet, which will be, from Equation 122,

$$G = \frac{2.04 \times 49 \times 3^2}{8^2} = 14 \text{ gallons per minute nearly}$$

On referring to Example 62 it will be seen that a head of 90 feet is required at the point B to obtain the necessary water supply to the house Hence the available head from B to A is

$$90 + 12 - 39.6,$$
$$= 62.4 \text{ feet}$$

Applying, therefore, Table XVIII we have

$$\frac{250 \times 14^2}{62.4} = 785,$$

and to this number corresponds a $1\frac{1}{4}$ pipe

To find the diameter of the portion BC of the pipe we have the available head $112 - 90 - 12 = 10$ feet, and the discharge is $14 + 5 = 19$ gallons per minute. Hence

$$\frac{5700 \times 19^2}{10} = 206{,}000 \text{ nearly},$$

a number which corresponds in Table XVIII to a $3\frac{1}{2}$ inch pipe

The jet will not be exactly 30 feet high, because the available diameters of pipes are not quite those required to produce a jet of this height under the conditions given Working backwards we find—

Loss of head from C to B,

$$\frac{5700 \times 19^2}{208{,}000} = 9.9 \text{ feet},$$

and the loss of head from B to A will be

$$\frac{250 \times 14^2}{810} = 60.5 \text{ feet.}$$

Hence the head at A will be

$$112 - 9.9 - 60.5 = 41.6$$

The velocity due to this head is

$$\sqrt{41.6 \times 64.4} = 51.8 \text{ feet per second},$$

hence the issuing velocity will be

$$51.8 \times 0.97 = 50.3$$

$$\text{Therefore height of jet} = \frac{0.90^2 \times 50.3^2}{64.4} = 31.9 \text{ feet}$$

When no water is being supplied to the house, the jet will be a little higher In this case the loss of head from C to B

$$= \frac{5700 \times 14^2}{208{,}000} = 5.4$$

Hence the head at A will be

$$112 - 5.4 - 60.5 = 46.1$$

The ratio of the head to the diameter of the nozzle is

$$\frac{46\cdot1 \times 12 \times 8}{3} = 1480 \text{ nearly.}$$

Hence from Table XXIV. the value of J is about 0·89.

Therefore the height of the jet will be

$$46\cdot1 \times (0\cdot97 \times 0\cdot89)^2 = 34\cdot3 \text{ feet.}$$

Strictly a slight correction ought to be made, because the issuing velocity of the jet is slightly increased. Hence the discharge will be greater—consequently the loss of head somewhat greater than calculated above; but as the formulæ used are only approximate it is not worth while going into such niceties.

APPENDICES.

APPENDIX I

Factors of Safety.

The following Table shows the Factors of Safety recommended by various eminent engineers for application in several cases that arise in practice

Authority	Nature of Structure	Nature of Load	Factor of Safety	Remarks
	Cast Iron.			
U	General	Dead	4	
U		*Live or varying*	6	*Stress of one kind only*
U			10	Equal alternate stresses of different kinds
U		Varying with shocks	15	
R		Dead	3 to 4	
R		Live	6 to 8	
B	Girders	Dead	3	
B		Live	6	
S		Live	6	
S	Water tanks	Dead	4	
S	Pillars		6	
S	subject to vibration	Live	8	
S	transverse shock		10	
	Wrought Iron			
U	*General*	*Dead*	3	
U		Live or varying	5	Stress of one kind only
U			8	Equal alternate stresses of different kinds
U		Varying with shocks	12	
R		Dead	3	
R		Live	4 to 6	
S	Girders	Dead	3	
S		Live with shocks	6	
B	Bridges	Mixed	4	
	Riveted joints	Dead	4	
U	Roofs		4	
S	Tension and compression bars	Live with shocks	6	
SS			4½ to 7	According to proportion of length to least diameter
S	Compression bars	Dead	4	
	Steel			
U	*General*	Dead	3	
U		Live and varying	5	Stress of one kind only
U			8	Equal alternate stresses of different kinds
U		Varying and shocks	12	
R		Dead	3	
R		Live	4 to 6	
C	Bridges	Mixed	4	
	Timber			
U	General	Dead	[illegible]	
U		Live and varying	10	Stress of one kind only
U			15	Equal alternate stresses of different kinds
U		Varying and shocks	20	
R			10	
S	Exposed to weather	Dead	10	
S	Under cover		8	
S	For temporary purposes		4	
	Brickwork and Masonry			
U	General	Dead	20	
U		Live	30	
R		Dead	4 to 10	
S	Arches		20	

B Board of Trade C Commissioners R Rankine S Stoney SS Shaler Smith. U Unwin.

The Factor of Safety determined upon for any particular case will depend upon what is accurately known of the strength and quality of the particular material to be used, upon the importance of the structure, etc.

APPENDIX II.

Note on Equilibrium.

When the forces acting on a body lie in a plane, there are three conditions of equilibrium, namely—

1. The algebraic sum of the resolved parts of the forces along some straight line is equal to zero.
2. The algebraic sum of the resolved parts of the forces along another straight line perpendicular to the former is also equal to zero.
3. The algebraic sum of the moments of the forces about any point is equal to zero.

By the resolved part of a force along a straight line, is meant the effect the force produces along that straight line. Thus if a force is parallel to a straight line, the resolved part is equal to the force; but if the force is perpendicular to the straight line, the resolved part is zero. When the force is inclined, the resolved part is equal to the force multiplied by the cosine of the angle included between the direction of the force and the straight line. Thus the resolved part of X along AB (the direction A to B being taken as positive), Fig. 29, is $+X \cos \theta$ and the resolved part of Y will be $-Y \cos \phi$.

By the moment of a force about a point, is meant the magnitude of the force multiplied by the length of the perpendicular drawn from the point to the direction of the force. The moment measures the power the force has of turning the body it is acting upon about the point; and if one direction of rotation (say like the hands of a watch) be taken as positive, then the reverse direction will be negative. Thus in Fig. 29 the moments of the forces X and Y about the point P will be $-Xa$ and $+Yb$.

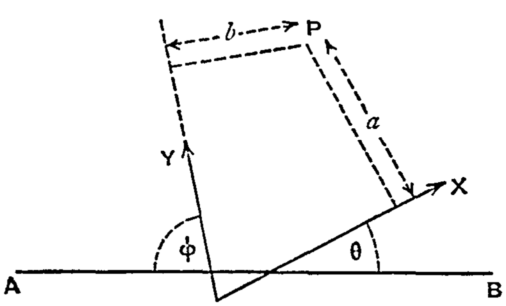


Fig. 29.

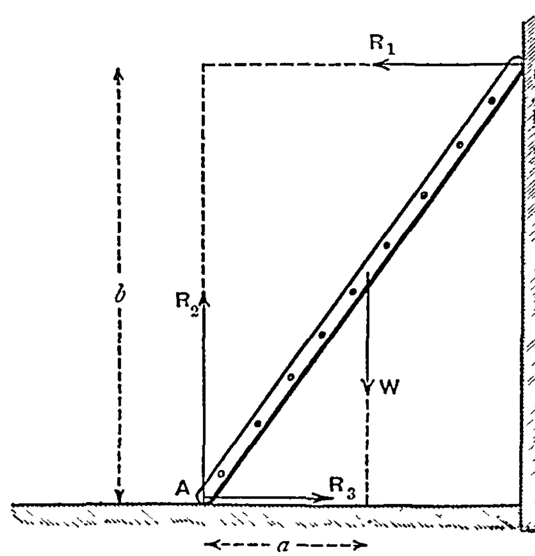


Fig. 30.

As an example, let the conditions of equilibrium be applied to the case of a ladder resting against a wall shown in Fig. 30.

Resolving horizontally, $R_3 - R_1 = 0$,

,, vertically, $R_2 - W = 0$,

Moments about A, $R_2 \times 0 - R_3 \times 0 + W \times a - R_1 \times b = 0$.

From which it appears, by solving these three equations, that

$$R_1 = \frac{a}{b} \quad W,$$
$$R_2 = W,$$
$$R_3 = \frac{a}{b} \quad W$$

Sometimes a solution is arrived at sooner by taking moments about more than one point, instead of resolving

APPENDIX III

To draw a Parabola.

Suppose the object to be to find the curve of bending moments for a

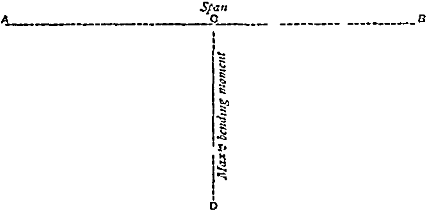


Fig 550

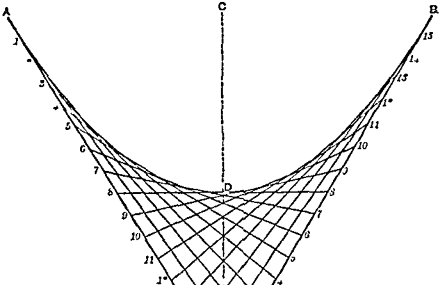


Fig 551

uniformly loaded beam. The data given are the two points of support AB (or, in other words, the span) and the maximum bending moment at the centre CD; that is, the three points A, B, and D (Fig. 550) are given, and it is required to draw a parabola through these three points.

Produce CD to F, making DF = CD; join AF and BF. Divide AF and BF into an even number of parts by repeated bisections, and number the points thus obtained as shown in Fig. 551. Then join 1, 1; 2, 2; 3, 3; etc. etc. These lines are tangents to the required parabola, which can then be drawn in. (The parabola has not been drawn in the figure, as it would confuse the construction, and the tangents sufficiently indicate the shape.)

APPENDIX IV.

To show that the Neutral Axis in a Beam passes through the Centre of Gravity of the Cross-section (*see p.* 44).

For a *rectangular* cross-section.

Let r_0 be the stress in the outside layers of fibres ($o'q'$ or $p'r'$, Fig. 62).

d, the depth of the beam.

x, the distance from the neutral axis of any layer of fibres situated above the neutral layer, such as $c'd'$, Fig. 62. Δx the thickness of the layer.

$b\Delta x$, the area of this layer of fibres (b being the width of the cross-section), x' and $b\Delta x'$ the same for a layer such as $c'f'$.

Then the total tension over the cross-section of layer $c'd'$ is, by assumptions 2 and 3, p. 44,

$$\frac{x}{\frac{1}{2}d} \times r_0 b\Delta x.$$

Hence the sum of the compressions above the neutral axis

$$= \frac{2r_0 b}{d}\Sigma x\Delta x,$$

and similarly the sum of the tensions below the neutral axis

$$= \frac{2r_0 b}{d}\Sigma x'\Delta x'.$$

But for equilibrium the tensions must be equal to the compressions. Hence

$$\Sigma x\Delta x - \Sigma x'\Delta x' = 0.$$

If now $\bar{x}$ be the distance of the centre of gravity from the neutral axis, we have, by taking moments about this axis,

$$\begin{aligned}\bar{x}(b\Sigma\Delta x + b\Sigma\Delta x') &= b\Sigma x\Delta x - b\Sigma x'\Delta x',\\ &= 0.\end{aligned}$$

that is, $\bar{x} = 0$, or the neutral axis passes through the centre of gravity.

The same reasoning can be applied to a cross-section of any shape, with the difference that the breadth of each layer, instead of being constant, varies.

APPENDIX V.

Diagrams of Shearing Stress.

These, for beams supported at the ends, are often differently arranged from those given at pages 58 to 60.

By many writers the shearing stresses acting in one direction, say thus $\uparrow\downarrow$, are called *positive*, and those acting in the other direction $\downarrow\uparrow$ *negative*, or *vice versa*

The diagram of positive shearing stress is shown on one side of the line representing the beam, and that for the negative shearing stress on the other

Thus in Fig 263 p 173 which represents the shearing stress caused by an uniformly distributed load, the diagram for the shearing stress of this direction $\uparrow\downarrow$ is below the line, that for the shearing stress $\downarrow\uparrow$ is above the line representing the beam, instead of the whole diagram of shearing stress being below the line as in Fig 80, p 59

Fig 255 p 167, is similarly arranged for a distributed and a concentrated load both diagrams from A to D being below the line, and both from D to B above the line instead of being as in Fig 91, p 59

Similarly in Fig 87, p 58, half the rectangle denoting the shearing stress from one abutment to C might be below the line and the other half above the line

In Fig 4 Appendix VI, the diagram of shearing stress with direction $\uparrow\downarrow$ may be called positive, and is above the line from A up to the point where the direction changes to $\downarrow\uparrow$, from this to F it may be called negative, and is below the line

The area of the shearing stress diagram between any point and either abutment is equal to the bending moment at the point

This is evident in Fig 82, p. 57, where (P being the point), the area of the shearing stress diagram at P (being the rectangle under PB) = PB × W = Wx and the bending moment at P $M_P = Wx$

In Fig 83 P being the point, the area of shearing stress diagram at P (being the triangle under PB) $= \frac{wx}{2} \times x = \frac{wx^2}{2}$

and $M_P = wx \times \frac{x}{2} = \frac{wx^2}{2}$

When the portions of shearing stress diagram between the point and the abutment chosen are of different signs or directions, the difference of these areas must be taken

Thus in Fig 90, if the area be taken of the diagram under PB we have to take the area of the triangle under BC *minus* the area of the triangle under PC

$$= \frac{wl}{2} \times \frac{l}{2} \times \tfrac{1}{2} - \left(\frac{l}{2} - x\right) \times \left(\frac{wl}{2} - wx\right) \times \tfrac{1}{2}$$
$$= \frac{wlx}{2} - \frac{wx^2}{2}$$

and

$$M_P = \frac{wl}{2} \times x - wx \times \frac{x}{2}$$
$$= \frac{wlx}{2} - \frac{wx^2}{2}$$

By working out other cases the student will find that the rule given above always holds good, and it is sometimes very useful

APPENDIX VI

Graphic method of finding the Bending Moments and Shearing Stresses in a loaded Beam

In Fig 1 AF is a beam carrying the loads W_1, W_2, W_3, W_4, at the points BCDE, through which draw vertical lines downwards as dotted

Draw another vertical line YZ (Fig 4) at a convenient distance. Along

YZ lay off W_1, W_2, W_3, W_4, on any convenient scale of loads. Take any point o, which is called the "*pole*," and draw the lines a, b, c, d, e, joining to it the extremities of the lines indicating the loads.

Bending Moments.—Starting from any convenient point X (Fig. 2) vertically under A, draw in succession a, b, c, d, e, parallel to a, b, c, d, e of Fig. 4, respectively—intercepted between the verticals from A to F of Fig. 1, and terminating at V on the vertical from F. Join XV, Fig. 2. The polygon thus formed is called the "*Funicular polygon.*" In Fig. 4 draw the horizontal line oH, and the dotted line oN—the latter parallel to XV. Then YN will represent R_A and NZ will represent R_F measured on the scale of loads.

Moreover in Fig. 2 the ordinate UT of the "*funicular polygon*" vertically under any point P measured on a proper scale represents the bending moment at P.

Take a foot as the unit of the linear scale, and a lb. as the unit of the scale of loads.

Then if Ho is equal to 1 foot on the linear scale, UT on the scale of loads will represent the number of foot pounds of the bending moment at this point.

If, however, Ho is more than one foot of the linear scale in length, then the length UT on the scale of loads must be multiplied by Ho to find the number of foot pounds of the bending moment. Of course any other units may be substituted for the foot and pound—for example, the inch and ton.

Shearing Stresses.—These are shown in Fig. 3, the construction of which is obvious.

The dotted horizontal lines are drawn through the extremities of the loads set off along YZ, and are intercepted between the verticals from Fig. 1 as shown.

A dotted horizontal line is drawn also through the lower extremity of R_A on Fig. 3, and at this point the shearing stress changes from $\uparrow\downarrow$ to $\downarrow\uparrow$ or $+$ to $-$.

The shearing stress at any point of AF is equal to the portion of the vertical from that point intercepted between the upper and lower lines of the shearing stress diagram (Fig. 3). Thus SQ is the shearing stress at the point P.

Example.—In Fig. 1 take AF = 20 feet.

AB = 4 feet.	W_1 = 80 lbs.
BC = 2 ,,	W_2 = 120 ,,
CD = 6 ,,	W_3 = 20 ,,
DE = 4 ,,	W_4 = 60 ,,
EF = 4 ,,	

Take the linear scale to be 10 feet = 1 inch, and the scale of loads 120 lbs. = 1 inch.

Lay off AB, BC, etc., on Fig. 1 on the linear scale, and W_1, W_2, W_3, W_4, Fig. 4 on the scale of loads. Complete the figures as described above.

Measure oH. It equals 10 feet on the linear scale.

Measure YN on the scale of loads. It measures 168 lbs., which should be equal to R_A. NZ measures 112 lbs., which should equal R_F.

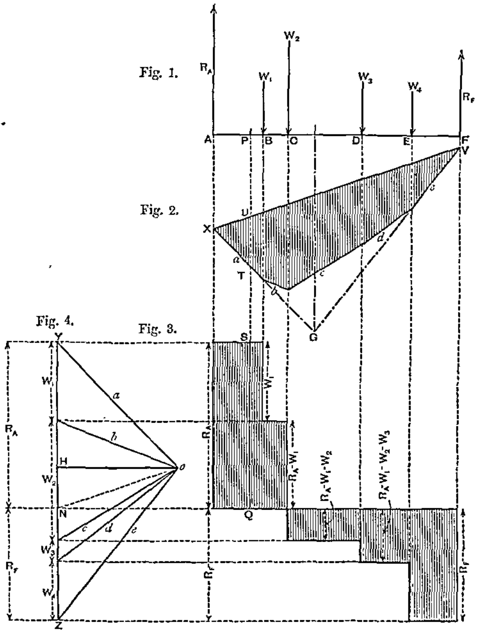
To check this by calculation (see page 60) we have

$$R_A = \frac{80 \times 16}{20} + \frac{120 \times 14}{20} + \frac{20 \times 8}{20} + \frac{60 \times 4}{20}$$

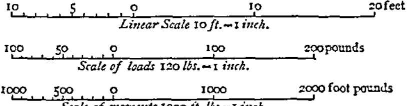
$$= \frac{3360}{20} = 168 \text{ lbs., which agrees.}$$

Similarly $R_F = 112$ lbs.

Fig. 1. Fig. 2. Fig. 3. Fig. 4.



SCALES.



Linear Scale 10 ft. = 1 inch.

Scale of loads 120 lbs. = 1 inch.

Scale of moments 1200 ft. lbs. = 1 inch.

To find the scale of bending moments we have

Ho in feet on the linear scale × number of lbs. per inch on load scale = the number of foot pounds in one inch on the scale of moments.

$$10 \text{ ft.} \times 120 = 1200 \text{ foot pounds}$$

$$\therefore \text{ on scale of moments } 1200 \text{ ft. lbs.} = 1 \text{ inch.}$$

Draw the scale.

Then to find the bending moment at any point P, draw PUT a vertical intercepted by the upper and lower sides of the polygon. UT will be the bending moment. Measuring UT on the scale of moment we find it = 500 foot pounds.[1]

Shearing Stress.—Prolonging PT through the shearing stress diagram we have the shearing stress at P = SQ = 168 lbs. By calculation we know that it equals R_A = 168 lbs., which agrees.

In the same way the bending moment and shearing stress at any other point can be found, and a similar process will enable the bending moments and shearing stresses for any other distribution of load to be found. When a load is uniformly distributed it may be divided into equal portions, and each of these treated as a load; but in this case the bending moments may be obtained more easily by drawing the parabola described at p. 34.

Centre of gravity or Resultant of system of parallel forces.—If the lines *a* and *e* be prolonged so as to meet at G, then a vertical drawn upwards through G will cut the beam at the centre of gravity of the loads—that is, the point through which their resultant would pass.

APPENDIX VII.

Bending moments, Breaking weights, and Shearing stresses for Beams and Cantilevers with various distributions of load.

The Table opposite shows for *Cantilevers* and *supported Beams* with different distributions of load the *Bending moments* and the *Breaking weights* for the cases pp. 28 to 40, and the *Shearing stresses* for the cases pp. 57 to 60.

Comparison of Concentrated and Distributed Loads.

From the table opposite it will be seen that if the *total* distributed weight wl be taken as W, then,

Case 1. *Cantilever.* Load at free end $W = \frac{fobd^2}{6l}$

Case 2. *Cantilever.* Uniformly loaded $wl = W = \frac{2fobd^2}{6l}$

Case 6. *Supported Beam.* Load in centre $W = \frac{4fobd^2}{6l}$

Case 7. *Supported Beam.* Load distributed $wl = W = \frac{8fobd^2}{6l}$

From this we see that

W in Case 2 is twice as great as in Case 1.

Hence *the load uniformly distributed over its length that a cantilever can bear is twice the load that it can bear at the free end.*

W in Case 7 is twice as great as in Case 6.

Hence *the load uniformly distributed over its length that a beam supported at the ends can bear is twice the load that it can bear at the centre.*

[1] If we had not made a scale of moments we should measure UT on scale of loads— x = 50 lbs.—and multiply by Ho = 10 feet. 50 lbs. × 10 feet = 500 foot pounds.

Again,

If a cantilever can bear W concentrated at its free end

It can bear 2W uniformly distributed over its length

A beam of the same length supported at both ends and loaded in the centre can bear 4W.

A beam of the same length supported at both ends and loaded uniformly throughout its length can bear 8W.

In all these cases the weight of the beam itself is omitted, being assumed to be so small compared with the loads as to be practically insignificant

APPENDIX VIII

Comparison of the Strength, Stiffness, and Shearing Strength, of Beams of Uniform Cross Section throughout their Length Symmetrical above and below the neutral axis and supported at both ends, with those fixed at one or both ends, showing the effects produced by fixing one or both ends.

Arrangement of Beam.	Arrangement of Load	Bending Moments	Effect of fixing on Strength	Deflection value of n in formula $\Delta = \frac{nWl^3}{EI}$ see p 66	Effect of fixing on Stiffness	Shearing Stress	Effect of fixing on Shearing Stress
Supported both ends	Centre	$\frac{Wl}{4}$ at centre		$\frac{1}{48}$		$\frac{W}{2}$ at ends	
Fixed both ends	Centre	$\frac{Wl}{8}$ at fixed ends $\frac{Wl}{8}$ at centre	Increased as 2 to 1	$\frac{1}{192}$	Increased as 4 to 1	$\frac{W}{2}$ at ends 0 in centre	No effect
Supported one end fixed at other	Centre	$\frac{3Wl}{16}$ at fixed end $\frac{5Wl}{32}$ at centre	Increased as 4 to 3	[illegible]	Increased as 2 3 to 1	$\frac{11}{16}$W at fixed end $\frac{5}{16}$W at supported end	Increased as $1\frac{3}{8}$ to 1 Reduced by $\frac{3}{8}$
Supported both ends	Uniformly distributed	$\frac{wl^2}{8}$ at centre		$\frac{5}{384}$		$\frac{wl}{2}$ at ends	
Fixed both ends	Do	$\frac{wl^2}{12}$ at fixed ends $\frac{wl^2}{24}$ at centre	Increased as 3 to 2	$\frac{1}{384}$	Increased as 5 to 1	$\frac{wl}{2}$ do	No effect
Supported one end, fixed at other	Do	$\frac{wl^2}{8}$ at fixed end $\frac{wl^2}{16}$ at centre of beam $\frac{9wl^2}{128}$ at centre of supported portion	No effect	$\frac{1}{185}$	Increased as 5 to 2 08	$\frac{5}{8}wl$ at fixed end $\frac{3}{8}wl$ at supported end	Increased as 5 to 4 Reduced by $\frac{1}{4}$

From the above table it will be seen that

The effect of fixing both ends of a beam as compared with leaving both ends supported is

With *Load at Centre* Strength doubled Stiffness increased as 4 to 1 Shearing stress not altered

With *Load uniformly distributed* Strength increased 50 per cent Stiffness increased as 5 to 1 Shearing stress not altered

The effect of fixing one end of a beam as compared with leaving both ends supported is

With *Load at Centre.* Strength increased 33 per cent. Stiffness increased as 2·3 to 1. Shearing stress increased at fixed end, reduced at supported end.

With *Load uniformly distributed.* Strength not altered. Stiffness increased as 5 to 2. Shearing stress increased at fixed end, reduced at supported end.

APPENDIX IX.

Deflection of Rectangular Beams under different Conditions of Load.

The formula (47), p. 68,

$$\Delta = \frac{12nWl^3}{Ebd^3} \quad . \quad . \quad . \qquad (47)$$

is for rectangular beams. It may, by the substitution of the different values of n from Table C, p. 67, be stated in a convenient form for different conditions of load.

Take the case of a beam fixed at one end, free at the other, with a load at the free end. Substituting in (47) from Table C for n, we have

$$\Delta = \frac{\frac{1}{3}Wl^3}{E\frac{bd^3}{12}} = \frac{4Wl^3}{Ebd^3}$$

Substituting similarly values of n for other cases we have the following Table.

Arrangement of Beam.	Load.	Formula.
Beam fixed one end, free other . .	at free end	$\Delta = \frac{4Wl^3}{Ebd^3}$
" " . .	uniform	$\Delta = \frac{3Wl^3}{2Ebd^3}$
Beam supported both ends . .	centre	$\Delta = \frac{Wl^3}{4Ebd^3}$
" " . .	uniform	$\Delta = \frac{5Wl^3}{32Ebd^3}$
Beam fixed both ends . . .	centre	$\Delta = \frac{1}{16}\frac{Wl^3}{Ebd^3}$
" " . . .	uniform	$\Delta = \frac{1}{32}\frac{Wl^3}{Ebd^3}$
Beam supported one end, fixed other .	centre	$\Delta = \frac{1}{9}\frac{Wl^3}{Ebd^3}$
" " " .	uniform	$\Delta = \frac{1}{15}\frac{Wl^3}{Ebd^3}$

APPENDIX X.

Relative Strength of Beams of different Sections.

The following are given as they may be of interest to the student, but they are of no practical value.

SQUARE BEAM, AND CYLINDRICAL POLE OF DIAMETER EQUAL TO SIDE OF BEAM

$$\overline{M} \text{ of } \textit{square beam} = \frac{f_o I}{y_o} = \frac{f_o \frac{bd^3}{12}}{\frac{d}{2}}$$

$$= \frac{f_o bd^2}{6}$$

In a square beam $b = d$ $\quad M = \frac{f_o d^3}{6} \qquad \text{I}$

$$\overline{M} \text{ of cylindrical pole} = \frac{f_o I}{y_o} = \frac{f_o \frac{\pi}{4} r^4}{r}$$

$$= \frac{f_o \pi r^3}{4}$$

but $r = \frac{d}{2}$ $\quad \overline{M} = \frac{f_o \pi \left(\frac{d}{2}\right)^3}{4}$

$$= \frac{f_o \pi d^3}{8 \times 4}$$

$$= \frac{f_o \times 3\ 14 \times d^3}{32}$$

$$= \tfrac{1}{10} f_o d^3 \qquad \text{II}$$

from I and II

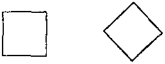
$\overline{M}$ square beam : $\overline{M}$ circular pole :: $\frac{1}{6} f_o d^3$: $\frac{1}{10} f_o d^3$

□ ○

$\frac{10}{60} f_o d^3$: $\frac{6}{60} f_o d^3$

10 to 6

THE STRENGTH OF A SQUARE BEAM SIDE VERTICAL IS TO THAT OF A SQUARE BEAM WITH DIAGONAL VERTICAL



7 : 5

A TRIANGULAR BEAM OF THE SAME AREA AND DEPTH AS A RECTANGULAR BEAM IS ONLY ONE HALF THE STRENGTH OF THE LATTER

The strength of a cylindrical pole to that of a square beam of the same area is as 1 *to* 1 18

The strength of a cylindrical beam to that of the strongest beam that can be cut out of it is as 1 53 *to* 1

APPENDIX XI

Continuous Beams of Uniform Section loaded uniformly with Points of Support equidistant from one another and in the same Straight Line.

On pp 74, 75 are shown the forms assumed by a uniformly loaded girder of *uniform strength* extending over two spans

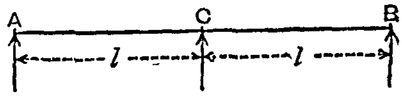
In practice, however, small continuous beams are generally of *uniform section.*

When such beams are *fixed* at the ends and continuous over one or more supports, each portion may be treated as a beam fixed at both ends (see p. 72, Case 2), and the moments of flexure, etc., will be found accordingly.

When, however, a beam is continuous over one or more supports, and

Total load $= 2wl$.

Fig. 1.



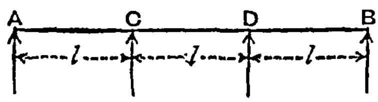
$R_A = \frac{3wl}{8}$ $R_C = \frac{10wl}{8}$ $R_B = \frac{3wl}{8}$

$M_A = 0$ $M_C = \frac{wl^2}{8}$ $M_B = 0.$

supported only at the ends—which is sometimes the case in practice—the

Total load $= 3wl$.

Fig. 2.



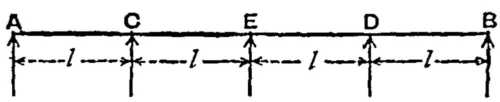
$R_A = \frac{4}{10}wl$ $R_C = \frac{11}{10}wl$ $R_D = \frac{11}{10}wl$ $R_B = \frac{4}{10}wl$

$M_A = 0$ $M_C = \frac{4}{5}\frac{wl^2}{8}$ $M_D = \frac{4}{5}\frac{wl^2}{8}$ $M_B = 0.$

resulting stresses are different, and are very difficult to find entirely by calculation.

Total load $= 4wl$.

Fig. 3.



$R_A = \frac{11}{28}wl$ $R_C = \frac{32}{28}wl$ $R_E = \frac{26}{28}wl$ $R_D = \frac{32}{28}wl$ $R_B = \frac{11}{28}wl$

$M_A = 0$ $M_C = \frac{6}{7}\frac{wl^2}{8}$ $M_E = \frac{4}{7}\frac{wl^2}{8}$ $M_D = \frac{6}{7}\frac{wl^2}{8}$ $M_B = 0.$

Reactions and Bending Moments.—Figs. 1 to 3 show the reaction and the bending moment at each point of support for beams continuous over from 2 to 4 spans.

Each span $= l$
Load on each span $= wl$

Deflection.—This will be a maximum in the two end spans, and may be obtained for those from Case 3, p. 73, and Appendix VIII., though if the two end spans are not reduced their inner ends will not be perfectly fixed and the actual deflection will be in excess of that calculated.

Points of Contrary Flexure.—These, if required to be known, may for each of the intermediate spans be found as in the case of a beam fixed at both ends (p. 72), or for each of the end spans, as in the case of a beam fixed at one end and supported at the other.

To find the Bending Moment from the Reactions.—When the reactions are known, the bending moment at any point may be found by the ordinary method described in Chapter III.

Thus in Fig 1

The bending moment at a section infinitely close to C will be

$$M_c = R_A \times AC - wl \times \frac{l}{2}$$

$$= \tfrac{3}{8}wl \times l - \frac{wl^2}{2}$$

$$= -\tfrac{1}{8}wl^2$$

Again in Fig 3

$$M_c = \tfrac{11}{28}wl \times l - \frac{wl^2}{2}$$

$$= -\tfrac{3}{28}\,wl^2$$

Beams continuous over several Supports.

The following table gives the reaction on each support caused by uniformly loaded beams of uniform section continuous over several supports

UNIFORMLY LOADED BEAMS CONTINUOUS OVER SEVERAL SUPPORTS

Table of Distribution of Loads

No of Span	Number of each Support and Load borne by it in terms of wl i e of load on each span									
	1st	2d	3d	4th	5th	6th	th	8th	9th	10th
1	$\frac{1}{2}$	$\frac{1}{2}$								
2	$\frac{3}{8}$	$\frac{10}{8}$	$\frac{3}{8}$							
3	$\frac{4}{10}$	$\frac{11}{10}$	$\frac{11}{10}$	$\frac{4}{10}$						
4	$\frac{11}{28}$	$\frac{32}{28}$	$\frac{26}{28}$	$\frac{32}{28}$	$\frac{11}{28}$					
5	$\frac{15}{38}$	$\frac{43}{38}$	$\frac{37}{38}$	$\frac{37}{38}$	$\frac{43}{38}$	$\frac{15}{38}$				
6	$\frac{41}{104}$	$\frac{118}{104}$	$\frac{108}{104}$	$\frac{108}{104}$	$\frac{108}{104}$	$\frac{118}{104}$	$\frac{41}{104}$			
7	$\frac{56}{142}$	$\frac{161}{142}$	$\frac{137}{142}$	$\frac{147}{142}$	$\frac{143}{142}$	$\frac{137}{142}$	$\frac{161}{142}$	$\frac{56}{142}$		
8	$\frac{152}{388}$	$\frac{440}{388}$	$\frac{374}{388}$	$\frac{392}{388}$	$\frac{388}{388}$	$\frac{392}{388}$	$\frac{374}{388}$	$\frac{440}{388}$	$\frac{152}{388}$	
9	$\frac{209}{530}$	$\frac{601}{530}$	$\frac{511}{530}$	$\frac{535}{530}$	$\frac{529}{530}$	$\frac{529}{530}$	$\frac{535}{530}$	$\frac{511}{530}$	$\frac{601}{530}$	$\frac{209}{530}$

Illustration —The principal rafter of the roof dealt with in Chapter XII is treated in page 214, first as if it were in two portions each supported at both ends, and next as a beam continuous over two spans In both cases it is uniformly loaded

In this example the maximum bending moment is the same under either supposition, being in the case of each supported portion $\frac{wl^2}{8}$ in the centre, and in the continuous beam $\frac{wl^2}{8}$ at the intermediate point of support

In the case, however, where a rafter extends over several points of support (see p 202, Fig 343) the maximum bending moment to which it is subjected will be very different if it is a continuous beam, compared with what it would be in a jointed and supported beam

Thus in Fig. 4, if it be assumed that there are joints at the points of

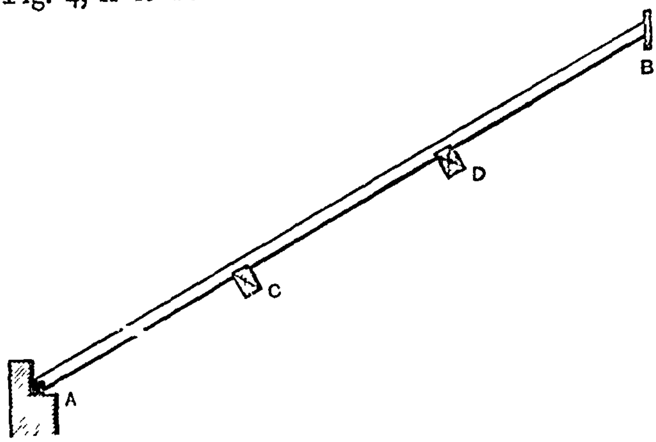


Fig. 4.

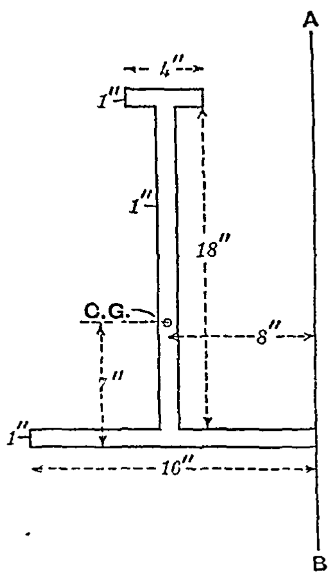
support, then AC, CD, DB are supported beams (see Case 7, p. 33), and the maximum bending moment is $\frac{wl^2}{8}$ at the centre of each.

If AB is assumed to be continuous, then (see Fig. 2 above) the maximum bending moment is at C or D, *i.e.* $\frac{4}{5} \times \frac{wl^2}{8}$ being less than $\frac{wl^2}{8}$, and therefore if the rafter is really continuous, all the points being accurately in a straight line a less scantling will do for it than if it were jointed and supported.[1]

APPENDIX XII.

Method of finding the Centre of Gravity of an unsymmetrical Cross Section of a Girder by Calculation.

Divide up the cross section into rectangles all parallel to each other and to the bottom edge of the section.



Rule.—Multiply the area of each rectangle by the distance of *its* centre of gravity from the bottom edge, and add these products together. Divide by the whole area of the section; the quotient is the distance of the centre of gravity from the bottom edge.

Example.—Thus the cross-section of the cast iron girder shown in the figure can be divided up into three rectangles, and the products are

$4 \times 1 \times 19{\cdot}5 + 18 \times 1 \times 10 + 16 \times 1 \times 0{\cdot}5 = 266.$

The whole area is $4 \times 1 + 18 \times 1 + 16 \times 1 = 38$. Therefore distance of CG from bottom edge $= \frac{266}{38} = 7$ inches.

Since the section is symmetrical vertically, the centre of gravity is determined as shown in the figure. Should, however, the figure be unsym-

[1] In reality the case lies between the two, because C and D are not *rigid* supports.

metrical in all directions, the above process must be repeated with reference to an axis perpendicular to the first one Thus in the present case, to find the distance of the centre of gravity from the axis AB the products are

$$4 \times 1 \times 8 + 18 \times 1 \times 8 + 16 \times 1 \times 8 = (4 + 18 + 16)8$$

Hence distance of centre of gravity

$$= \frac{(4 + 18 + 16)8}{4 + 18 + 16} = 8 \text{ inches.}$$

APPENDIX XIII.

Graphic Method of finding Centre of Gravity of unsymmetrical Cross Section of Girder.

Set off *abc*, Fig 2, respectively equal to the areas of the portions ABC of the girder, and join the extremities of the lines representing these with any point *x*, called the pole

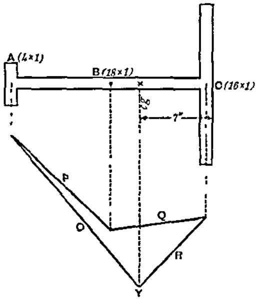


Fig 1.

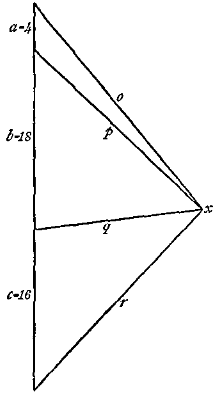


Fig 2.

Draw the polygon OPQR having its sides parallel to *opqr* respectively, and intercepted between verticals drawn through the centres of gravity of ABC From the intersection of R and O at Y draw a vertical. The point at which this cuts the centre line of the girder will be its centre of gravity

The figures show that this agrees with the calculations above (App XII)

APPENDIX XIV.

Moment of Inertia (I) of Various Sections.[1]

Section.	Value of I.
(1) *Rectangle* (breadth b, depth d)	$\dfrac{bd^3}{12}$
(2) *Isosceles triangle* (base b, height d)	$\dfrac{bd^3}{36}$
(3) *Circle* (radius r)	$\dfrac{\pi}{4}r^4$
(4) *Annulus* (R outer radius, r inner radius) . . .	$\dfrac{\pi}{4}(R^4 - r^4)$
(5) *Semicircle* either thus  or thus  . . .	$0{\cdot}11r^4$

(6) T *iron* $\dfrac{b(x^3 - z^3) + t(y^3 + z^3)}{3}$ (see Fig. 1).

(7) I girder divided into rectangles as in Fig. 2—

$$\tfrac{1}{12}\{ga^3 + k(b^3 - a^3) + l(c^3 - b^3) + m(d^3 - c^3)\}.$$

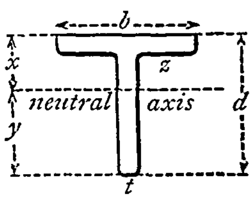


Fig. 1.

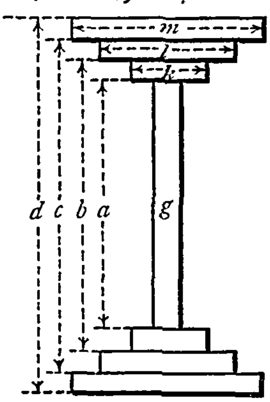


Fig. 2.

GENERAL RULE FOR ANY SECTION MADE UP OF A NUMBER OF SIMPLE FIGURES.

a. *Find the moment of inertia of each of the simple figures about an axis, traversing its centre of gravity parallel to the neutral axis of the complex figure.*

b. *Multiply the area of each simple figure by the square of the distance between its centre of gravity and centre of gravity of the whole figure.*

Add the results so found (by a *and* b *above) for the moment of inertia of the whole figure.* (RANKINE.)

I *for Rolled Beam.*—Applying No. 7 formula to find I for the section of the rolled iron girder, p. 83, we have

$$\begin{aligned} &\tfrac{1}{12}\{\tfrac{1}{2} \times 9^3 + 4(10^3 - 9^3)\}, \\ &= \tfrac{1}{12}\{\tfrac{1}{2} \times 729 + 4(1000 - 729)\}, \\ &= 120{\cdot}7. \end{aligned}$$

[1] In each case the moment of inertia is taken about an axis parallel to the neutral axis passing through the centre of gravity of the section.

π is the ratio between the circumference and diameter of a circle, and its numerical value is 3·1416.

I *for Cast Iron Girder* —Applying Rankine's rule to the section of the cast iron girder, Fig 3, we have

a I of upper flange $= \dfrac{4 \times 1^3}{12} = \dfrac{4}{12}$,

I of web $= \dfrac{1 \times 18^3}{12} = \dfrac{5832}{12}$,

I of lower flange $= \dfrac{16 \times 1^3}{12} = \dfrac{16}{12}$,

Total $\dfrac{5852}{12} = 487\ 6$

b Area × square of distance of centre of gravity from the neutral axis

Of upper flange $= 4 \times 12\ 5^2 = 625$,
Web $= 18 \times 3^2 = 162$,
Lower flange $= 16 \times 6\ 5^2 = 676$,

Total of a and $b = 1950 = \text{I}$

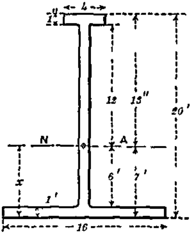


Fig 3

It should be observed that the thickness assumed for the lower flange, namely, 1 inch, is rather little for the width, but it simplifies the example

APPENDIX XV

Formulæ for finding approximately the Weight of Girders.

The weight of a girder can be found with a fair degree of accuracy by either of the following formulæ, the first being, however, the more accurate

Weight of girder in tons

$$= \frac{W \times L^2}{Cr_c D - L^2} \quad \text{(Prof Unwin s formula)},$$

$$= \frac{W \times L}{560} \quad \text{(Anderson's formula)},$$

where

W = distributed, or *equivalent* distributed, load to be borne by the girder, in tons.
L = span in feet
C = a factor whose value is from 1400 to 1500 for small girders, and from 1500 to 1800 for large bridges.
r_c = stress in compression boom (generally about 4 tons per square inch)
D = effective depth of girder in feet.

APPENDIX XVI

Rules for drawing Maxwell's Diagrams (*see p* 183)

The diagram of the truss itself is called the *frame diagram* The diagram of the forces is called the *stress diagram* or "*diagram of forces* '

In the stress diagram lines are drawn parallel to the several lines of the frame diagram, thus

1. Draw each *known force* in succession in the direction in which you know it acts.

2. Then from the two "*open ends*" draw the two *unknown forces* parallel to their directions on the frame. The first unknown force working in this direction $\rightarrow$ is at the end of the last known force, that is, at the end to which the arrow mentioned in next paragraph points.

3. Place arrows on the *stress diagram* beginning with the known forces, and make them all follow each other pointing in the same direction.

4. By transferring these arrows in imagination back to the *frame*, and considering them with reference to the point in question, you know whether each member is in compression or tension.

The above rules are applied to each joint of the frame in succession.

Example.—Thus in the example illustrated in Figs. 305 to 308, Plate II.

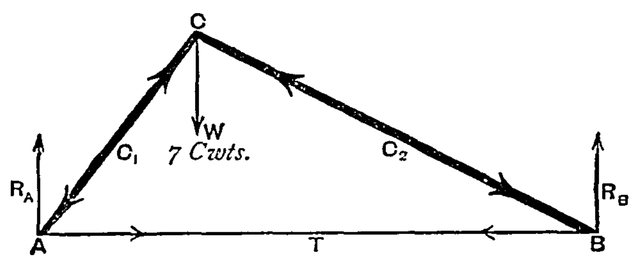


Fig. 1. *Frame Diagram.*

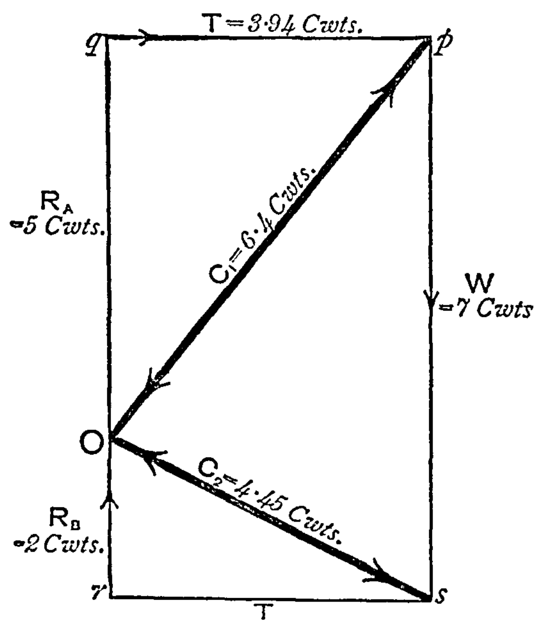


Fig. 2. *Stress Diagram.*

we know (pp. 180, 181) that W = 7 cwts. R_A = 5 cwts. R_B = 2 cwts.

Commencing at joint A in the frame (Fig. 1) draw for the stress diagram the known R_A = 5 cwts., and mark upon it the arrow head showing its direction, *i.e.* upwards. Then working in the direction $\rightarrow$ draw from its end *q*, to which the arrow points, the

unknown T parallel to AB of the frame, and from the other end O draw C_1 parallel to AC These intersect at p, and by measurement

$$qp = 3\ 94 \text{ cwts} = T$$
$$Op = 6\ 4 \text{ cwts} = C_1$$

Now place arrows on the stress diagram (Fig 2) We have marked the arrow on R_1 pointing upwards—following round in the same direction the arrow on T points towards p and that on C_1 towards O Transferring these to the frame we see that C_1 is in compression pushing towards A, and T in tension pulling from A

Proceeding to the next *joint C* on the frame, we have C_1 and W known and C_2 unknown

On the stress diagram we have C_1 already drawn Draw W vertically $= 7$ cwts., and draw a line from the end s of W parallel to C_2, this will be the line sO

We know the direction of the arrow for W is downwards Following on in the same direction the arrow for C_2 will point toward O and that for C_1 toward p The latter is in a different direction from what it was before, because C_1 is now in compression toward C

Transferring these arrows in imagination to the frame, we find C_1 C_2 both in compression

We come now to the last *joint, B,* and have R_2 C_2 T all known

Draw $R_2 = 2$ cwts and a line $T = 3{\cdot}94$ cwts. from s parallel to AB of frame This will be the line sr

Again the arrows are placed—on R_2 pointing upwards, on C_2 toward s, and on T towards r. Transferring these to the frame we find C_2 in compression toward B and T in tension from B

If a line drawn through s parallel to AB and equal in length to 3 94 cwts did not come exactly to r, it would be certain that some mistake had been made in the drawing

In such a case the diagram is said not to "*close*" properly, and this property of not closing when there is anything wrong in the diagram is very valuable, because the error thus shown to exist can be investigated and corrected, whereas in using *formulæ* there is often nothing to show the existence of an error, which remains undetected and makes the result incorrect The non closing may be due only to inaccurate drawing If this is suspected, the stress upon one or more bars may be checked by Ritter's method (see p 185)

APPENDIX XVII

Bow's System of lettering Maxwell's Diagrams (*see p* 184)

The following words quoted are from Mr Bow's own description of his system [1]

"This plan of lettering consists in assigning a particular letter to each enclosed area or space in, and also to each space (enclosed or not) around or bounding the truss, and attaching the same letter to the angle or point of concourse of lines which represents the area in the diagram of forces Any linear part of the truss or any line of action of an external force applied to it is to be named from the two letters belonging to the two spaces it separates · and the corresponding line in the reciprocal diagram of forces which represents the force acting in that part or line, will have its *extremities* defined by the same two letters"

Example [2]—Figs 1 and 2 give an example of a truss and a diagram of forces so treated

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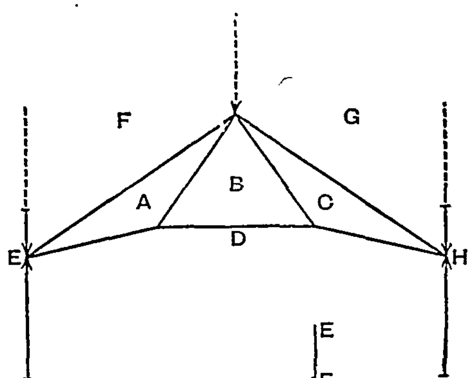
[1] From Bow's *Economics of Construction.* [2] *Ibid*, but condensed.

The spaces *in* the truss are lettered ABC, those around the truss DEFGH. These letters are attached to the corresponding points of concourse in Fig. 2.

Fig. 1. *Truss.*



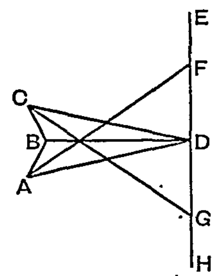


Fig. 2. *Diagram of Forces.*

Each line in the truss is lettered according to the letters of the spaces it separates. Thus the line separating spaces A and F is AF and is shown as AF on the force diagram below. Similarly the line separating spaces B and C is BC.

If any triangle in the truss be taken, such as A, the stresses on its sides as shown on the force diagram will be found to radiate from A. Thus, in Fig. 2, AB, AF, AD radiate from A.

Examples of the application of Bow's method to girders will be found at p. 194.

APPENDIX XVIII.

Calculation of the Stresses upon the Iron Roof (Example 42, p. 209) by the Method of Sections.

Referring to Example 42, p. 209, we now proceed to find the stresses by means of the method of sections (see p. 185).

Stresses due to Permanent Loads.—Commencing as before at joint A (Fig. 356, Plate III.), we take the section shown in Fig. 398, which cuts through only two bars; therefore to find the stress in AF we can take moments about any point in AD, round m for instance. $R_A = 1{\cdot}17$ tons (see p. 209). We find by measurement that the lever arm of R_A is 8·53 feet, and of the stress in AF, 2·39 feet.

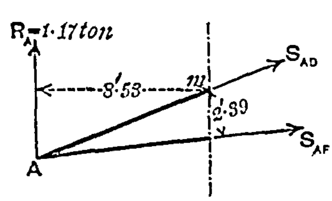


Fig. 398.

Hence

$$8{\cdot}53 \times 1{\cdot}17 - 2{\cdot}39 \times S_{AF} = 0,$$

or

$$S_{AF} = \frac{8{\cdot}53 \times 1{\cdot}17}{2{\cdot}39} = +4{\cdot}17 \text{ tons.}$$

Similarly to find S_{AD} we can take moments about any point in AF. We find, for instance,

lever arm of R_A = 10·35 feet,

" S_{AD} = 2·72 feet.

Hence

$$10.35 \times 1.17 + 2.72 \times S_{AD} = 0$$

or

$$S_{AD} = -4.45 \text{ tons}$$

so that AD is in compression

To find S_{DF} we can take the section shown in Fig 399 and the lever arms are marked on the figure Taking moments about A we have

$$10 \times 0.78 + 10.78 \times S_{DF} = 0$$

$$S_{DF} = -0.72 \text{ ton}$$

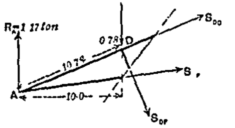


Fig 399

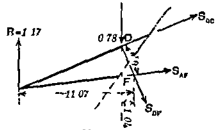


Fig 400

The same section will enable us to find S_{DC} the lever arms are as shown in Fig 400 for

R_A, 11.07 feet
W_1 1.04
S_{DC} 2.90

and therefore

$$11.07 \times 1.17 \quad 1.04 \times 0.78 + 2.90 \times S_{DC} = 0$$

whence

$$S_{DC} = \quad 4.20 \text{ tons}$$

Lastly to find S_{FG} and S_{FC} we take the section shown in Fig 401, and obtain the lever arms as marked whence

$$20 \times 1.17 - 10 \times 0.78 - 6.67 \times S_{FG} = 0,$$

$$S_{FG} = +2.34 \text{ tons}$$

For S_{FC} we must take moments about the intersection of FG and DC, thus

$$3.25 \times 1.17 + 6.75 \times 0.78 - 4.69 \times S_F$$

$$S_{FC} = +1.93 \text{ ton}$$

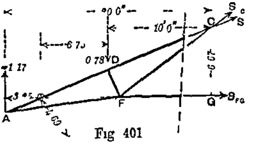


Fig 401

On examination (see Table H) it will be found that the two methods do not in all cases give exactly the same values for the stress The differences are due to slight errors in the drawing of the diagram and in measuring the lever arms

Stresses due to Wind Pressure —*It does not appear necessary to describe the* working of this method again in detail and therefore the various sections to be taken are simply marked on the figures together with the turning points and corresponding lever arms and the calculations are given below for each stress

Case 1 *Wind on left—Reactions parallel to wind pressure* —S_{AD} Section 1 1 *Fig 402* Turning point *a*

$$2.69 \times S_{AD} + 10.0 \times 0.88 = 0$$

$$S_{AD} = -3.27 \text{ tons}$$

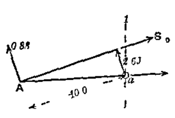


Fig 402

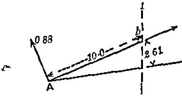


Fig 403

S_{AF}: Section 1, 1, Fig. 403. Turning point *b*.

$$-2{\cdot}61 \times S_{AF} + 10{\cdot}0 \times 0{\cdot}88 = 0,$$
$$S_{AF} = +3{\cdot}37 \text{ tons.}$$

S_{DF}: Section 2, 2, Fig. 404. Turning point A.

$$AD \times S_{DF} + AD \times 0{\cdot}96 = 0,$$
$$S_{DF} = -0{\cdot}96 \text{ ton.}$$

'hat is, the compression in DF is equal to the wind pressure acting at D.

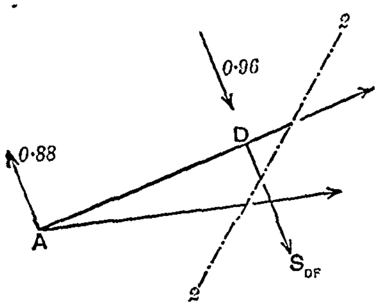


Fig. 404.

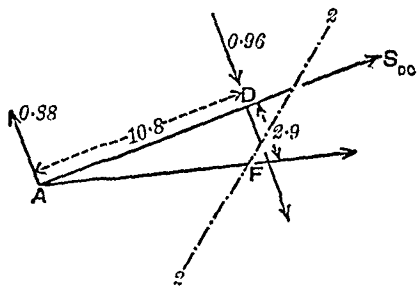


Fig. 405.

S_{DC}: Section 2, 2, Fig. 405. Turning point F.

$$2{\cdot}9 \times S_{DC} + 10{\cdot}8 \times 0{\cdot}88 = 0,$$
$$S_{DC} = -3{\cdot}28 \text{ tons.}$$

Thus we see (allowing for errors of measurement) that

$$S_{AD} = S_{DC}.$$

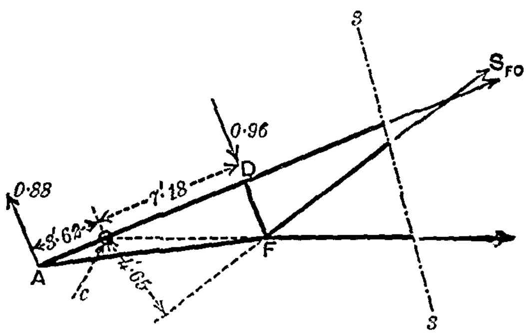


Fig. 406.

S_{FC}: Section 3, 3, Fig. 406. Turning point *c*.

$$-4{\cdot}65 \times S_{FC} + 3{\cdot}62 \times 0{\cdot}88 + 7{\cdot}18 \times 0{\cdot}96 = 0,$$
$$S_{FC} = +2{\cdot}17 \text{ tons.}$$

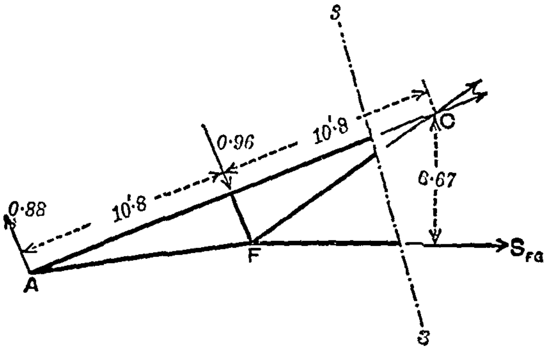


Fig. 407.

S_{FG} Section 3, 3, Fig 407 Turning point C

$$-6\ 67 \times S_{FG} - 10\ 8 \times 0\ 96 + 21\ 6 \times 0\ 88 = 0,$$
$$S_{FG} = +1\ 30 \text{ ton}$$

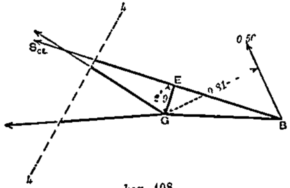


Fig 408

S_{CE} · Section 4, 4, Fig 408 Turning point G.

$$-2\ 9 \times S_{CE} - 9\ 81 \times 0\ 56 = 0,$$
$$S_{CE} = -1\ 89 \text{ ton}$$

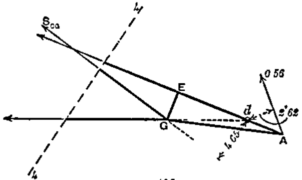


Fig 409

S_{CG} Section 4, 4, Fig 409 Turning point d

$$4\ 65 \times S_{CG} - 2\ 62 \times 0\ 56 = 0,$$
$$S_{CG} = +0\ 32 \text{ ton}$$

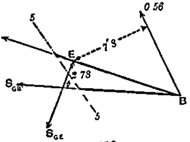


Fig 410.

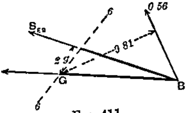


Fig 411.

S_{GE} Section 5, 5, Fig 410 Turning point B

$$-10\ 8 \times S_{GE} - 0 \times 0\ 56 = 0,$$
$$S_{GE} = 0$$

S_{GB} Section 5, 5, Fig 410 Turning point E

$$+2\ 78 \times S_{GB} - 7\ 8 \times 0\ 56 = 0,$$
$$S_{GB} = +1\ 57 \text{ ton}$$

S_{EB} Section 6, 6 Fig 411 Turning point G

$$-2\ 9 \times S_{EB} - 9\ 81 \times 0\ 56 = 0,$$
$$S_{EB} = -1\ 9 \text{ ton}$$

Again we find slight discrepancies between the values found by the two methods, and, as before, these discrepancies are due to errors in measurement.

Case 2. Wind on right—Reactions parallel to normal wind pressure.—The stresses can be deduced from Case 1.

Case 3. Wind on left—Reaction at free end vertical.—It should be noted that P_A and R_A being no longer exactly opposite in direction (as they were in Case 1), we cannot now use their difference only, but must introduce them separately into the equation.

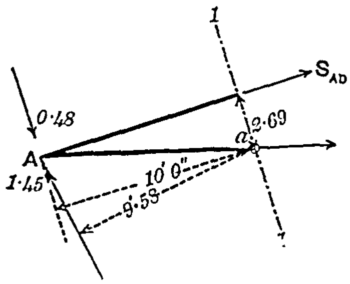


Fig. 412.

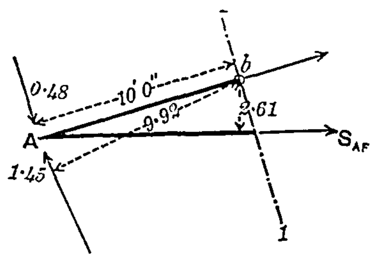


Fig. 413.

S_{AD}: Section 1, 1, Fig. 412. Turning point *a*.

$$+2{\cdot}69\times S_{AD}-10{\cdot}0\times 0{\cdot}48+9{\cdot}58\times 1{\cdot}45=0,$$

$$S_{AD}=-3{\cdot}38 \text{ tons.}$$

S_{AF}: Section 1, 1, Fig. 413. Turning point *b*.

$$-2{\cdot}61\times S_{AF}-10{\cdot}0\times 0{\cdot}48+9{\cdot}92\times 1{\cdot}45=0,$$

$$S_{AF}=+3{\cdot}67 \text{ tons.}$$

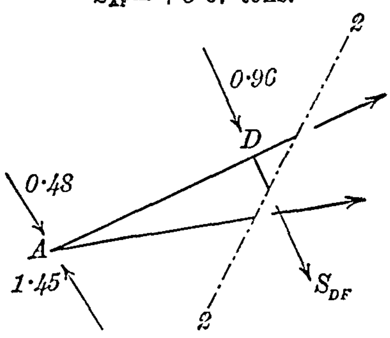


Fig. 414.

S_{DF}: Section 2, 2, Fig. 414. Turning point A.

$$AD\times S_{DF}+AD\times 0{\cdot}96=0,$$

$$S_{DF}=-0{\cdot}96 \text{ ton,}$$

which is the same value as already obtained under Case 1. In fact, since this stress does not depend on the reaction at the abutment, but only on the load at the joint (as is evident from the equation of moments), it follows that it is unnecessary to recalculate it each time.

S_{DC}: as in Case 2.

$$S_{CD}=S_{AD}=-3{\cdot}38 \text{ tons.}$$

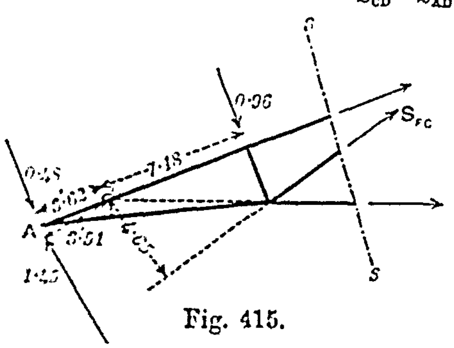


Fig. 415.

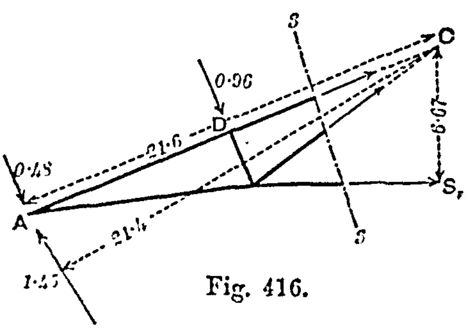


Fig. 416.

S_{FC}: Section 3, 3, Fig 415. Turning point *c*.

$$-4{\cdot}65\times S_{FC}+7{\cdot}18\times 0\ 96+3{\cdot}51\times 1{\cdot}45-3\ 62\times 0\ 48=0,$$
$$S_{FC}=+2\ 2 \text{ tons}$$

S_{FG}: Section 3, 3, Fig. 416. Turning point C.

$$-6{\cdot}67\times S_{FG}-10\ 8\times 0\ 96-21\ 6\times 0{\cdot}48+21{\cdot}4\times 1\ 45=0,$$
$$S_{FG}=+1{\cdot}54$$

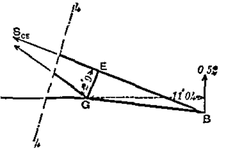


Fig. 417.

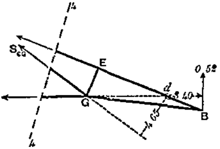


Fig. 418.

S_{CE}: Section 4, 4, Fig 417. Turning point G

$$-2{\cdot}9\times S_{CE}-11{\cdot}04\times 0{\cdot}52=0,$$
$$S_{CE}=-1\ 98 \text{ ton.}$$

S_{CG}: Section 4, 4, Fig. 418 Turning point *d*

$$+4\ 65\times S_{CG}-3\ 40\times 0\ 52=0,$$
$$S_{CG}=+0\ 38 \text{ ton.}$$

S_{FG} could also be found very easily from this section

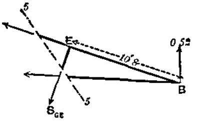


Fig. 419.

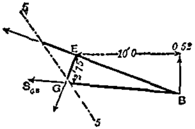


Fig. 420.

S_{GE}: Section 5, 5, Fig. 419. Turning point B.

$$-10{\cdot}8\times S_{GE}-0\times 0{\cdot}52=0,$$
$$S_{GE}=0.$$

S_{GB}: Section 5, 5, Fig 420. Turning point E

$$+2{\cdot}75\times S_{GB}-10\ 0\times 0\ 52=0,$$
$$S_{GB}=+1{\cdot}88 \text{ ton.}$$

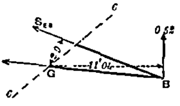


Fig 421.

S_{EB}: Section 6, 6, Fig. 421. Turning point G.

$$-2{\cdot}9\times S_{EB}-11{\cdot}04\times 0{\cdot}52=0,$$
$$S_{EB}=-1{\cdot}98 \text{ ton.}$$

Case 4. Wind on right—Reaction at free end vertical.

S_{AD}: Section 1, 1, Fig. 422. Turning point F.

$$2 \cdot 9 \times S_{AD} + 5 \cdot 40 \times 0 \cdot 87 = 0,$$
$$S_{AD} = -1 \cdot 62 \text{ ton.}$$

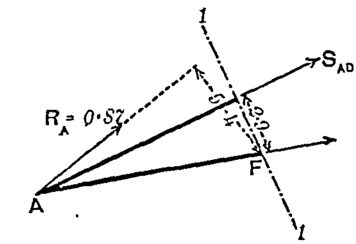


Fig. 422.

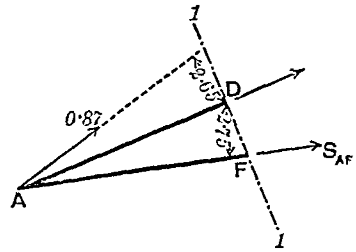


Fig. 423.

S_{AF}: Section 1, 1, Fig. 423. Turning point D.

$$-2 \cdot 75 \times S_{AF} + 2 \cdot 65 \times 0 \cdot 87 = 0,$$
$$S_{AF} = +0 \cdot 84 \text{ ton.}$$

S_{DF}; the stress in DF is clearly 0.

S_{DC}: Similarly, as in Case 3,

$$S_{DC} = S_{AD} = -1 \cdot 62 \text{ ton.}$$

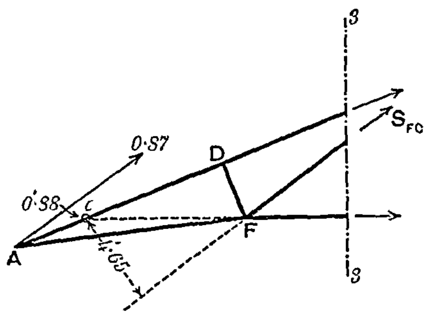


Fig. 424.

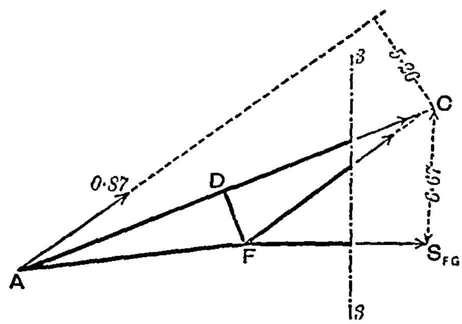


Fig. 425.

S_{FC}: Section 3, 3, Fig. 424. Turning point c.

$$-4 \cdot 65 \times S_{FC} + 0 \cdot 88 \times 0 \cdot 87 = 0,$$
$$S_{FC} = +0 \cdot 16 \text{ ton.}$$

S_{FG}: Section 3, 3, Fig. 425. Turning point C.

$$-6 \cdot 67 \times S_{FG} + 5 \cdot 26 \times 0 \cdot 87 = 0,$$
$$S_{FG} = +0 \cdot 69 \text{ ton.}$$

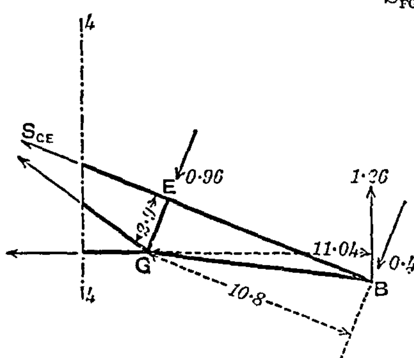


Fig. 426.

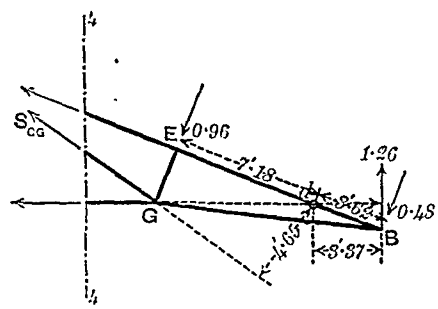


Fig. 427.

S_{CE}: Section 4, 4, Fig. 426. Turning point G.

$$-2 \cdot 9 \times S_{CE} - 0 \times 0 \cdot 96 - 11 \cdot 04 \times 1 \cdot 26 + 10 \cdot 8 \times 0 \cdot 48 = 0,$$
$$S_{CE} = -3 \cdot 0 \text{ tons.}$$

S_{CG}: Section 4, 4, Fig. 427. Turning point d.

$$+4 \cdot 65 \times S_{CG} - 7 \cdot 18 \times 0 \cdot 96 + 3 \cdot 62 \times 0 \cdot 48 - 3 \cdot 37 \times 1 \cdot 26 = 0,$$
$$S_{CG} = +2 \cdot 02 \text{ tons}$$

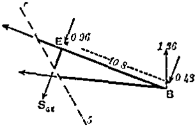


Fig 428

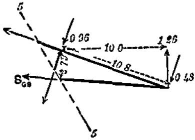


Fig 429

S_{GE} Section 5, 5, Fig 428 Turning point B

$$-10\ 8 \times S_{GE} - 10\ 8 \times 0\ 96 = 0,$$

$$S_{GE} = -0\ 96 \text{ ton}$$

S_{GB}. Section 5, 5, Fig 429 Turning point E

$$+2\ 75 \times S_{GB} + 10\ 8 \times 0\ 48 - 10\ 0 \times 1\ 26 = 0,$$

$$S_{GB} = +2\ 7 \text{ tons}$$

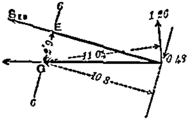


Fig 430

S_{EB} Section 6, 6, Fig 430 Turning point G

$$-2\ 9 \times S_{EB} - 11\ 04 \times 1\ 26 + 10\ 8 \times 0\ 48 = 0,$$

$$S_{EB} = -3\ 0 \text{ tons}$$

APPENDIX XIX

Centre of Pressure in Masonry Joints (*see p* 219)

It can be shown by an investigation, similar to, but more complicated than that given at p 219, that the position of the centre of pressure for a joint of *any section whatever, when the pressure vanishes along one edge, depends on* the moment of inertia of the section Rankine[1] gives the result as follows —

Let A denote the area of the joint

y, the distance from the centre of gravity of the figure of the joint to the edge farthest from the centre of pressure

h, the total breadth of the joint in the same direction

I, the moment of inertia of that figure, computed as for the cross section of a beam relatively to a neutral axis traversing the centre of gravity at right angles to the direction of the deviation to be found

δ, the deviation to be found

Then
$$\delta = \frac{I}{Ay}.$$

[1] *A Manual of Civil Engineering*, by W. T M Rankine, F R S, etc, p 378, 10th Edition

If the joint is rectangular, as shown in Fig. 431, p. 218, and the centre of pressure is situated in GH, then :

$$A = BD \,.\, GH,$$

$$y = \frac{GH}{2},$$

$$h = GH,$$

$$I = \frac{1}{12} \,.\, BD \,.\, GH^3,$$

$$\delta = \frac{GH}{2} - GC.$$

Hence
$$GC = \frac{GH}{2} - \frac{2 \,.\, BD \,.\, GH^3}{12 \,.\, BD \,.\, GH^2},$$

or
$$GC = \tfrac{1}{3} GH, \text{ as before.}$$

The following values of GC for a few sections will be found useful in practice :—

Description of Section.	Value of GC.
Rectangular	$\frac{1}{3}$GH
Circular	$\frac{3}{8}$GH
Hollow rectangular chimneys .	$\frac{1}{4}$GH approximate
,, circular chimneys .	$\frac{1}{3}$GH approximate

APPENDIX XX.

Arch with Unsymmetrical Load.

The following memorandum by Mr. Henry Fidler describes how the line of least resistance in an arch with an unsymmetrical load may be drawn, by the method devised by Professor Fuller, and modified by Professor Perry.

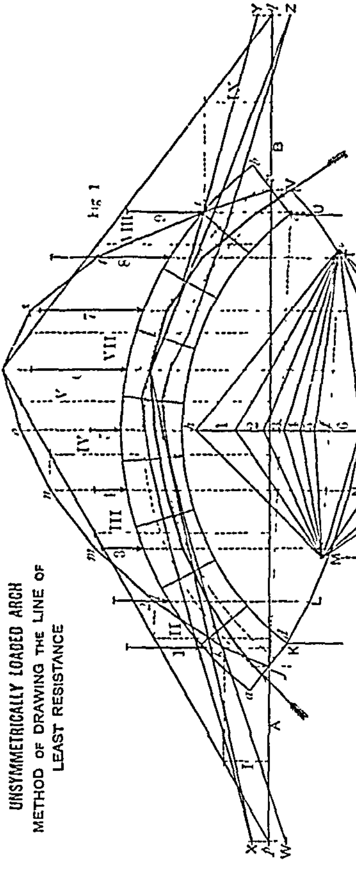
Let *abcd* be a segmental arch divided into any convenient number of imaginary voussoirs (in this case 9), and loaded unequally.

The positions and intensities of the loads are indicated by the vertical arrows 1, 2, 3, 4, etc., and the direction of each vertical force is supposed to pass through the centre of gravity of each voussoir.

The centre third of the depth of the arch ring is indicated by the dotted arcs, and it is required to draw the polygon of forces corresponding to the load and contained within the boundaries of the central third.

With any pole *e*, draw the force diagram, Fig. 2, shown below the arch, plotting the vertical loads 1, 2, 3, etc. to any convenient scale. Draw the polygon of forces JKLMNORSTUV in the usual way, terminating with the closing line JV, to which *ef* is parallel, giving the vertical components of abutment reactions *fg*, *fh*, for the right and left hand abutments respectively.

For the sake of convenience repeat the polygon of forces *jklmnorstuv* above the arch to an increased vertical scale if necessary, plotting the vertical ordinates from a horizontal base AB. On AB take any two convenient points *p*, *q*, and join *pr*, *qr*.



UNSYMMETRICALLY LOADED ARCH

METHOD OF DRAWING THE LINE OF LEAST RESISTANCE

Timber Beams, *supported at ends and uniformly loaded*

STRENGTH.—For Fir

The *safe* distributed dead load in cwts $= \frac{bd^2 \text{ in inches}}{L \text{ in feet}}$ (I)[1]

(See pp. 50 51.) This assumes the modulus of rupture $f_r = 5040$ lbs., or 45 cwts, and a factor of safety of 5

For *English Oak* and *Pitch Pine* multiply the result of (I) by $1\frac{1}{2}$

For *Teak* multiply by 2, and for *Greenheart* by 3

STIFFNESS.—When a beam supported at the ends is loaded with the distributed *safe* weight found by (I) then (see Equation 45, p. 67)

$$\text{Deflection at centre} = \tfrac{1}{40}\,\frac{S^2 \text{ in feet}}{d \text{ in inches}} \qquad \text{(II)}$$

For *Fir, Oak, Teak, Greenheart*

The distributed dead load in cwts. to cause a deflection of $\frac{1}{40}''$ per foot of span $= \frac{bd^3 \text{ in inches}}{S \text{ in feet}}$ (III)[2]

(See Equation 47, p. 68.) For dead load in centre to cause the same deflection, multiply the result of III by $\frac{5}{8}$

In II and III the modulus of elasticity is taken at 1,210 000 for large scantlings

Cast iron Girders *supported at ends and uniformly loaded*

The *safe* distributed dead load in *tons* $= \frac{a\,d}{S}$ (IV)

a being the effective area of tension flange in inches—*d* the depth of girder in inches—S the span in feet.

(See Equation 57, p. 95.) This assumes $c = 6\frac{3}{4}$ tons and factor of safety = 4

Wrought iron, rolled, or Plate Girders, *supported at ends and uniformly loaded*

The safe distributed dead load in *tons* $= \frac{3ad}{S}$ (V)

(See Equation 54, p. 87.) *a* being the effective area of compression flange in inches. The ultimate resistance to compression is taken at 18 tons per square inch the factor of safety as 4. The limiting stress per square inch is therefore $4\frac{1}{2}$ tons

Open Webbed Girders

Load is as in (V) above

Stress on any bar in web

$$= \frac{\text{shearing stress at the point} \times \text{length of bar}}{\text{depth of girder}} \qquad \text{(VI)}$$

(See p. 194.)

Retaining Walls

For water (see p. 235) mean thickness $= \frac{1}{2}$ of height (VII)

For average earth (see p. 244) „ $= \frac{1}{3}$ „ (VIII)

Arches, *thickness of*, from 10 to 25 span

Masonry—Block, 1 inch for every foot of span (IX)

Rubble, $1\frac{1}{4}$ inch „ „ (X)

Brick, $\frac{1}{2}$ brick for every 5 feet of span (XI)

---

[1] *P. E. Aide Memoire* [2] Seddon

Water Supply.—*Pipes flowing full* (see p. 267).

G = Gallons supplied per *minute.*
H = Head in feet.
L = Length of pipe in feet.
d = diameter of pipe in *inches.*

$$G = 28\sqrt{\frac{d^5 H}{L}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(XII.)}$$

$$d = \frac{1}{4}\sqrt[5]{\frac{LG^2}{H}} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad \text{(XIII.)}$$

For $\frac{1}{2}''$ pipes take $\frac{5}{8}$, for $1''$ pipes take $\frac{4}{5}$, and for pipes of over $12''$ diameter take $\frac{11}{10}$, of the result found by XII.

For $\frac{1}{2}$ pipes take d $\frac{1}{3}$ larger than as found by XIII.

Ultimate Resistance, etc. etc., of materials.

Showing the weight per cubic foot and the ultimate resistance of the materials used under stress in building construction, to Tension, Compression, and Shearing, also the Modulus of Rupture and Modulus of Elasticity for each.

Material.	Ultimate resistance *in tons per square inch* to							Elastic limit, tons per square inch, ordinary.	f_r Modulus of Rupture in lbs. per square inch			E Modulus of Elasticity in lbs. per square inch.			Weight per cubic foot in lbs. (dry).
	Tension			Compression.			Shearing								
	From	To	Ordinary.	From	To	Ordinary.	Ordinary.		From	To	Ordinary	From	To	Ordinary.	
Cast iron	5	15	9	20	64	48	8	none			40,000	12,000 000	23,000 000	17,000 000	434 to 456 av. 444
Wrought iron bars	15	31	25	16	22	20	20	10 to 16			42,000 for plate girders	24,000,000	29,000,000	29 000,000	474 to 487 av. 480
Wrought iron plates, grain lengthways	20	25	21												··
Wrought iron plates, grain crossways	19	23	20												··
Wrought iron angle irons	17	28	24												··
Mild steel (for Structures)	26	32	28				24	15 to 20						30,000,000	457 to 493 av. 490
	Ultimate resistance in lbs per sq inch.														
	Tension			Compression.											
	From	To	Ordinary	From	To	Ordinary.									
Oak, British	5800	19,800	10,000	3,946	10,000	8,000			5,960	13,700	10,000	444,080	1,750 000	1,200,000	39 to 69
Fir (red), Baltic	2240	14,000	7,000	2,500	10,000	6 000			3,683	9,540	6,600	752,420	2,274,000	1,440,000	25 to 48
Pitch Pine	4200	7,650	5,000	6,090	6,790	6,400			5,700	15 000	11,000	755,240	2,274,000	1,500,000	31 to 58
American Yellow Pine	1800	2 800	2,000	1,570	5,600	4,000			5,650	9,230	6,800	621,050	1,590,000	1,500,000	26 to 37
Elm, English	4070	14,000	5,000	5,600	10,300	6 000			4 320	9,700	6,000	250 8[illegible]0	699,800	600 000	33 to 48
Beech	4950	22,000	11,000	7,720	9,360	8,000			9,000	12,000	10 000	1,350,000	1,354,000	1,350 000	9 to 53
Teak	2500	21,000	10 000	4 240	12,000	9 000			8 120	19 000	13,000	530 990	2,414,000	1,500 000	41 to 57
Greenheart	7980	9,380	8,800	12,900	15,540	13,000			16,360	27,000	20 000	[illegible]	2 656 400	1 100,000	67 to 73

N B.—The figures in the above table are taken from various authorities. Those given in the columns headed "ordinary" are not averages, but show the *figures that* may be used for ordinary work. The figures for timber in the columns headed "To" are from experiments upon very small and perfect specimens as given by Barlow, Bevan, Tredgold, etc. Those in the columns headed "From" *are from more* recent experiments by Laslett, Kirkaldy, etc., on large scantlings—many of these are taken from the table in Seddon's *Builder's Work*.

When nothing special is known about the material to be used, the figures in the columns headed "Ordinary" may be taken. If the timber is known to be very good, and well seasoned, and is to be used *in small scantlings, free from* defects, then higher figures may be used, after consulting the columns headed "To," which however are very extreme. If the material is likely to be inferior, unseasoned, and to be used in scantlings full of defects, smaller *figures approximating to those* in the columns headed "From" may be used. In the column for weight of timber, the higher figures are for unseasoned, the lower for well seasoned material.

TABLE IA.

Safe Resistance of Materials.

This Table only includes the materials most generally in use, and the values given can be depended upon as quite safe for the ordinary quality of materials.[1]

SAFE RESISTANCE TO TENSION.

Cast iron	1·5 ton per sq. inch	Oak, English	16 cwts. per sq. inch	Portland cement, neat, after 9 mon.	1·0 cwt., per sq. inch
Wrought iron built-up girders	5 tons do.	Fir, Baltic	12 do., do.	B'kwork in cement[2]	0·10 cwt., do.
Wrought iron rolled joists	5 do., do.	Pine, American yellow	8 do., do.	Do. in mortar (hydraulic lime)[2]	0·06 do., do.
Steel, mild	6·5 do., do.	Pine, pitch	10 do., do.	Concrete, Portland cement, 5 to 1	0·8 do., do.
		Teak Moulmein	10 do., do.		

SAFE RESISTANCE TO COMPRESSION.

Cast iron	8 tons per sq. inch	Pine, pitch	10 cwts. per sq. inch	Bricks, ordinary stock	0·8 cwt. per sq. inch
Wrought iron built-up girders	4 do., do.	Teak, Moulmein	12 do., do.	Brickwork in mortar (good)[3]	0·5 do., do.
Wrought iron rolled joists	5 do., do.	Portland cement, neat, after 9 months	9 do., do.	Brickwork in cement[3]	0·8 do., do.
Steel, mild	6·5 do., do.	Mortar, common	0·5 cwt. do.	Granite, Aberdeen	8 cwts. do.
Oak, English	13 cwts., do.	Concrete, lime	0·5 do., do.	Limestone, granular	7 do., do.
Fir, such as Riga or Dantzic	10 do., do.	,, Portland cement, 5 to 1	2 cwts. do.	..	..
Pine, American yellow	6 do., do.	Concrete, Portland cement, 10 to 1	1 cwt. do.	Sandstone, ordinary	4 do., do.
		..	..	Masonry, rubble	0·4 cwt. do.

SAFE RESISTANCE TO SHEARING.

Cast iron	2·4 tons per sq. inch	Fir, such as Riga or Dantzic, along the grain	1·8 cwt. per sq. inch	Oak, English, across grain	5 cwts. per sq. inch
Wrought iron	5 do., do.			,, ,, along the grain	2 do., do.
Steel, mild	5·5 do., do.				

SAFE RESISTANCE TO BEARING.

Cast iron	10 tons per sq. inch	Steel, mild	8 tons per sq. inch	Fir, such as Riga or Dantzic	12 cwts. per sq. inch
Wrought iron	5 do., do.	Oak, English	25 cwts. do.	..	..

SAFE MODULUS OF RUPTURE (RECTANGULAR BEAMS).

Cast iron	8·5 tons per sq. inch	Oak, English	16 cwts. per sq. inch	Pine, American yellow	10 cwts. per sq. inch
Wrought iron	6·5 do., do.	Fir, such as Riga or Dantzic	11 do., do.	Teak, Moulmein	20 do., do.
		..			

[1] For more detailed information see Part III.

[2] These figures represent the *adhesion* of fresh mortar to bricks.

The best hydraulic mortar cannot be safely taken as adhering to the best stock bricks after six months with a greater ultimate force than 86 lbs. per square inch. The adhesion to soft place bricks is only about 18 lbs. per square inch.

Portland cement mortar (1 to 1) will at twelve months exert an adhesive resistance of about 0·5 cwt. per square inch in stock brickwork, but experimental results show variations dependent apparently upon the nature of the surface of the bricks cemented.

[3] *The resistance of brickwork to cracking or crushing* is much less than that of the bricks alone. After allowing three to six months (according to the mortar) for setting, good stock brickwork will begin cracking at a pressure of 200, 400, or 700 lbs. per square inch, according as it is laid in gray Chalk lime, Lias lime, or Portland cement mortar. For *ultimate crushing* from one and a half time to twice these pressures would be required. (*Wray and Seddon.*) For the results of experiments on the Strength of Brick Piers see Part III, pp. 116 to 121.

TABLE II

Safe loads for rectangular beams of Northern Pine or Baltic Fir, for a breadth of one inch

Beam supported at both ends, loaded uniformly Calculated from the formula $\frac{wl^2}{8} = \frac{1}{6} f_s b d^2$, taking *safe* $f_s = 1440$ lbs. per square inch

Span in feet.	Depth in Inches and *safe* Load in Cwts.																		Span in feet.
	3	4	5	5½	6	6½	7	7½	8	8½	9	9½	10	10½	11	12	13	14	
4	3.2	5.7	8.9	10.8	12.8	15.1	17.5	20.1	22.8	25.8	28.9	32.3	35.7	39.4	43.3	51.5	60.4	70.0	4
5	*2.6*	*4.6*	*7.1*	*8.7*	*10.3*	*12.1*	*14.0*	*16.1*	*18.3*	*20.7*	*23.1*	*25.9*	*28.6*	*31.6*	*34.6*	*41.2*	*48.1*	*56.0*	5
6	2.1	3.8	6.0	7.2	8.6	10.0	11.7	13.4	15.2	17.2	19.3	21.5	23.8	26.3	28.9	34.2	40.3	46.7	6
7	*1.8*	*3.3*	*5.1*	*6.2*	*7.3*	*8.6*	*10.0*	*11.5*	*13.0*	*14.7*	*16.5*	*18.5*	*20.4*	*22.6*	*24.7*	*29.4*	*34.5*	*40.0*	7
8	1.6	2.9	4.5	5.4	6.4	7.6	8.7	10.0	11.4	12.9	14.5	16.1	17.8	19.7	21.6	25.7	30.2	35.0	8
9	*1.4*	*2.5*	*4.0*	*4.8*	*5.7*	*6.7*	*7.8*	*8.9*	*10.1*	*11.5*	*12.8*	*14.4*	*15.9*	*17.6*	*19.2*	*22.8*	*26.8*	*31.2*	9
10	1.3	2.3	3.6	4.3	5.1	6.1	7.0	8.0	9.1	10.3	11.5	12.9	14.3	15.8	17.3	20.6	24.1	28.0	10
12	*1.1*	*1.9*	*3.0*	*3.6*	*4.3*	*5.0*	*5.8*	*6.7*	*7.6*	*8.6*	*9.6*	*10.8*	*11.9*	*13.2*	*14.4*	*17.1*	*20.2*	*23.3*	12
14	.91	1.6	2.6	3.1	3.7	4.3	5.0	5.8	6.5	7.4	8.3	9.2	10.2	11.3	12.3	14.7	17.2	20.0	14
16	*.79*	*1.4*	*2.2*	*2.7*	*3.2*	*3.8*	*4.4*	*5.0*	*5.7*	*6.4*	*7.2*	*8.1*	*8.9*	*9.9*	*10.8*	*12.8*	*15.1*	*17.6*	16
18	.70	1.3	2.0	2.4	2.9	3.4	3.9	4.5	5.1	5.7	6.4	7.2	7.9	8.8	9.6	11.4	13.4	15.5	18
20	*.63*	*1.1*	*1.8*	*2.2*	*2.6*	*3.0*	*3.5*	*4.0*	*4.6*	*5.1*	*5.7*	*6.5*	*7.1*	*7.9*	*8.6*	*10.3*	*12.1*	*14.0*	20
25	.51	.92	1.4	1.7	2.0	2.4	2.8	3.2	3.6	4.1	4.6	5.2	5.7	6.3	6.9	8.2	9.6	11.2	25
30	*.43*	*.77*	*1.2*	*1.4*	*1.7*	*2.0*	*2.3*	*2.7*	*3.0*	*3.4*	*3.8*	*4.3*	*4.7*	*5.3*	*5.8*	*6.9*	*8.0*	*9.3*	30

Example—Find the safe uniformly distributed load that can be applied to a beam supported at both ends, 20′ 0″ long, 11″ deep, and 7″ broad

From the Table the safe load for 1″ breadth is 8.6 cwts. Hence the safe distributed load = 8.6 × 7 = 60.2 cwts

Factors for other cases

Fixed at one end, free at the other $\left\{\begin{array}{l}\text{loaded at free end, } \frac{1}{8}, \\ \text{loaded uniformly, } \frac{1}{4}\end{array}\right.$ *Fixed* at both ends $\left\{\begin{array}{l}\text{load at centre, } 1.0, \\ \text{loaded uniformly, } 1\frac{1}{2}\end{array}\right.$

Supported at both ends, load at centre, $\frac{1}{2}$

The comparative strength of some *other kind of woods* is as follows —

Larch, $\frac{3}{4}$, English oak, $1\frac{1}{2}$, Teak, $1\frac{3}{4}$, Beech, $1\frac{1}{4}$, Ash, $1\frac{1}{2}$, Greenheart, 2

TABLE III.

Deflection of rectangular beams of Northern Pine or Baltic Fir, *supported at both ends, loaded uniformly,* when the stress in the extreme fibres is 1·440 lbs. per square inch (*i.e.* when the loads are those given in Table II.)

Taking modulus of elasticity = 1,440,000 lbs.

Span in feet.	Depth in Inches and Deflection in Inches.																		Span in feet.
	3	4	5	5½	6	6½	7	7½	8	8½	9	9½	10	10½	11	12	13	14	
4	·16	·12	·10	·09	·08	...	...	...	...	...	...	...	...	...	..	...	...	...	4
5	·25	·19	·15	·14	·12	·11	·11	·10	·09	·09	·08	·08	...	...	...	...	...	...	5
6	·36	·27	·22	·20	·18	·17	·15	·14	·13	·13	·12	·11	·11	·10	·10	·09	·08	·07	6
7	·49	·37	·29	·27	·24	·23	·21	·20	·18	·17	·16	·15	·14	·14	·13	·12	·12	·10	7
8	·64	·48	·38	·35	·32	·30	·27	·25	·24	·23	·21	·20	·19	·18	·17	·16	·15	·13	8
9	·81	·61	·49	·44	·40	·37	·35	·33	·30	·29	·27	·26	·24	·23	·22	·20	·18	·17	9
10	...	·75	·60	·55	·50	·46	·43	·40	·37	·35	·34	·32	·30	·28	·27	·25	·23	·21	10
12	...	...	·86	·79	·72	·67	·62	·57	·54	·51	·48	·46	·43	·41	·39	·36	·33	·31	12
14	...	...	...	...	·98	·91	·84	·78	·73	·69	·65	·62	·59	·56	·53	·49	·45	·42	14
16	...	...	...	...	...	...	1·10	1·03	·96	·91	·85	·81	·77	·73	·70	·64	·59	·55	16
18	...	...	...	...	...	...	...	...	...	1·14	1·08	1·02	·97	·93	·88	·81	·75	·69	18
20	...	...	...	...	...	...	...	...	...	...	...	...	1·20	1·14	1·09	1·00	·92	·86	20
25	...	...	...	...	...	...	...	...	...	...	...	...	...	...	...	1·56	1·44	1·31	25
	3	4	5	5½	6	6½	7	7½	8	8½	9	9½	10	10½	11	12	13	14	

Factors for other cases:

Fixed at one end, free at the other $\begin{cases}\text{load at any point, 3·2,}\\ \text{loaded uniformly, 2·4.}\end{cases}$ *Fixed* at both ends $\begin{cases}\text{load at the centre, 0·4,}\\ \text{loaded uniformly, 0·3.}\end{cases}$

Supported at both ends, load at the centre, 0·8.

The deflection for *other intensities of stress* can be found by proportion.

Example—(combining Tables II. and III.)—Find the dimensions of a beam of Northern pine to support a distributed load of 70 cwts. over a span of 10 feet, the deflection being $\frac{1}{40}''$ per foot of the span.

The deflection is to be $\frac{10}{40}$ inches = 0·25 inch. From the above Table the necessary depth is 12″. From Table II. the safe load for this span and depth is 20·6 cwts. Hence required breadth $= \frac{70}{20·6} = 3\frac{1}{2}''$ nearly; thus scantling required is $3\frac{1}{2}'' \times 12''$.

TABLE IV

Deflection of wrought-iron rolled beams, ***supported at both ends, loaded uniformly,*** **when the stress in the extreme fibres is** ***5 tons*** **per square inch**

Taking modulus of elasticity = 26,000,000 lbs

Span in feet.	Depth in Inches and Deflection in Inches																	Span in feet.
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	
6	13	10	08	07	06	05	04											6
8	23	18	14	12	10	09	08	07	06	06	05	05	05					8
10	36	28	22	18	16	14	12	11	10	09	08	08	07	07	06	06	05	10
12		40	32	26	23	20	18	16	14	13	12	11	11	10	09	09	08	12
14		54	43	36	31	27	24	22	20	18	17	15	14	13	13	12	11	14
16			54	45	40	35	33	29	26	24	22	21	19	18	17	16	14	16
18			71	59	51	45	40	36	32	30	27	25	24	22	21	20	17	18
20				73	63	55	49	44	40	37	34	31	29	27	26	24	21	20
22					76	67	59	53	48	44	41	38	35	33	31	30	25	22
24						80	70	63	58	53	49	45	42	40	37	35	30	24
26							83	74	68	62	57	53	50	47	44	41	36	26
28									78	72	66	62	57	54	51	48	41	28
30												71	66	62	58	55	47	30
	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	

FACTORS for other cases

Fixed at one end free at the other { load at any point, 3 2, loaded uniformly, 2 4 } *Fixed* at both ends { load at the centre, 0 4, loaded uniformly, 0 3

Supported at both ends, load at the centre, 0 8

The deflection for *other intensities of stress* can be found by proportion and for BUILT UP BEAMS by multiplying by 1 5

Example—Find the deflection of a rolled iron beam 12 inches deep 16 feet span, fixed at both ends and loaded uniformly, when the maximum stress is 5 tons per square inch

Deflection = 0 3 × 0 24 = 0 072 inch

TABLE V.

Strength of Struts.[1]

l = length of strut in *inches*, κ = least radius of gyration in inches, *i.e.* the moment of inertia about the neutral axis of the cross section divided by the area of the cross section.

r_c = *safe* stress per square inch of cross section in *tons*.

$\frac{l}{\kappa}$ See p. 333.	r_c Cast iron.		Wrought iron.		Mild steel.		$\frac{l}{\kappa}$ See p. 333.	r_c Cast iron.		Wrought iron.		Mild steel.	
	Ends rounded.	Ends fixed.	Ends rounded.	Ends fixed.	Ends rounded.	Ends fixed.		Ends rounded.	Ends fixed.	Ends rounded.	Ends fixed.	Ends rounded.	Ends fixed.
10	*8·68*	8·85	*4·00*	4·00	*5·33*	5·34	**120**	*0·93*	2·37	*1·57*	2·82	*1·83*	3·46
15	*8·41*	8·76	*3·98*	4·00	*5·26*	5·31	**125**	*0·86*	2·19	*1·47*	2·74	*1·71*	3·32
20	*8·07*	8·65	*3·92*	3·99	*5·20*	5·29	**130**	*0·80*	2·03	*1·39*	2·66	*1·61*	3·21
25	*7·58*	8·46	*3·88*	3·98	*5·13*	5·24	**135**	*0·75*	1·90	*1·32*	2·58	*1·50*	3·09
30	*6·98*	8·21	*3·80*	3·95	*5·02*	5·20	**140**	*0·70*	1·78	*1·24*	2·48	*1·42*	2·96
35	*6·32*	7·91	*3·72*	3·92	*4·90*	5·15	**145**	*0·66*	1·66	*1·17*	2·40	*1·36*	2·85
40	*5·68*	7·56	*3·64*	3·89	*4·76*	5·09	**150**	*0·61*	1·56	*1·10*	2·32	*1·28*	2·72
45	*5·02*	7·19	*3·54*	3·86	*4·58*	5·03	**160**	*0·56*	1·40	*0·98*	2·14	*1·13*	2·51
50	*4·43*	6·82	*3·44*	3·82	*4·40*	4·98	**170**	*0·49*	1·25	*0·88*	2·00	*1·01*	2·32
55	*3·84*	6·46	*3·31*	3·78	*4·22*	4·92	**180**	*0·43*	1·14	*0·80*	1·84	*0·91*	2·13
60	*3·35*	6·10	*3·17*	3·73	*4·02*	4·83	**190**	*0·39*	1·03	*0·72*	1·70	*0·83*	1·97
65	*2·92*	5·75	*3·04*	3·68	*3·80*	4·75	**200**	*0·36*	0·93	*0·66*	1·57	*0·75*	1·83
70	*2·57*	5·39	*2·90*	3·63	*3·59*	4·67	**210**	*0·32*	0·84	*0·58*	1·46	*0·68*	1·68
75	*2·23*	5·02	*2·76*	3·55	*3·37*	4·56	**220**	*0·30*	0·77	*0·55*	1·35	*0·62*	1·55
80	*1·96*	4·68	*2·60*	3·48	*3·15*	4·45	**230**	*0·28*	0·70	*0·50*	1·26	*0·58*	1·44
85	*1·74*	4·33	*2·46*	3·40	*2·96*	4·35	**240**	*0·25*	0·64	*0·46*	1·18	*0·53*	1·34
90	*1·56*	4·00	*2·33*	3·32	*2·77*	4·25	**250**	*0·23*	0·59	*0·43*	1·11	*0·49*	1·25
95	*1·42*	3·66	*2·18*	3·22	*2·56*	4·13	**260**	*0·22*	0·56	*0·40*	1·04	*0·46*	1·16
100	*1·29*	3·35	*2·03*	3·17	*2·40*	4·00	**270**	*0·20*	0·52	*0·37*	0·97	*0·42*	1·08
105	*1·17*	3·07	*1·92*	3·08	*2·24*	3·88	**280**	*0·19*	0·49	*0·35*	0·91	*0·39*	1·01
110	*1·07*	2·80	*1·79*	3·00	*2·08*	3·74	**290**	*0·18*	0·46	*0·32*	0·86	*0·37*	0·96
115	*0·99*	2·57	*1·67*	2·91	*1·95*	3·61	**300**	*0·17*	0·43	*0·30*	0·80	*0·35*	0·92

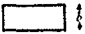
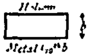
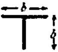
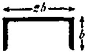
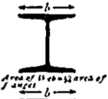
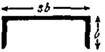
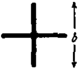
[1] This Table has been compiled from a similar table given in "Fidler on practical strength of columns" (*Min. Pro. Inst. C.E.* vol. lxxxvi.)

For values of $\frac{l}{k}$ see next page.

For practical purposes the value of $\frac{l}{\kappa}$ can be found with sufficient accuracy from

$$\frac{l}{\kappa}=n\frac{l}{b},$$

where n and b have the following values for various sections in general use —

Section	n	Section	n	Section	n
	3.5		6.0		4.9
	2.5		4.9		3.9
	3.0		4.2		3.8
	4.0		3.9		4.3
	3.1		4.2		4.9

Example.—Find the safe stress for an L wrought iron strut, 3″ × 3′ × ½″, 10′ 0″ long, the ends being considered rounded. We have $b=3$, $n=4.9$, $l=120$, hence

$$\frac{l}{\kappa}=4.9\ \frac{120}{3}=196$$

From the Table the safe stress per square inch is 0.70 ton nearly. Hence

$$\text{Safe stress}=0.70\times(3\times\tfrac{1}{2}+2\tfrac{1}{2}\times\tfrac{1}{2})=1.9\ \text{ton nearly}$$

TABLE VI.

Strength of Wooden Struts.

Rounded Ends.

Wood supposed to be average quality fir, such as is used in roof work.

R = ratio of length ÷ least dimension of cross section,

r_c = *safe* stress per square inch in *cwts.*

If both ends are *fixed* take value of r_c corresponding to $\frac{R}{2}$.

If one end is *fixed* and the other *rounded* take mean of values of r_c corresponding to $\frac{R}{2}$ and R.

R	r_c	R	r_c	R	r_c	R	r_c
5	9·55	19	3·85	33	1·56	54	0·58
6	9·25	20	3·60	34	1·43	56	0·55
7	8·85	21	3·38	35	1·35	58	0·52
8	8·30	22	3·18	36	1·28	60	0·49
9	7·75	23	2·92	37	1·22	62	0·46
10	7·20	24	2·75	38	1·16	64	0·43
11	6·70	25	2·60	39	1·11	66	0·41
12	6·25	26	2·41	40	1·06	68	0·39
13	5·80	27	2·25	42	0·96	70	0·37
14	5·40	28	2·11	44	0·87	72	0·35
15	5·05	29	1·96	46	0·79	74	0·33
16	4·75	30	1·82	48	0·72	76	0·31
17	4·45	31	1·71	50	0·66	78	0·29
18	4·15	32	1·60	52	0·62	80	0·27

Example.—Find the *safe* stress that can be applied to a wooden strut 3″ × 2″ and 10 feet long.

$$R = \frac{10 \times 12}{2} = 60\ ;\ r_c = 0·49\ ;\ \text{safe stress} = 3 \times 2 \times 0·49 = 2·9\ \text{cwts.}$$

TABLE VII.

Strength of Timber Columns.

The following Table has been deduced by Mr. Stoney, C.E., from a series of experiments made by Mr. Brereton, C.E., on square columns of American yellow pine (*Pinus strobus*).

Ends adjusted as in ordinary practice.

Ratio of length to side of column	10	15	20	25	30	35	40	45	50
Breaking weight in tons per sq. foot of section . . .	120	118	115	100	90	84	80	77	75

TABLE VIII

Strength of Rivets (Iron Rivets in Iron Plates)

Safe resistance to shearing, 4 tons per square inch
Safe resistance to bearing, 8 tons per square inch

Diam of rivet.	Resistance to single shear—tons.	Resistance to bearing of one rivet in plates of various thicknesses—tons							
		1/16"	1/4"	3/8"	1/2"	5/8"	3/4"	7/8"	1"
1/2"	0·78	0·25	1·00	1·50	2·00	2·50	3·00	3·50	4·00
5/8"	1·23	0·31	1·25	1·87	2·50	3·12	3·75	4·37	5·00
3/4"	1·77	0·37	1·50	2·25	3·00	3·75	4·50	5·25	6·00
7/8"	2·40	0·44	1·75	2·62	3·50	4·37	5·25	6·12	7·00
15/16	2·76	0·47	1·87	2·81	3·75	4·69	5·62	6·56	7·50
1"	3·14	0·50	2·00	3·00	4·00	5·00	6·00	7·00	8·00
1 1/8"	3·98	0·56	2·25	3·37	4·50	5·62	6·75	7·87	9·00
1 1/4"	4·91	0·62	2·50	3·75	5·00	6·25	7·50	8·75	10·00

TABLE IX.[1]

Dimensions of Eyes of Wrought Iron Tension Bars.

Ratio of diameter of pin to width of bar or diameter of bar (if round) (not to be < 0·67)	Ratio of *area* of metal at side of eye to *area* of bar		Ratio of maximum thickness of bar to width (Pin in *single shear*)
	Hammered eye (Metal section at back of eye = that of bar)	Hydraulic forged *eye* (Same section at back as at sides of eye)	
0·67	0·66	0·74	0·21
0·75	0·67	0·75	0·25
1·00	0·75	0·75	0·38
1·25	0·76	0·80	0·54
1·33	0·79	0·85	0·61
1·50	0·83	0·93	0·70
1·75	0·84	1·00	0·88
2·00	0·88	1·13	1·08

[1] This Table is taken from *Instruction in Construction* (revised edition) and is *founded on that given by Mr Shaler Smith*

TABLE X.[1]

Weight of Angle Iron and Tee Iron.

1 foot in length.

Thickness.	Sum of the Width and Depth in Inches.										
	1½	1⅝	1¾	1⅞	2	2⅛	2¼	2⅜	2½	2⅝	2¾
Inches.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.	lbs.
⅛	·57	·62	·68	·73	·78	·83	·88	·94	·99	1·04	1·09
3/16	·81	·89	·97	1·05	1·13	1·21	1·29	1·37	1·45	1·52	1·60
¼	1·04	1·15	1·25	1·36	1·46	1·56	1·67	1·77	1·88	1·98	2·08
5/16	1·24	1·37	1·50	1·63	1·76	1·89	2·02	2·15	2·28	2·41	2·54
	2⅞	3	3⅛	3¼	3⅜	3½	3⅝	3¾	3⅞	4	4¼
⅛	1·14	1·20	1·25	1·30	1·35	1·41	1·46	1·51	1·56	1·62	1·72
3/16	1·68	1·76	1·84	1·91	1·99	2·07	2·15	2·23	2·30	2·38	2·54
¼	2·19	2·29	2·40	2·50	2·60	2·71	2·81	2·92	3·02	3·13	3·33
5/16	2·67	2·80	2·93	3·06	3·19	3·32	3·45	3·58	3·71	3·84	4·10
⅜	3·13	3·28	3·44	3·59	3·75	3·91	4·06	4·22	4·38	4·53	4·84
7/16	3·57	3·75	3·93	4·11	4·29	4·48	4·66	4·84	5·02	5·20	5·56
	4½	4¾	5	5¼	5½	5¾	6	6¼	6½	6¾	7
3/16	2·70	2·85	3·01	3·16	3·32	3·48	3·63	3·79	3·95	4·10	4·26
¼	3·54	3·75	3·96	4·17	4·38	4·58	4·79	5·00	5·21	5·42	5·63
5/16	4·36	4·62	4·88	5·14	5·40	5·66	5·92	6·18	6·45	6·71	6·97
⅜	5·16	5·47	5·78	6·09	6·41	6·72	7·03	7·34	7·66	7·97	8·28
7/16	5·92	6·29	6·65	7·02	7·38	7·75	8·11	8·48	8·84	9·21	9·57
½	6·67	7·08	7·50	7·92	8·33	8·75	9·17	9·58	10·00	10·42	10·83
9/16	7·38	7·85	8·32	8·79	9·26	9·73	10·20	10·66	11·13	11·60	12·07
	7¼	7½	7¾	8	8¼	8½	8¾	9	9¼	9½	9¾
¼	5·83	6·04	6·25	6·46	6·67	6·88	7·08	7·29	7·50	7·71	7·92
5/16	7·23	7·49	7·75	8·01	8·27	8·53	8·79	9·05	9·31	9·57	9·83
⅜	8·59	8·91	9·22	9·53	9·84	10·16	10·47	10·78	11·09	11·41	11·72
7/16	9·93	10·30	10·66	11·03	11·39	11·76	12·12	12·49	12·85	13·22	13·58
½	11·25	11·67	12·08	12·50	12·92	13·33	13·75	14·17	14·58	15·00	15·42
9/16	12·54	13·01	13·48	13·94	14·41	14·88	15·35	15·82	16·29	16·76	17·23
⅝	13·80	14·32	14·84	15·36	15·89	16·41	16·93	17·45	17·97	18·49	19·01
	10	10½	11	11½	12	12½	13	13½	14	14½	15
⅜	12·03	12·66	13·28	13·91	14·53						
7/16	13·95	14·67	15·40	16·13	16·86	17·59	18·31	19·04	19·77	20·50	21·22
½	15·83	16·67	17·50	18·33	19·17	20·00	20·84	21·67	22·50	23·34	24·17
9/16	17·70	18·63	19·57	20 51	21·44	22·38	23·31	24·25	25·19	26·12	27·06
⅝	19·53	20·57	21·61	22·66	23·70	24·74	25·78	26·83	27·87	28·91	29·95
¾	23·13	24·38	25·63	26·88	28·13	29·37	30·63	31·88	33·13	34·38	35·63
	12	12½	13	13½	14	15	16	17	18	19	20
⅝	23·70	24·74	25·78	26·83	27·87	29·95	32·03	34·12	36·20	38·28	40·36
¾	28·13	29·37	30·63	31·88	33·13	35·63	38·13	40·63	43·13	45·63	48·13
⅞	32·45	33·91	35·36	36·82	38·28	41·19	44·12	47·02	49·95	52·87	55·78
1	36·67	38·33	40·00	41·67	43·33	46·67	50·00	53·33	56·67	60·00	63·33

Note.—When the base or the web tapers in section, the mean thickness is to be measured.

[1] From D. K. Clark's *Rules and Tables.*

TABLE XI

Web Plates—thickness of.

Table[1] *for determining the thickness of web plate*, suitable for a given shearing stress per foot depth of girder, at varying widths of plate between supports (whether ⌊ irons of flanges or vertical stiffeners) The Table is founded on the formula

$$T = \frac{48t}{1 + \dfrac{h}{9t}}$$

(Unwin s *Iron Bridges and Roofs*), where T is the stress in tons, t the thickness of web plate, h the unsupported width of plate in feet

To use the Table—Find the least unsupported width of plate in the top row of figures, and underneath it find the number nearest in excess of the given shearing stress in tons per foot depth of girder The corresponding thickness of plate will be that required

For example, a girder 3 feet deep has a maximum shearing stress of 18 tons = 6 tons per foot of depth the distance between the ⌊ irons is 30 inches. Under the number 30 we find 6 3 tons, and the corresponding thickness is $\frac{1}{2}''$, which is to be the thickness at abutments

Thickness of web in inches	NET UNSUPPORTED WIDTH OF PLATE IN INCHES WHETHER BETWEEN FLANGE ⌊ IRONS OR VERTICAL STIFFENERS													
	12	15	18	21	24	27	30	33	36	39	42	45	48	51
$\frac{1}{4}$	4 3	3 1	2 4	1 9	1 5	1 2	1 0	8	7	6	5	45	4	36
$\frac{5}{16}$	7 0	5 5	4 3	3 4	2 8	2 2	1 8	1 5	1 3	1 2	1 0	9	8	7
$\frac{3}{8}$	10 0	8 0	6 5	5 3	4 3	3 5	3 0	2 6	2 2	1 9	1 7	1 5	1 3	1 2
$\frac{7}{16}$	13 3	10 9	9 0	7 5	6 3	5 3	4 5	3 9	3 4	2 9	2 6	2 3	2 0	1 8
$\frac{1}{2}$	16 7	14 2	12 0	10 2	8 7	7 4	6 3	5 5	4 8	4 2	3 7	3 3	3 0	2 7
$\frac{9}{16}$	20 1	17 4	15 1	13 0	11 2	9 8	8 5	7 4	6 5	5 7	5 1	4 6	4 2	3 8
$\frac{5}{8}$	23 4	20 7	18 3	16 0	14 0	12 3	10 8	9 5	8 4	7 5	6 7	6 0	5 4	4 9
$\frac{11}{16}$	26 7	24 1	21 6	19 2	17 0	15 0	13 4	11 9	10 6	9 5	8 5	7 6	6 8	6 3
$\frac{3}{4}$	30 0	27 4	24 9	22 4	20 1	17 9	16 1	14 5	13 0	11 7	10 5	9 5	8 6	7 8

Obviously the Table can be used for determining the thickness of web plate at *any* part of the girder, as well as at the abutments, when the shearing stress is known, and it is easy to find thicknesses for unsupported widths not given in the Table by estimation between the widths above and below The safe compression shearing stress used in the formula is 4 tons per square inch, and as the assumed diagonal pillar, 12 inches wide, is not independent but continuous with the adjacent parts, the stresses given in the Table are well on the safe side If, therefore, the given stress is but little over a tabular number it will be safe to use the corresponding thickness instead of the next greater

If, for example, the stress is 11 tons per foot of depth, and the unsupported width 30 inches, it will be quite safe to use a web plate $\frac{5}{8}$ thick

TABLE XII[2]

Weight of Roof Framing, Ceilings, etc

CEILINGS

Lath and plaster ceiling	8 } lbs per square foot horizontal
Ceiling joists	3 } surface covered

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[1] Kindly given to the writer by Mr C Light

[2] The weights of framing of iron roofs are taken from various sources, those for wooden roofs from *Instruction in Construction* by Col Wray R E

Framing, etc., Wooden Roofs.

Common rafters	3	lbs. per square foot of roof surface.
Collar beams	2	

Framing for wooden roofs, including purlins and ridge boards, but exclusive of tie beams—

20 feet span, king post, rise ¼ span	2	lbs. per square foot of roof surface.
30 ,, ,, ,,	2	,, ,,
40 ,, queen post ,,	2	,, ,,
50 ,, ,, ,,	3	,, ,,
60 ,, ,, ,,	4	,, ,,

Tie beams—

20 feet span, king post	11 lbs. per foot run of tie.
30 ,, ,,	20 ,, ,,
40 ,, queen post	18 ,, ,,
50 ,, ,,	20 ,, ,,
60 ,, ,,	30 ,, ,,

Framing of Iron Roofs.

Description.	Span centres of end pins or centres of ribs.	Distance apart of Principals.	Nature of Covering.	Weight per square foot of horizontal area in lbs.			
				Of Principal.	Of Purlins.	Total Iron-work.[1]	Total with covering
	ft. in.	ft. in.					
Trussed Roofs.	20 0	8 0	Slates	2·0	From	5·5	16·5
,, ,,	30 0	to	on	2·5	2·5 to	6·0	17·0
,, ,,	40 0	12 0	boarding	3·0	4·0	6·5	17·5
,, ,,	50 0	...	...	4·5	...	7·5	18·5
,, ,,	60 0	...	...	5·5	...	8·5	19·5
Do. type, Fig. 392.	116 0	25 0	Zinc	8·3	...	...	...
Bowstring Roofs.							
Birkenhead	91 4 98 4	25 0	Zinc and glass	6·0	5·5	13	21
Charing Cross	165 10	35 0	do.	10·4	...	...	37 ?
Cannon Street	192 4	33 6	Slate and glass	16·4	...	...	37
Birmingham	212 0	24 0	Corrugd. iron	11·0	...	20	25
Liverpool— Lime Street	212 0	32 0	Zinc and glass	14·5	...	...	38
Arched Roofs.							
Glasgow— St. Enoch	198 0[3]	36 10	...	13·2	12·2	32	...
Manchester— Central Station	210 0[3]	35 0					
St. Pancras	240 0[3]	29 4	Slates and glass	16·7	...	...	36
Hinged Arches.							
Cologne— Central Depot	208 3	27 10	Corrugd. iron and glass	...	...	32·8	...
Jersey City	252 8	58 0[2]	do.	...	...	...	30 ?
Philadelphia	259 8	50 2[2]	"Tin" and glass				
Paris Exn.— Palais des Machines	362 9	70 6	Zinc and glass	17·8	14·3	32·8	40·5
Chicago Exn.— Liberal Arts	368 0	50 0	...	...	...	...	...

[1] For ordinary trusses up to 60 ft. span these weights do not include wind bracing, ironwork to Louvres, standards, etc. or guttering. [2] Centres of pairs of arches. [3] Clear spans.

TABLE XIII[1]

Weight of Roof Coverings.

	Per square foot
Lead covering, including laps, but not boarding or rolls	5½ lbs to 8½ lbs
Zinc covering „ *14 to 16 zinc gauge*	1¼ „ *to* 1¾ „
Corrugated iron, galvanised, 16 W G	3½ „
„ „ „ 18 W G	2½ „
„ „ „ 20 W G	2 „
Sheet iron, 16 W G	2½ „
„ „ 20 W G	1½ „
Slating laid with a 3 inch lap including nails, but not battens or iron laths—	
Slates, Doubles, 13 *inches* × 9 *inches*, at 18 *cwts.* per 1200	8¼ „
„ Ladies, 16 inches × 8 inches, at 31 5 cwts per 1200	8¼ „
„ Countesses, 20 inches × 10 inches, at 50 cwts per 1200	8 „
„ Duchesses, 24 inches × 12 inches, at 77 cwts per 1200	8½ „
Tiles, plain, 11 inches × 7 inches, laid with a 3 inch lap and pointed with mortar, including laths and absorbed rain	18 „
„ pan, *13½ inches × 9½ inches, laid with a 3 inch lap* and pointed with mortar, including laths and absorbed rain	12 „
„ Italian (ridge and furrow), not including the boarding, but including mortar and absorbed rain	14 „
Slate battens, 3½ inches × 1 inch—	
For Doubles	2 „
For Countesses	1¼ „
Boarding, ¾ *inch thick*	2½ „
„ 1 „	3½ „
„ 1¼ „	4¼ „
Wrought Iron Laths, angle irons—	
For Duchess Slates	2 „
For Countess „	1¾ „
Cast Iron plates, ⅜ inch thick	15 „
Thatch, including battens	6½ „

TABLE XIV

Wind Pressure.

Wind pressure normal to a roof surface from Hutton's formula, viz $P_N = P(\sin i)^{1.84 \cos i - 1}$, taking P = 50 lbs per square foot

Pitch of Roof (i)	10°	15°	20	21° ¾ or ⅕ span	25°	26° ½ or ¼ span	30°	33 ½ or ⅓ span	35°	40	45°	50
Normal wind pressure in lbs per sq foot (P_N)	12 1	18	22·6	25 2	28 8	30 2	33 0	36 6	37 8	41 6	43	47 6

[1] This table is taken from *Instruction in Construction*, by Col Wray, R E

TABLE XV.

Scantlings for Wooden Roofs.

The roofs are supposed to be of Baltic fir covered with Countess slates laid on inch boards; the maximum horizontal wind force is taken at 45 lbs. per foot super acting only on one side of the roof at a time, equivalent to a normal wind pressure of 30 lbs. per square foot for a pitch of 30°, and 40 lbs. per square foot for a pitch of 45°.

The common rafters to be 1 ft. from centre to centre, but in sheltered positions they may be placed 1 ft. apart in the clear.

A ROOFS WITHOUT CEILINGS.—Pitch up to 30°.

Nature of Roof.	Span in feet.	Common Rafters. Walls capable of resisting thrust.* Collar placed ½-way up.	Common Rafters. Walls not capable of resisting thrust.† Collar placed ¼-way up.	Collar. In fixing the collar to the rafter the latter should not be cut into.	Remarks.
		Bdth. Depth.			* The following *solid* walls are strong enough, when built in Lias lime mortar 1 to 2, to resist the thrust of roof; allowance must be made for door and window openings. Height to be taken from level of the floor below roof. Stone Walls: 16″ thick, not over 15 ft. high. Brick Walls: Span of roof 10′ { 9″ thick, not over 7′ high. 14″ ,, ,, 14′ ,, } Span of roof 18′ { 9″ ,, ,, 5′ ,, 14″ ,, ,, 10′ ,, }
Couple.	8	2″×3″	—	—	
	10	2″×3½″	—	—	
	12	2″×4″	—	—	
Collar Beam.	8	1¾″×2½″	2″×3¼″	2″×2½″	† *When the walls are not capable of resisting the thrust of the roof*, place the collar low down; but if the collar is required half-way up, the scantlings must be increased as follows:— *Rafters*, add one-fourth to both breadth and depth; *Collar*, add ¾″ to depth; but it would be better to use the scantlings for walls capable of taking the thrust, and make some arrangement to prevent the walls from spreading, such as tying the wall plates together at intervals.
	10	1⅓″×2½″	2″×4″	2″×2½″	
	12	1¼″×2¼″	2″×4½″	2″×2¾″	
	14	1¾″×3″	2¼″×5″	2″×3″	
	16	2″×3¼″	2¼″×5½″	2″×3½″	
	18	2″×3¾″	2¼″×6″	2″×4″	

King Post.		Tie Beam.‡ Depth includes 3″ for joints.	Principal Rafters.	King Post.	Struts.	Straining beam.	Purlins.§ 10 ft. bearing.	Common Rafters.
Trusses 10′ centre to centre.	20	3″×4½″	3″×5″	3″×2¾″	3″×3″	—	5″×7½″	2″×3½″
	22	3″×4¾″	3″×5½″	3″×2¾″	3″×3½″	—	5″×7¾″	2″×3¾″
	24	3½″×4½″	3½″×5¼″	3½″×2¾″	3½″×3½″	—	5″×8″	2″×4″
	26	3½″×4¾″	3½″×5¼″	3½″×2¾″	3½″×4″	—	5″×8¼″	2″×4¼″
	28	4″×4½″	4″×5½″	4″×2¾″	4″×4″	—	5″×8½″	2″×4½″
	30	4″×4¾″	4″×6″	4″×2¾″	4″×4½″	—	5″×8¾″	2″×4¾″
Queen Post.				Queen Posts.				
Queen Post. Trusses 10′ centre to centre.	32	4½″×4½″	4½″×4¾″	4½″×2¾″	4½″×2½″	4½″×5½″	5″×7½″	2″×3½″
	34	4½″×4¾″	4½″×5″	4½″×2¾″	4½″×2¾″	4½″×6″	5″×7¾″	2″×3¾″
	36	4¾″×4¾″	4¾″×5″	4¾″×2¾″	4¾″×3″	4¾″×6¼″	5″×8″	2″×4″
	38	4¾″×4¼″	4¾″×5¼″	4¾″×2¾″	4¾″×3¼″	4¾″×6¾″	5″×8″	2″×4″
	40	4¾″×5″	4¾″×5½″	4¾″×2¾″	4¾″×3¼″	4¾″×7¼″	5″×8¼″	2″×4¼″
	42	5″×5″	5″×5½″	5″×2¾″	5″×3¼″	5″×7½″	5″×8½″	2″×4½″
	44	5″×5¼″	5″×5¾″	5″×2¾″	5″×3½″	5″×8″	5″×8½″	2″×4¾″
	46	5¼″×5¼″	5¼″×5¾″	5¼″×2¾″	5¼″×3½″	5¼″×8¼″	5″×8¾″	2″×5″

TABLE XV —*Continued*

B — ROOFS WITH CEILINGS.—Pitch up to 30°

Nature of Roof	Span in feet.	Common Rafters		Collar *	* In fixing the collar to the rafter, the latter should not be cut into. As regards walls capable or not capable of resisting the thrust of roof see remarks under A.			
Collar Beam	8 to 18	Add ½ to depths given in A		Add ¾ to depths given in A				
		Tie beam ‡	**Principal Rafters**	**King Post**	**Struts**	**Straining beam**	**Purlins § 10 ft bearing**	**Common Rafters**
King Post Trusses 10′ centre to centre	20	4 × 7	4 × 4	4 × 3	4 × 2½	—	5 × 7½	2 × 3½
	22	4″ × 7½″	4″ × 4½	4 × 3	4 × 3	—	5 × 7¾	2 × 3¾
	24	4¼ × 8″	4¼ × 4¾	4¼ × 3	4¼ × 3	—	5 × 8″	2″ × 4
	26	4½ × 8½	4½ × 5	4½ × 3	4½ × 3″	—	5 × 8¼″	2 × 4¼
	28	4½″ × 9	4½ × 5½	4½ × 3	4½ × 3½	—	5 × 8½	2 × 4½″
	30	4¾ × 9½	4¾ × 5¾	4¾ × 3	4¾ × 3¾	—	5 × 8¾	2″ × 4¾
				Queen Posts				
Queen Post Trusses 10′ centre to centre	32	4¾ × 7½″	4¾ × 5½	4¾ × 3	4¾ × 2½	4¾ × 6¾	5 × 7¾	2″ × 3½
	34	4¾ × 7¾″	4¾ × 5¾	4¾ × 3″	4¾″ × 2¾	4¾ × 7½	5 × 7¾	2 × 3¾
	36	4¾ × 8¼	4¾ × 6¼	4¾ × 3	4¾ × 3	4¾ × 8¼	5 × 8″	2 × 4
	38	5 × 8½	5 × 6	5 × 3	5 × 3	5 × 8¼	5 × 8	2 × 4
	40	5 × 9″	5 × 6½	5 × 3½	5 × 3¼	5 × 9	5 × 8¼	2 × 4¼
	42	5¼ × 9	5¼ × 6¼	5¼ × 3¼	5¼ × 3	5½ × 9	5 × 8½″	2″ × 4¼
	44	5½ × 9½	5½ × 6¾	5½ × 3½	5½ × 3¼	5½ × 9½	5 × 8½	2″ × 4¾
	46	5½ × 10″	5½ × 7¼	5½ × 4	5½ × 3½	5½ × 10	5 × 8¾	2 × 5

FOR ROOFS OF 45° PITCH —Add 1″ to the depth of common rafters, purlins, and struts, and ½″ to the depth of the principal rafters, as given in A and B

‡ The joint of the tie beam with the principal rafter should be placed *immediately* over the supporting wall. If this cannot be conveniently done, the depth of the tie beam should be increased one or two inches.

§ If the purlins, instead of being placed immediately over the joints, are placed at intervals along the principal rafter, increase the depth of the latter, given in the Tables, as follows

King post roof $\begin{cases} \textit{without} \text{ ceiling } 2'' \\ \textit{with} \quad\quad ,, \quad 1\tfrac{1}{2}'' \end{cases}$ Queen post roof $\begin{cases} \textit{without} \text{ ceiling } 1\tfrac{1}{4}'' \\ \textit{with} \quad\quad ,, \quad \tfrac{1}{2}'' \end{cases}$

The purlins if placed 2 feet apart and with 10 feet bearing may be made 3″ × 6.

The scantlings of the principal rafters, struts, and straining beam can be *slightly* modified by means of the following rough rule "For every ¼″ deducted from the *lesser dimension of the scantling, add ½″ to the other dimension, and vice versâ*. For the tie beam, purlins, and common rafters, so long as the depth is about double the breadth, ¼″ deducted from the breadth requires ¼″ to be added to the depth.

This Table is derived from the War Office practice.

TABLE XVa

Coefficients of Friction.

ϕ = Angle of Repose $f = \tan\phi$ = coefficient of friction

	ϕ	f		ϕ	f
Masonry and brickwork (dry)	31° to 35°	·6 to ·7	Wood on stone	22°	·4
„ with wet mortar	25¼°	·47	Iron on stone	35° to 16¾	·7 to ·3
„ with slightly damp mortar	36½°	·74	Wood on wood (dry)	14° to 26½	·25 to ·5
on dry clay	27°	·51	Metals on metals (dry)	8½° to 11½	·15 to ·2
„ on moist clay	18¼°	·33	Smoothest and best greased surfaces	1¾° to 2°	·03 to ·036

[1] Rankine's values

TABLE XVI.

Retaining Walls.[1]

Angle of repose (ϕ) of various earths, and value of $K = 0{\cdot}7 \tan \frac{90-\phi}{2}$.

Description of Earth.	ϕ	K.	Description of Earth.	ϕ	K.
Fine dry sand	37 to 31	0·35 to 0·40	Loamy earth, consolidated and dry	40°	0·33
Sand, wet	26	0·44	Clay, dry	29	0·41
„ very wet	32	0·39	„ damp, well drained	45	0·29
Vegetable earth, dry	29	0·41	„ wet	16	0·53
„ „ moist	45 to 49	0·29 to 0·26	Gravel, clean	48	0·27
„ „ very wet	17	0·52	„ with sand	26	0·44
„ „ consolidated and dry	49	0·26	Loose shingle	39	0·33

TABLE XVII.

Weights of Earths, Stone, etc.[2]

	Pounds avoirdupois per cubic foot.		Pounds avoirdupois per cubic foot.
Basalt	180	Concrete, lime	118
Bathstone	123	Earth, vegetable	90
Brick, common stock	115	„ loamy	80-100
„ red facing	130	„ semi-fluid	110
„ fire	150	Granite, Aberdeen	164
Brickwork, in mortar	110	Gravel, Thames	112
„ in cement	112	Limestone, lias	156
Cement, Portland	87	Lime, ordinary quick, stone	53
Chalk, solid	112 to 175	Masonry, rubble	140
		„ ashlar, Portland	150
		„ „ Granite	160
Clay, with gravel	130	Mortar, new	110
„ ordinary	120	Portland stone	145
Concrete, Portland cement	137 to 142	Sandstone, Craigleith	145
		Slate, Welsh	181
		Sand	119

[1] Prof. Unwin's "Railway Construction" as regards the value of ϕ.
[2] From *R. E. Aide Memoire.*

TABLE XVIIA

Table of Thickness[1] required for Arches, Semicircular Arches, and Arches of 120°.

	Ordinary vaults, bridges, etc						Remarks
	Semicircular			Arches of 120°			
	Block Stone	Brick	Rubble Stone	Block Stone	Brick	Rubble Stone	
	′ ″		′ ″	′ ″		′	
Span of 5 feet and less	0 8	1	0 10	0 9	1	0 10½	
" 6 "	0 9	1	0 11	0 10	1	1 0	
" 8 "	0 10	1	1 0	0 11	1½	1 1½	
" 10 "	0 11	1¼	1 1½	1 0	1½	1 3	The arches are assumed to be built in well-made hydraulic lime mortar
" 12 "	1 0	1½	1 3	1 1	1½	1 4½	
" 14 "	1 1	1½	1 4	1 2	1½	1 6	
" 16 "	1 2	1½	1 5	1 3	2	1 7	
" 18 "	1 3	2	1 6	1 4	2	1 8	
" 20 "	1 4	2	1 7	1 5	2	1 9	
" 22 "	1 5	2	1 8	1 6	2	1 10½	
" 24 "	1 6	2	1 9	1 7	2½	2 0	

[1] War Department practice (*Wray*)

TABLE XVIII

HYDRAULICS

Flow of Water in Pipes running full (gallons per min.)

Calculated from Darcy's formula.

Value of the expression $\frac{L}{H} \cdot G^2$ *	Corresponding diameter of pipe in inches.	Internal diameter of pipe allowing for incrustation †	Value of the expression $\frac{L}{H} G^2$ *	Corresponding diameter of pipe in inches.	Internal diameter of pipe allowing for incrustation †
1	0 32	*3/8*	34,600	2 14	2½
5	0 48	½	60,000	2 36	*2¾*
18	0 54	*5/8*	92,000	2 57	3
48	0 64	*¾*	140,000	2 79	*3¼*
117	0 75	*7/8*	208,000	3 00	*3½*
250	0 86	1	296,000	3·21	*3¾*
370	0 96	*1⅛*	420,000	3 43	4
610	1·07	1¼	760,000	3 86	*4½*
1,410	1 18	*1⅜*	1,310,000	4 29	5
2,260	1 29	1½	2,150,000	4 71	*5½*
3,400	1 39	*1⅝*	3,400,000	5 14	6
5,020	1 50	1¾	7,500,000	6 00	7
7,400	1 61	*1⅞*	16,500,000	7 00	8
10,200	1 71	2	32,800,000	8 00	9
20,800	1 93	*2¼*	60,200,000	9 00	10

* L = length of pipe ; H = available head , G = discharge in gallons per minute

† The allowance made for incrustation is

⅛th of diameter for pipes under 6 inches in diameter,

1″ for pipes over 6 inches diameter

The pipes that are not market sizes are printed in italics

Example.—The length of a pipe is 500 feet, the available head is 30 feet, and the discharge is to be 600 gallons per minute. What ought the diameter to be, allowing for incrustation?

We have $$\frac{L}{H} . G^2 = \frac{500}{30} . 600^2 = 6,000,000.$$

A 7″ pipe would therefore be required.

The discharge from this pipe when new will be

$$\sqrt{\frac{16,500,000 \times 30}{500}} = 995 \text{ gallons per minute.}$$

And when incrusted

$$\sqrt{\frac{7,500,000 \times 30}{500}} = 670 \text{ gallons per minute.}$$

TABLE XIX.

HYDRAULICS.

Flow of Water in Pipes running full (cubic feet per min.)

Calculated from Darcy's formula.

Value of the expression $\frac{L}{H} . F^2$. *	Corresponding diameter of pipe in inches.	Value of the expression $\frac{L}{H} . F^2$. *	Corresponding diameter of pipe in inches.
6·6	4	14,380	18
17·6	5	18,800	*19*
53	6	24,900	*20*
115	7	31,500	*21*
227	8	39,600	*22*
427	9	49,400	*23*
730	10	61,500	24
1,190	11	75,700	*25*
1,850	12	91,700	*26*
2,740	*13*	110,600	*27*
4,020	*14*	133,400	*28*
5,740	15	158,300	*29*
7,880	*16*	188,000	30
10,650	*17*	...	...

* L = length of pipe ; H = available head ; F = discharge in cubic feet per second. No allowance is made for incrustation.

The pipes that are not market sizes are printed in italics.

This Table is used in the same way as Table XVIII.

TABLE XX[1]

Cast Iron Pipes—Thickness, Weight, and Strength

Ins. in Diameter	Thickness by formula.[2]	Nearest thickness in six-teenths of an inch.	Net weight per foot run for thickness in Column 3.	Length of pipe equal in weight to the socket.	Weight of a 9 feet length of pipe	Bursting pressure per sq inch reckoned on Column 3	Factor of safety for Normal pressure of 300 ft. of water or 133 lbs. per sq inch
Inches.	Inches.	Whole Six-teenths.	lbs.	feet.	cwts.	lbs.	Times.
2	·31	$\frac{5}{16}$	7·09	60	(6 feet) ·418	4900	36
$2\frac{1}{2}$	·33	$\frac{3}{8}$	10·6	61	(6 feet) ·625	3920	30
3	·35	$\frac{3}{8}$	12·4	62	1·06	3920	30
4	·375	$\frac{3}{8}$	16·1	62	1·38	2940	22
5	·41	$\frac{7}{16}$	23·4	63	2·01	2744	21
6	·45	$\frac{7}{16}$	27·7	63	2·38	2290	17
7	·47	$\frac{1}{2}$	36·8	64	3·17	2210	17
8	·50	$\frac{1}{2}$	41·7	64	3·59	1960	15
9	·53	$\frac{9}{16}$	52·8	65	4·55	1960	15
10	·56	$\frac{9}{16}$	58·3	66	5·03	1764	13
11	·59	$\frac{9}{16}$	63·9	66	5·51	1604	12
12	·62	$\frac{5}{8}$	77·5	67	6·69	1633	12
13	·65	$\frac{5}{8}$	83·6	67	7·22	1508	11
14	·70	$\frac{11}{16}$	99·1	68	8·56	1540	12
15	·71	$\frac{11}{16}$	105·9	68	9·15	1440	11
			cwts.				
16	·75	$\frac{3}{4}$	1·10	69	10·06	1470	11
18	·81	$\frac{13}{16}$	1·43	70	13·87	1415	10·6
20	·87	$\frac{7}{8}$	1·60	71	15·54	1372	10·3
21	·90	$\frac{7}{8}$	1·68	72	16·33	1307	10
24	·99	1	1·91	73	18·58	1307	10
27	1·09	$1\frac{1}{16}$	2·61	75	25·45	1234	9·3
30	1·18	$1\frac{3}{16}$	3·24	77	31·65	1241	9·3
33	1·27	$1\frac{1}{4}$	3·75	78	36·67	1190	8·9
36	1·36	$1\frac{3}{8}$	4·50	80	44·10	1198	8·9
39	1·45	$1\frac{7}{16}$	5·11	82	50·18	1156	8·7
42	1·55	$1\frac{9}{16}$	5·98	83	58·78	1167	8·8
45	1·65	$1\frac{5}{8}$	6·67	85	65·70	1133	8·5
48	1·74	$1\frac{3}{4}$	7·63	87	75·31	1143	8·6

Note to Table—Flanges.—The additional weight for a pair of flanges is reckoned as equivalent to that of a lineal foot of pipe, equal to 11 per cent extra for 9 feet lengths.

[1] From D. K. Clark's *Rules and Tables*

[2] The thicknesses in this column are obtained from the following formula deduced by Mr Clark from Mr Bateman's practice—

$$t = \cdot25 + \frac{Hd}{6000}$$

where t = thickness of the pipe in inches,
H = the head of pressure in feet of water,
d = the inside diameter of the pipe in inches

TABLE XXI.

HYDRAULICS.

Loss of Head due to Bends.

When mean velocity of flow is 1 foot per second. For other velocities, multiply by the square of the velocity.

$\frac{\text{Radius of Bend}}{\text{Diameter of Pipe}}$.	Loss of Head for each Degree of Change of Direction.
1	0·000025
1·25	0·000018
1·5	0·000015
2·0	0·000013
3	0·000011
4	0·000011
5	0·000011

Example.—Find the loss of head caused by a bend of 20°, of 2 inches radius in a 1″ pipe; velocity of flow, 4 feet per second.

$$\frac{\text{Radius of bend}}{\text{Diameter of pipe}} = \frac{2}{1}. \quad \text{Hence loss of head} = 0·000{,}013 \times 20^\circ \times 4^2.$$

TABLE XXII.[1]

HYDRAULICS.

Loss of Head due to Elbows.

$= eV^2$, where e can be found from the following Table.

A	10°	20°	30°	40°	50°	60°	70°	80°	90°
e	0·0001	0·0005	0·0011	0·0022	0·0036	0·0056	0·0083	·0115	·0152

[1] From Hurst's *Pocket Book.*

TABLE XXIII
HYDRAULICS
Pipes flowing partially full

Relation between wetted perimeter, $P=\frac{\pi\phi}{360}$ D, and cross section of flow, $A=\frac{1}{2}\left(\frac{\pi\phi}{360}-\frac{\sin\phi}{2}\right)$ D^2, where ϕ is the angle subtended at the centre by the wetted perimeter, and D is the diameter of the pipe (see p 282)

$\frac{P}{D}$	$\frac{A}{D^2}$	$\frac{P}{D}$	$\frac{A}{D^2}$	$\frac{P}{D}$	$\frac{A}{D^2}$	$\frac{P}{D}$	$\frac{A}{D^2}$	$\frac{P}{D}$	$\frac{A}{D^2}$
0·1	0·00025	0·7	0·052	1·3	0·260	1·9	0·551	2·5	0·745
0·2	0·00137	0·8	0·075	1·4	0·307	2·0	0·594	2·6	0·760
0·3	0·0061	0·9	0·103	1·5	0·357	2·1	0·633	2·7	0·771
0·4	0·0105	1·0	0·136	1·6	0·406	2·2	0·669	2·8	0·779
0·5	0·020	1·1	0·174	1·7	0·456	2·3	0·699	2·9	0·782
0·6	0·034	1·2	0·203	1·8	0·505	2·4	0·725	3·0	0·785

TABLE XXIV
HYDRAULICS
Jets—Factors to find initial velocity.

$\frac{H}{d}$ *	Factor (J)
300	0·98
600	0·95
1000	0·92
1500	0·89
1800	0·84
2800	0·77
3500	0·71
4500	0·50

* H = head at nozzle, d = diameter of nozzle

NOTATION AND WORKING STRESSES.

TABLE XXV.

NOTATION.

The following is a list of the notation employed in this volume :—

General.

r_b, safe resistance to bearing in lbs., cwts., or tons per square inch.
r_c, " " compression " " "
r_s, " " shearing " " "
r_t, " " tension " " "
E, modulus of elasticity.
R_T or R_t, *total* safe resistance to tension.
R_c, " " compression.
R_s, " " shearing.
R_B, " " bearing.

Beams.

b, breadth in inches.
d, depth "
f_o, modulus of rupture.
I, moment of inertia.
L, span in feet.
l, " inches.
M, bending moment generally.
M_c, " " at centre of beam.
M_p, " " at any point P.
$\overline{M}$, moment of resistance generally.
$\overline{M}_P$, moment of resistance at point P.
R_A, reaction at abutment A.
R_B, " " B.
r_o, limiting stress in lbs., cwts., or tons per square inch on the outside fibres of a beam.
S_p, shearing stress at point P.
W, concentrated weight or load on a beam, also *total* distributed load.
w, distributed load per inch or per foot run.
y_c, distance of neutral axis from extreme fibre on compression side.
y_o, distance of neutral axis from extreme fibre on compression side, subject to stress r_o.
y_t, distance of neutral axis from extreme fibre on tension side.
Δ, maximum deflection in inches.

Tension and compression bars.

A, *effective* cross-sectional area.
d, "least diameter" of a long column.

Riveted joints.

b, breadth of the bars or plates to be jointed.
d, diameter of rivet holes.
k, number of rivets in the first row.

N, number of plates on each side to be connected
n_1, number of rivets in each end group
n_2, „ „ between the joints
n_3, number of rivet holes to be deducted from the cross section of cover plates.
T_b, total safe resistance to bearing
T_s, „ „ shearing
t, thickness of plates.
t_2, „ „ cover plates
α, number of rivets required for bearing
β, „ „ shearing

Plate girders.

A_c, effective area of *compression* flange
A_t, „ „ *tension* „
D, „ depth, i e distance between centres of gravity of flanges
t_w, thickness of web

Braced or framed girders.

$C_{4\,5}$, compression in the bar a_4,a_5 of the top boom
d, depth, i e distance between centres of gravity of flanges
$B_{4\,14}$, stress (tension or compression) in brace a_4,a_{14}
n, number of triangles
$T_{13\,14}$, tension in the bar a_{13},a_{14} of the lower boom

Roofs.

H_A, horizontal reaction at abutment A due to wind
h_A, „ „ A „ temperature
H_B, „ „ B „ wind
h_B, „ „ B „ temperature
P_H, horizontal wind pressure
P_N, normal „
P_V, vertical „
R_A, reaction at abutment A.
R_B, „ „ B
S_{AD}, stress (tension or compression) in member AD
V_A, vertical reaction at abutment A due to wind
V_B, „ „ B „

Masonry structures.

p, intensity of pressure at any point of a bed joint
$p_{(max)}$, greatest intensity of compression on a bed joint
r_0 *see* BEAMS

Retaining walls.

H, height of wall
K, constant depending on ϕ (angle of repose)
T, thickness of wall with vertical sides.
T_1, mean thickness
W, weight of a cubic foot of the material of the wall
ϕ, angle of repose of the earth to be retained
w, weight of a cubic foot of the earth to be retained

Arches.

H,	horizontal thrust.
P_1, P_2, etc.,	total pressure on joints between voussoirs.
V,	vertical component of pressures.
w_1, w_2, etc.,	loads on each voussoir.
W,	total load on an arch.

Hydraulics.

A,	area of cross section of flow.
D,	diameter of a pipe in feet.
d,	" " inches.
F,	discharge in cubic feet per second.
G,	discharge in gallons per minute.
H,	effective head of water in feet, *i.e.* the actual head reduced by the various losses.
H_B,	loss of head due to bends.
H_E,	" " elbows.
H_o,	" " orifice of entry.
H_v,	" " velocity.
J,	factor for jets.
L,	length of a pipe in feet.
R,	hydraulic mean depth.
S,	slope of a pipe.
V,	*mean* velocity of flow in feet per second.

TABLE XXVI.

WORKING STRESSES.

The undermentioned are the working stresses that have been used in this volume :—

Cast Iron.

Girders . .	Tension	$1\frac{1}{2}$ tons per square inch	(see p. 94).
	Compression	8 " "	(").
	Shearing	2·4 " "	(").

Wrought Iron.

Rolled beams . .	Tension	5 tons per square inch	(see p. 82).
	Compression	4 " "	(").
	Shearing	4 " "	(").
Built-up girders .	Tension	5 " "	(see p. 156).
	Compression	4 " "	(" 156).
Roofs . . .	Tension	5 " "	(" 108).
	Compression	(depends on ratio of length of struts to least diameter)	(" 332).
Tension bars and rods	Tension	5 tons per square inch	(" 106).
Riveted joints .	Shearing	4 " "	(" 123).
	Bearing	8 " "	(" ").

Pin joints	Shearing	4	tons per square inch	(see p. 140)
	Bearing	5	" "	(" 141)
Screws	Shearing	2	" "	(" 146)
Mild Steel.				
Rolled beams	Tension	6½	tons per square inch	(see p. 93)
	Compression	6½	" "	(" 93)
Timber.				
Fir (superior)	Tension	12	cwts. per square inch	(see p. 150)
	Compression	10	" "	(" ")
	Shearing	1·3	" "	(" ")
	Bearing	12	" "	(" ")
Fir (inferior)	Tension	10	" "	(" 152)
	Compression	7	" "	(" ")
	Shearing	1·3	" "	(" ")
	Bearing	7	" "	(" ")
Oak	Tension	16	" "	(" 328)
	Compression	13	" "	(" ")
	Shearing	5	" "	(" ")
	Bearing	25	" "	(" ")
Brickwork.				
In Mortar	Compression	0·5	cwts. per square inch	(see p. 223)
	Adhesion	0·06	" "	(" 227)
In Cement	Compression	0·8	" "	(" 259)
	Adhesion	0·1	" "	(" 328)

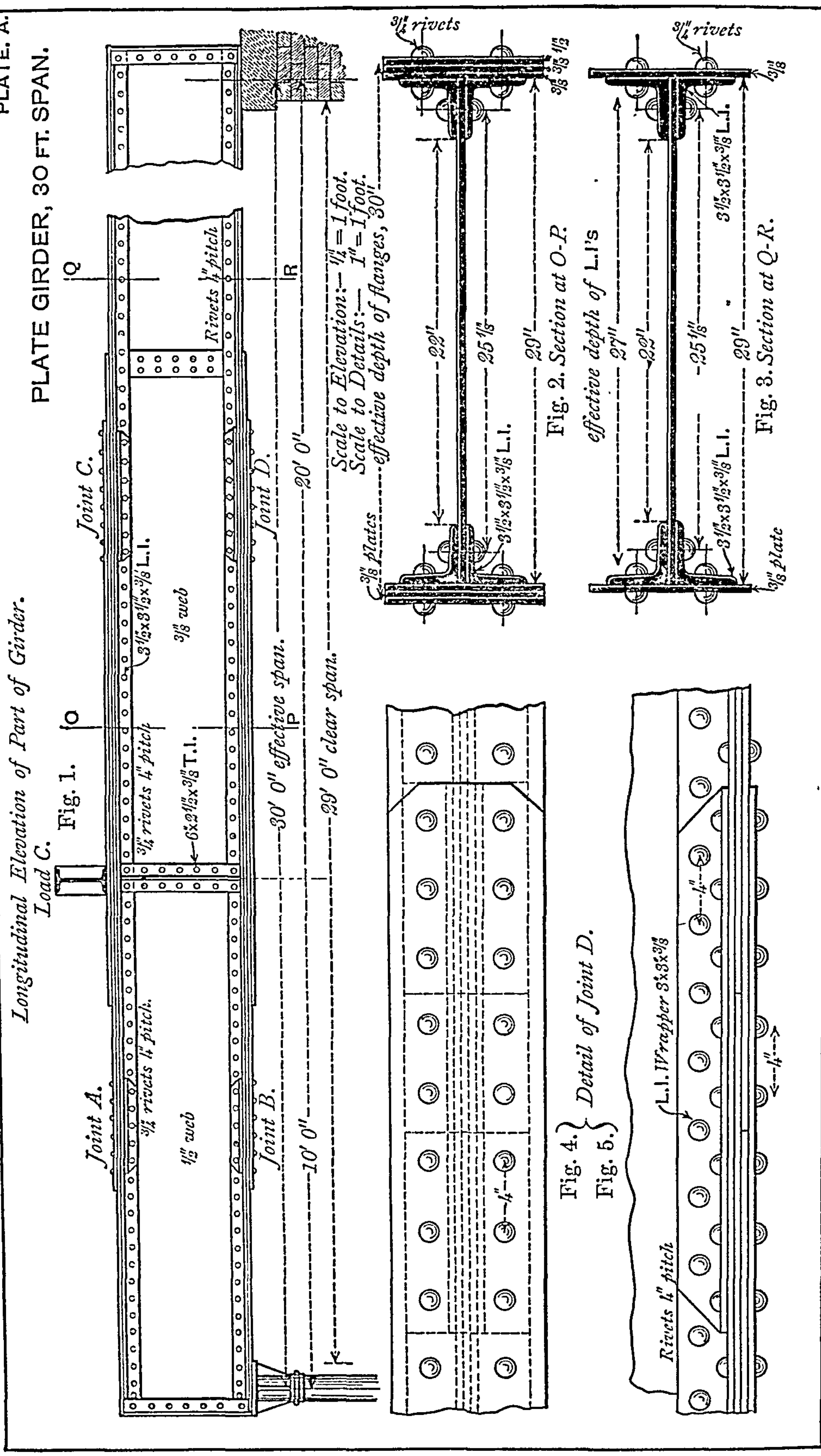


PLATE. A.
PLATE GIRDER, 30 FT. SPAN.
Longitudinal Elevation of Part of Girder.
Load C.
Fig. 1.
Joint A.
Joint B.
Joint C.
Joint D.
3/4" rivets 4" pitch.
1/2" web
3/8" web
3 1/2"x3 1/2"x3/8" L.I.
6"x2 1/2"x3/8" T.I.
Rivets 4" pitch
10' 0"
20' 0"
30' 0" effective span.
29' 0" clear span.
Scale to Elevation:— 1/4" = 1 foot.
Scale to Details:— 1" = 1 foot.
effective depth of flanges, 30"
O
P
Q
R
3/4" rivets
3/8" plates
22"
25 1/8"
29"
Fig. 2. Section at O-P.
effective depth of L.I's
27"
3/8" plate
Fig. 3. Section at Q-R.
Fig. 4. Fig. 5. Detail of Joint D.
4"
L.I. Wrapper 3 1/2"x3 1/2"x3/8"
Rivets 4" pitch

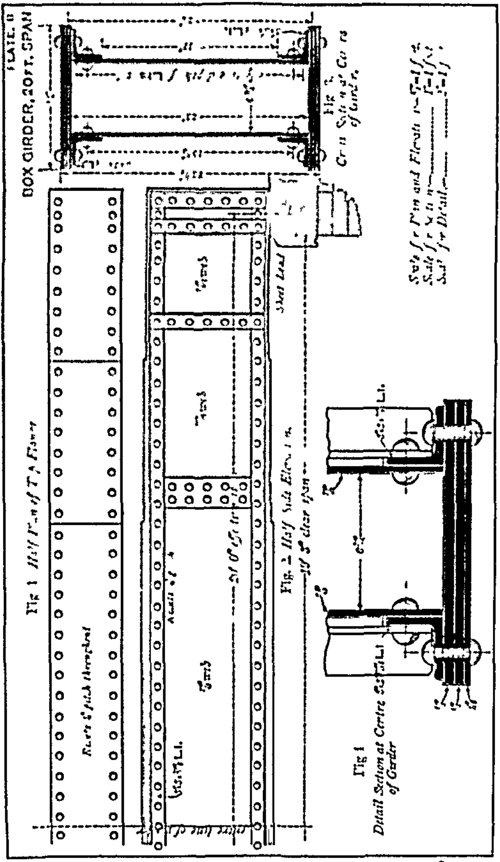
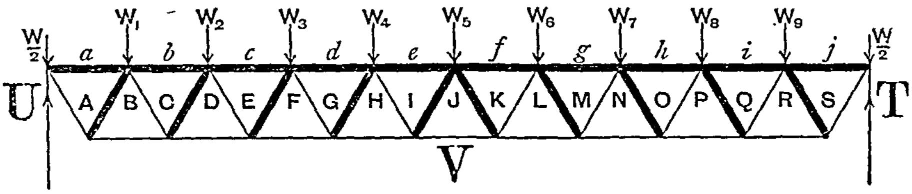


PLATE. II
BOX GIRDER, 20 FT. SPAN
Fig. 1 Half Plan of Top Flange
Fig. 2 Half Side Elevation
Fig. 4
Detail Section at Centre of Girder

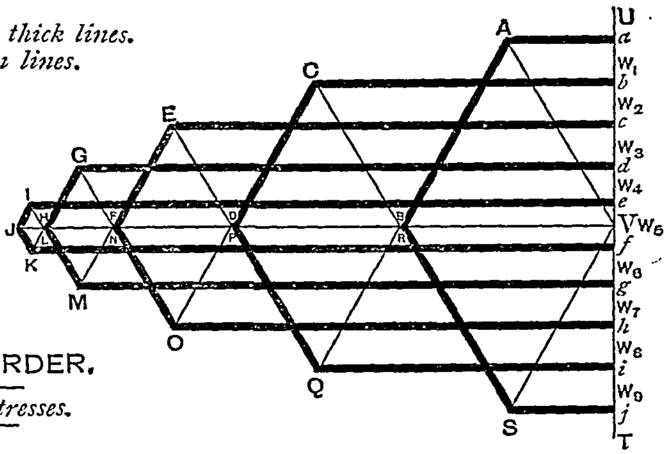
PLATE. C.

Fig. 1.



Compressions in thick lines.
Tensions in thin lines.

Fig. 2.



WARREN GIRDER.

Diagram of Stresses.

Scale, 12 tons = 1 inch.

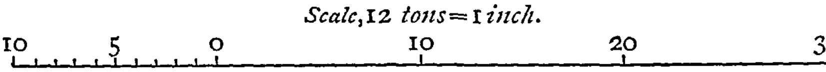
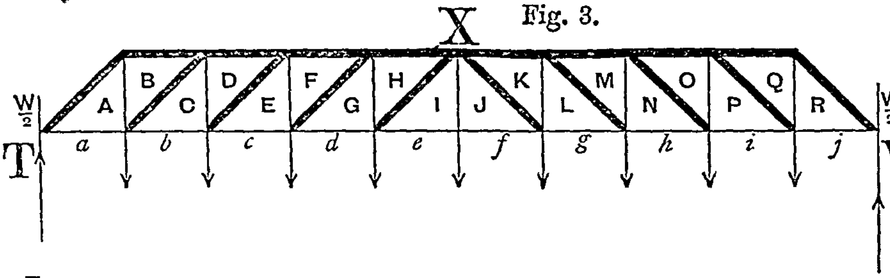


Fig. 3.



WHIPPLE MURPHY GIRDER.

Diagram of Stresses.

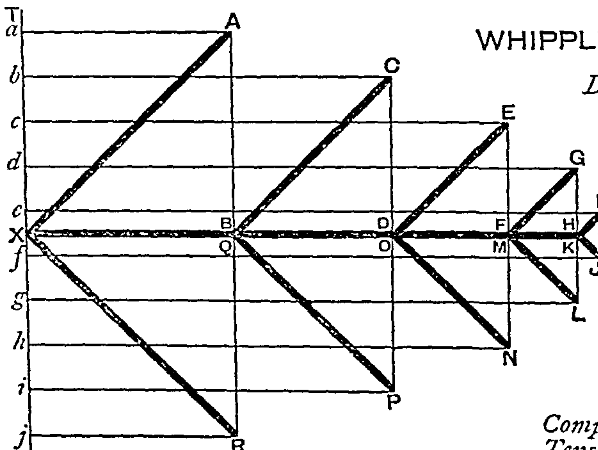
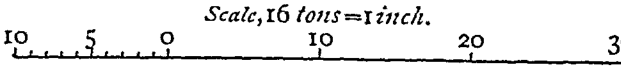


Fig. 4.

Compressions in thick lines.
Tensions in thin lines.

Scale, 16 tons = 1 inch.



S

END OF PART IV.

PLATE I

Compressions Red
Tensions Blue

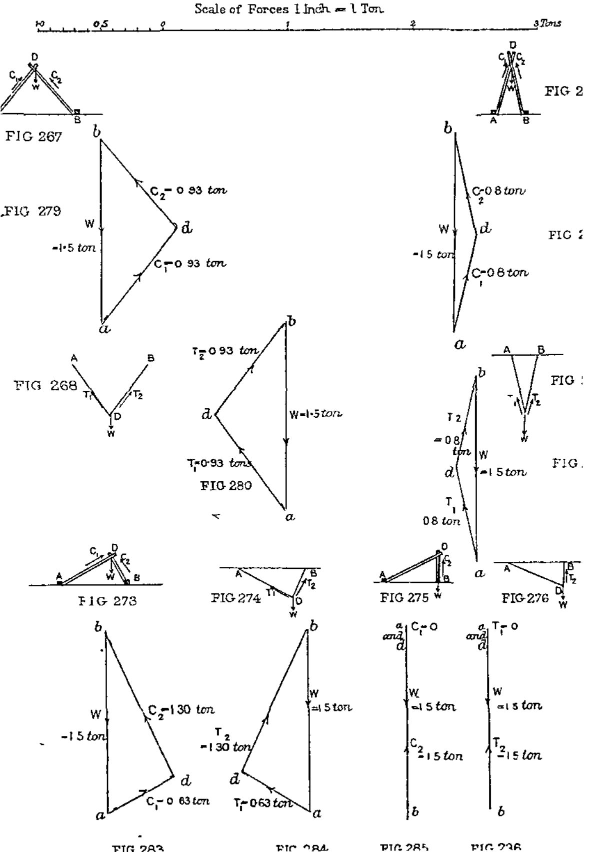


PLATE 2

Compressions Red
Tensions Blue

Scale of Forces 1 Inch = 3 Cwt

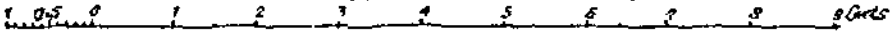
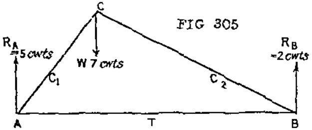


FIG 305



FRAME DIAGRAM
FIG 305

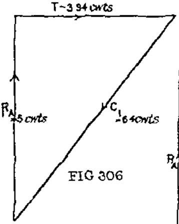
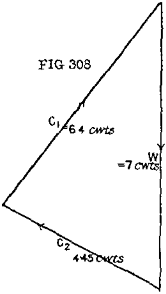
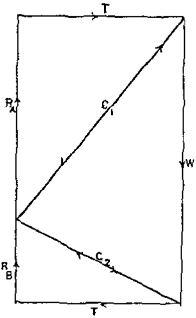


FIG 306

FIG 308





FORCE DIAGRAM
FOR FIG 305

FIG 309

FIG 307

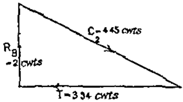
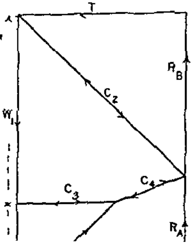
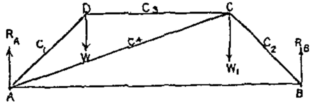


FIG 311



FORCE DIAGRAM
FOR FIG 310

FIG 310



FRAME DIAGRAM

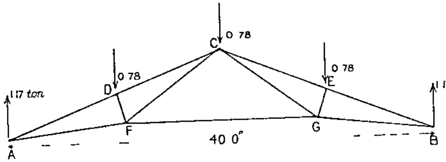


FIG 356

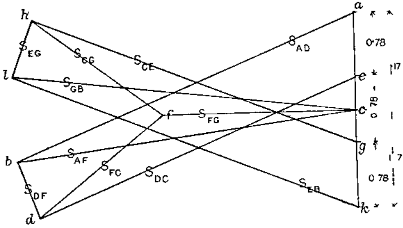


FIG 357

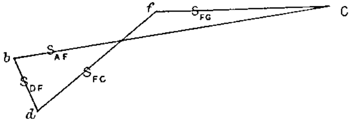


FIG 358

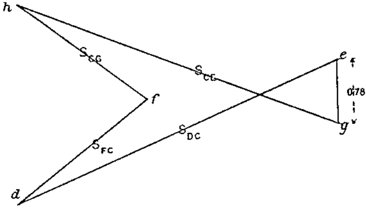


FIG 359

PLATE 4

Compressions Red
Tensions Blue

Scale of Forces 1" = 1 Ton

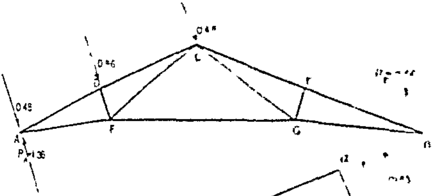


FIG 361

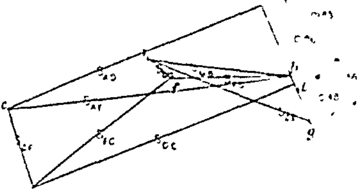


FIG 362

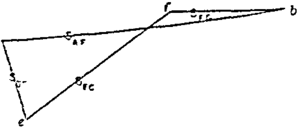


FIG 363

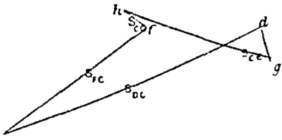
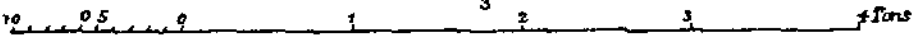


FIG 364

PLATE 5

Compressions Red
Tensions Blue

Scale of Forces $\frac{2}{3}$ Inch = 1 Ton.



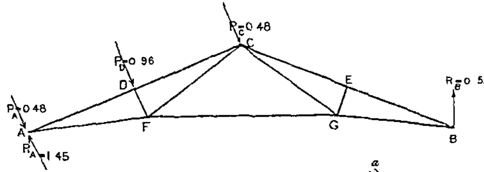


FIG 368

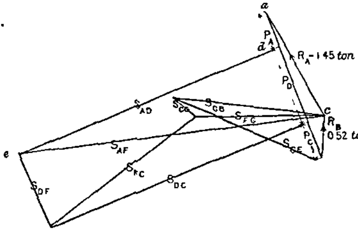


FIG 369

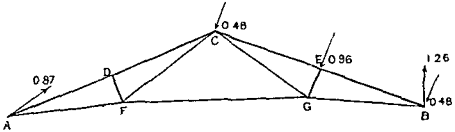


FIG 371

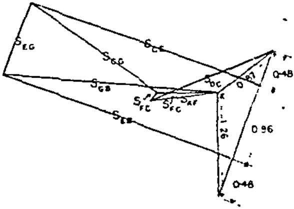


FIG 372

Compressions Red
Tensions Blue

FIG 376

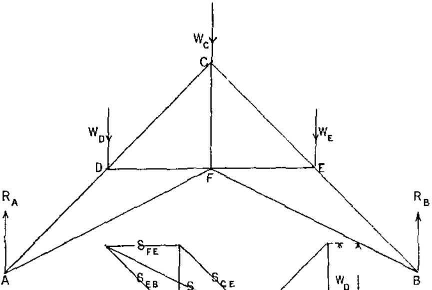


FIG 377

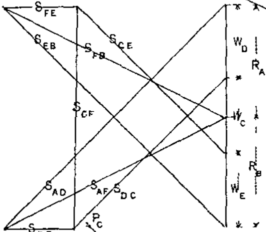


FIG 378

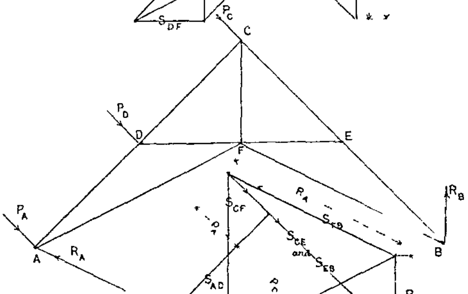


FIG 379

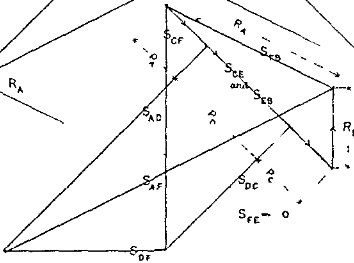


PLATE 7

Compressions Red
Tensions Blue

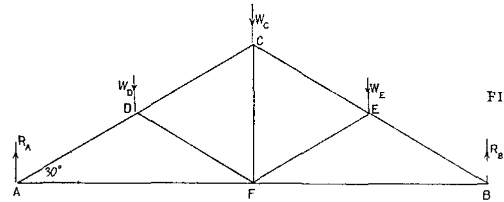


FIG 380

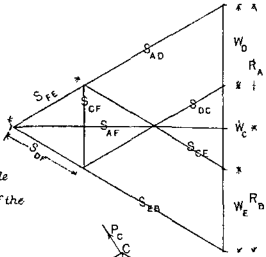


FIG 381

Note *The direction of S_{FE} and S_{AD} and of S_{DF} and S_{EB} coincide because the pitch of the roof is 30°*

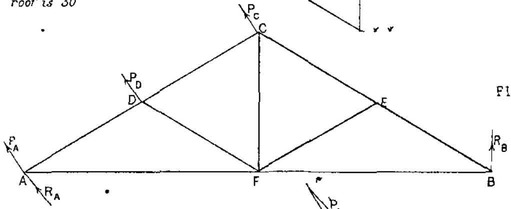


FIG 382

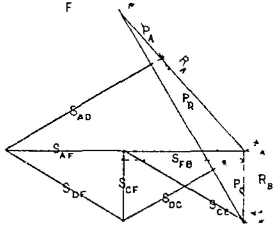


FIG 383

$S_{FE} = 0$ $\quad$ $S_{EB} = S_{CE}$

Compressions Red
Tensions Blue

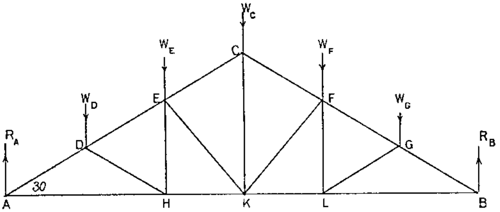
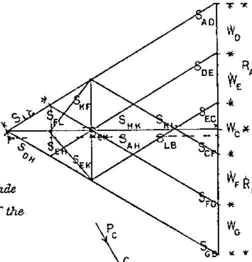


FIG 384



Note *The direction of* S_{GB} *and* S_{DH} *and of* S_{AD} *and* S_G *coincide because the pitch of the roof is 30*

FIG 385

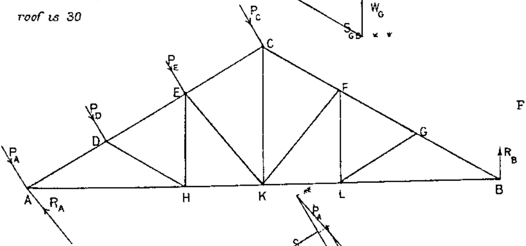


FIG 386

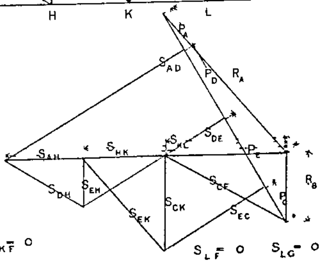


FIG 387

$S_{CF} = S_{FG} = S_{GB}$

$S_{LB} = S_{KL}$ $S_{KF} = 0$

$S_{LF} = 0$ $S_{LG} = 0$

PLATE 9

Compressions Red
Tensions Blue

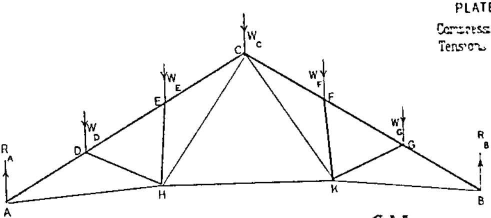


FIG 398

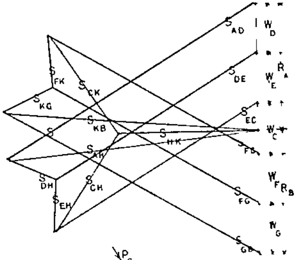


FIG 399

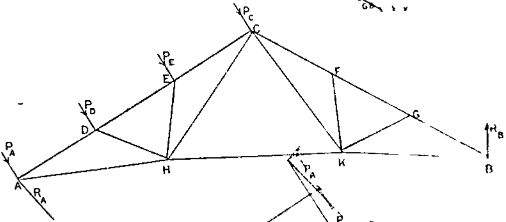


FIG 400

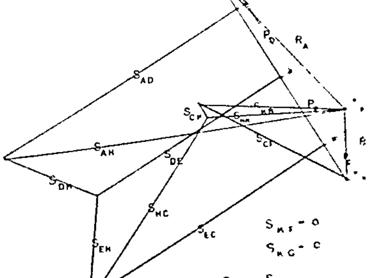


FIG 401

$S_{KF} = 0$

$S_{KG} = 0$

PLATE 10

Compressions Red
Tensions Blue

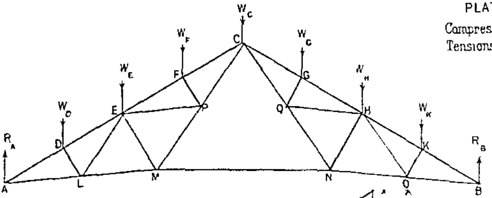


FIG 392

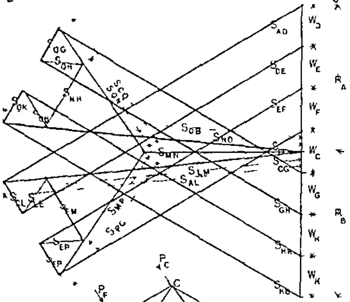


FIG 393

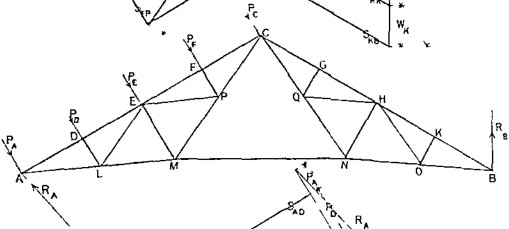


FIG 394

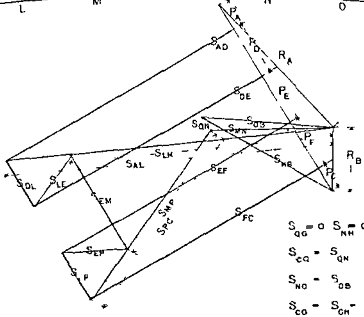


FIG 395

$S_{QG} = 0$ $S_{NH} = 0$ $S_{OK} = 0$

$S_{CQ} = S_{QN}$

$S_{NO} = S_{OB}$

$S_{CG} = S_{GH} = S_{HK} = S_{KB}$